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Full Length Research Paper

Eigen spectra for Manning-Rosen potential including a Coulomb-like tensor interaction

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We have obtained analytically the approximate energy equation and the corresponding wave functions of the Dirac equation for the Manning-Rosen potential coupled with a Coulomb-like tensor under the condition of the pseudo-spin symmetry using the parametric generalization of the Nikiforov-Uvarov method. The special cases of the Manning-Rosen potential for spin-1/2 particle were briefly discussed. Numerical results calculated for per special cases.

Key words: Dirac equation, Manning-Rosen potential, Coulomb-like tensor.

INTRODUCTION

It is well known, that the exact solutions of the Dirac equation for some physical potentials are important, since it contains all the necessary information for the quantum system under consideration. For example, some authors have solved approximately the Dirac equation for some potentials like the Eckart potential (Zou et al., 2005), Resen-Morse potential (Qiang et al., 2007), Woods-Saxon potential (Xu and Zhu, 2006), Scarf potential (Motavali, 2009), etc. The spin and pseudo-spin symmetry concepts in nuclear theory (Arima et al., 1969; Hecht and Adler, 1969) have been used to explain the features of deformed nuclei (Bohr et al., 1982), super-deformation (Dudek et al. 1987), and also, to establish an effective nuclear shell-model scheme (Troltenier et al., 1995). Ginocchio (1997, 1999), showed that pseudo-spin symmetry is exact when the sum of the vector potential $V_v(r)$ and scalar potential $V_s(r)$ is equal to zero or a constant, then pseudospin symmetry occurs in the Dirac equation. Of course, in real nuclei, $V_v(r)+V_s(r)\neq con.$ pseudo-spin symmetry is only an approximation. As an important physical potential, the Manning-Rosen potential (Wei and Dong, 2010, 2008; Manning and Rosen, 1933) can be used to describe molecular vibration with the form:

$$V(r)=\frac{V_1}{(e^{\alpha r}-q)^2}+\frac{V_2}{e^{\alpha r}-q}, \qquad (1)$$

where $V_1=[\alpha'(\alpha'-1)]\hbar^2/2M\beta^2$, $V_2=-A\hbar^2/2M\beta^2$ (α' and A are two dimensionless parameters) and $\alpha=1/\beta$ (parameter β is related to the range of the potential).

Recently, Wei and Dong (2010, 2008) studied the pseudo-spin and spin symmetries solutions of the Dirac equation with the Manning-Rosen Potential. Chen et al. (2009) investigated the solutions of the Dirac-Manning-Rosen problem with the spin and pseudo-spin symmetries. The s-wave bound state energy eigenvalues and the corresponding s-wave scattering solutions have been obtained by function analysis method (Chen et al., 2007). Tensor potentials have been introduced into the Dirac equation with the substitution. In this way, a spin-orbit coupling term was added to the Dirac Hamiltonian (Hamzavi et al., 2010a; Ackay and Tezcan, 2009; Hamzavi et al., 2010b; Zarrinkamar et al., 2010; Akcay, 2009; Aydogdu and Sever, 2010; Hamzavi et al., 2010c; Ikdair et al., 2010).

The motivation of the present work was to solve the Dirac equation under the pseudo-spin for Manning-Rosen potential including a Coulomb-like tensor potential (Hamzavi et al., 2010a),

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$$U(r) = -\frac{H}{r}, \quad H = \frac{Z_a Z_b e^2}{4\pi\varepsilon_0}, \quad r \ge R_c \,, \tag{2}$$

where R_c =7.78 fm is the Coulomb radius, Z_a and Z_b denote the charges of the projectile *a* and the target nuclei *b*, respectively.

The parametric generalization of the Nikiforov-Uvarov (NU) method has been used to solve the Dirac equation with this potential. We have also considered the special cases of this potential in the Dirac equation and the energy eigenvalues equation and the corresponding unnormalized eigenfunctins which include the number physical potentials which have been obtained.

NU METHOD

We give a brief description of the conventional NU method (Nikiforov and Uvarov, 1988). This method is based on solving the second order differential equation of hypergeometric-type by means of special orthogonal functions:

$$\psi_n''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\psi_n'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi_n(s) = 0 \,, \tag{3}$$

where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials, at the most of the second degree, and $\tilde{\tau}(s)$ is a polynomials, at most of the first degree. If we take the following factorization $\psi_n(s) = \phi(s)\, y_n(s)$, Equation 3 becomes:

$$\sigma(s) y_n''(s) + \tau(s) y_n'(s) + \lambda y_n(s) = 0 \,, \tag{4}$$

where

$$\sigma(s) = \pi(s)\frac{d}{ds}(\ln \phi(s)) \,, \tag{5}$$

$$\tau(s) = \tilde{\tau}(s) + 2\pi(s), \qquad \tau'(s) < 0 \,, \tag{6}$$

where $\pi(r)$ is a polynomial of at most one order.

The $y_n(s)$ which is a polynomial of degree can be expressed in terms of the Rodrigues relation:

$$y_n(s) = \frac{a_n}{\rho(s)}\frac{d^n}{ds^n}\left[\sigma^n(s)\rho(s)\right] , \tag{7}$$

where a_n is a normalization constant, and the weight function $\rho(s)$ must satisfy the differential equation:

$$\omega'(s) - \left(\frac{\tau(s)}{\sigma(s)}\right)\omega(s) = 0, \quad \omega(s) = \sigma(s)\rho(s) \,. \tag{8}$$

The function $\pi(s)$ and the parameter λ in the earlier equations are defined as follows:

$$\pi(s) = \frac{\sigma'(s) - \tilde{\tau}(s)}{2} \pm \sqrt{\left(\frac{\sigma'(s) - \tilde{\tau}(s)}{2}\right)^2 - \tilde{\sigma}(s) + q\sigma(s)} \,, \tag{9}$$

$$\lambda = q + \pi'(s) \,. \tag{10}$$

The determination of *q* is the essential point in the calculation of $\pi(s)$. It is simply defined by setting the discriminate of the square root which must be zero. The eigenvalues equation has been calculated from the aforementioned equations as:

$$\lambda = \lambda_n = -n\tau'(s) - \frac{n(n-1)}{2}\sigma''(s). \qquad n = 0,1,2,\ldots \tag{11}$$

For a more simple application of the method, we develop a parametric generalization of the NU method valid for any potential under consideration by an appropriate coordinate transformation $s = s(r)$. Thus, we obtain another generalized hypergeometric equation (Hamzavi et al., 2010b):

$$\left[s^2(1-\alpha_3 s)^2 \frac{d^2}{ds^2} + s(1-\alpha_3 s)(\alpha_1 - \alpha_2 s)\frac{d}{ds} + [-\xi_1 s^2 + \xi_2 s - \xi_3] \right]\psi_n(s) = 0 \,. \tag{12}$$

We may solve this as follows. Comparing Equation 12 with 3, yields:

$$\tilde{\tau}(s) = \alpha_1 - \alpha_2 s \,, \ \sigma(s) = s(1-\alpha_3 s) \,,$$
$$\tilde{\sigma}(S) = -\xi_1 s^2 + \xi_2 s - \xi_3 \,. \tag{13}$$

Substituting these into Equation 13 into 9, we find:

$$\pi(s) = \alpha_4 + \alpha_5 s \pm \left[(\alpha_6 - k\alpha_3)s^2 + (\alpha_7 + k)s + \alpha_8\right]^{\frac{1}{2}} \tag{14}$$

with the following parameters

$$\alpha_4 = \frac{1}{2}(1-\alpha_1) \,, \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3) \,, \qquad \alpha_6 = \alpha_5^2 + \xi_1 \,,$$
$$\alpha_7 = 2\alpha_4\alpha_5 - \xi_2 \,, \ \alpha_8 = \alpha_4^2 + \xi_3 \,. \tag{15}$$

We obtain the parameter *k* from the condition that the function under the square root should be the square of a polynomial:

$$k_{1,2} = -(\alpha_7 + 2\alpha_3\alpha_8) \pm 2\sqrt{\alpha_8\alpha_9} \,, \tag{16}$$

where

$$\alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6 \,. \tag{17}$$

For each k the following π's are obtained. The function $\pi(s)$ becomes:

$$\pi(s)=\alpha_4+\alpha_5 s-[(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})s-\sqrt{\alpha_8}], \qquad (18)$$

for the k-value

$$k=-(\alpha_7+2\alpha_3\alpha_8)-2\sqrt{\alpha_8\alpha_9}. \qquad (19)$$

We also have from $\tau(s)=\tilde{\tau}(s)+2\pi(s)$:

$$\tau(s)=\alpha_1+2\alpha_4-(\alpha_2-2\alpha_5)s$$
$$-2[(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})s-\sqrt{\alpha_8}]. \qquad (20)$$

Thus, we impose the following condition to fix the k-value:

$$\tau'(s)=-(\alpha_2-2\alpha_5)-2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})$$
$$=-2\alpha_3-2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})<0. \qquad (21)$$

When Equation 10 is used with Equations 20 and 21, the following equation is derived:

$$n[(n-1)\alpha_3+\alpha_2-2\alpha_5]+(2n+1)(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})$$
$$-\alpha_5+\alpha_7+2\alpha_3\alpha_8+2\sqrt{\alpha_8\alpha_9}=0. \qquad (22)$$

By using Equation 8, we have:

$$\rho(s)=s^{\alpha_{10}-1}(1-\alpha_3 s)^{\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1}, \qquad (23)$$

and together with Equation 7, we have:

$$y_n(s)=P_n^{\left(\alpha_{10}-1,\ \frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s), \qquad (24)$$

where

$$\alpha_{10}=\alpha_1+2\alpha_4+2\sqrt{\alpha_8}, \qquad (25)$$

and

$$\alpha_{11}=\alpha_2-2\alpha_5+2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}), \qquad (26)$$

and $P_n^{(\alpha,\beta)}$ are Jacobi polynomials. By using Equation 5, we get:

$$\phi(s)=s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}}, \qquad (27)$$

and the total wave function become:

$$\Psi(s)=s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}}P_n^{\left(\alpha_{10}-1,\ \frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1\right)}(1-2\alpha_3 s), \qquad (28)$$

where $\alpha_{12}=\alpha_4+\sqrt{\alpha_8}$ and $\alpha_{13}=\alpha_5-(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8})$.

In some problems, the situation appears where $\alpha_3=0$. For such problems, the solution given in Equation 28 becomes:

$$\Psi(s)=s^{\alpha_{12}}e^{\alpha_{13}s}L_n^{\alpha_{10}-1}(\alpha_{11}s). \qquad (29)$$

In some cases, one may need a second solution of equation 12. In this case, if the same procedure is followed, by using:

$$k=-(\alpha_7+2\alpha_3\alpha_8)+2\sqrt{\alpha_8\alpha_9}, \qquad (30)$$

the solution becomes:

$$\Psi(s)=s^{\alpha_{12}^*}(1-\alpha_3 s)^{-\alpha_{12}^*-\frac{\alpha_{13}^*}{\alpha_3}}$$
$$\times P_n^{\left(\alpha_{10}-1,\ \frac{\alpha_{11}^*}{\alpha_3}-\alpha_{10}^*-1\right)}(1-2\alpha_3 s), \qquad (31)$$

and the energy spectrum is:

$$n[(n-1)\alpha_3+\alpha_2-2\alpha_5]+(2n+1)(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8})$$
$$+\alpha_7+2\alpha_3\alpha_8-2\sqrt{\alpha_8\alpha_9}+\alpha_5=0. \qquad (32)$$

Pre-defined α parameters are:

$$\alpha_{10}^*=\alpha_1+2\alpha_4-2\sqrt{\alpha_8},$$
$$\alpha_{11}^*=\alpha_2-2\alpha_5+2(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8}),$$
$$\alpha_{12}^*=\alpha_4-\sqrt{\alpha_8},$$
$$\alpha_{13}^*=\alpha_5-(\sqrt{\alpha_9}-\alpha_3\sqrt{\alpha_8}). \qquad (33)$$

SOLUTION OF THE DIRAC EQUATION

According to the report which have been given in researches (Hamzavi et al., 2010a, 2010b; Ackay et al., 2009; Zarrinkamar et al., 2010; Akcay and Tezcan, 2009), the Dirac equation of a nucleon with mass M moving in a scalar and a vector potential including tensor interaction for spin-1/2 particles can be written as ($\hbar=c=1$):

$$[\vec{\alpha}\cdot\vec{P}+\beta(M+V_s(r))-i\beta\vec{\alpha}\cdot\hat{r}U(r)]\psi_{nk}(\vec{r})$$
$$=[E-V_v(r)]\psi_{nk}(\vec{r}), \qquad (34)$$

where α and β are the 4 × 4 matrices, E is the relativistic energy of the system, $\vec{P} = -i\vec{\nabla}$ is the three-dimensional momentum operator. For spherical nuclei, the nucleon angular momentum J and $\hat{K} = -\beta(\hat{\sigma}.\hat{L}+1)$ commute with the Dirac Hamiltonian, where $\hat{\sigma}$ and $\hat{L}$ are the Pauli matrix and orbital angular momentum, respectively.

The Dirac spinors wave functions can be classified according to their angular momentum j and k,

$$\psi_{nk}(\vec{r}) = \frac{1}{r}\begin{bmatrix} F_{nk}(r) Y^{l}_{jm}(\theta,\varphi) \\ iG_{nk}(r) Y^{\tilde{l}}_{jm}(\theta,\varphi) \end{bmatrix}, \tag{35}$$

where n is the radial quantum number, and m is the projection of the total angular momentum on the z-axis. The eigenvalues of $\hat{K}$ are $k = \pm(j+(1/2))$ with ' - ' for aligned spin ($s_{1/2}$, $p_{3/2}$, etc.) and '+' for unaligned spin ($p_{1/2}$, $d_{3/2}$, etc).

Substituting Equation 35 into Equation 34 and using the following relations (Bjorken and Drell, 1964):

$$(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = \vec{A}.\vec{B} + i\vec{\sigma}.(\vec{A}\times\vec{B}), \tag{36}$$

$$(\vec{\sigma}.\vec{P}) = \vec{\sigma}.\hat{r}(\hat{r}.\vec{P} + i\frac{\vec{\sigma}.\vec{L}}{r}), \tag{37}$$

and properties:

$$(\vec{\sigma}.\vec{L})Y^{\tilde{l}}_{jm}(\theta,\phi) = (k-1)Y^{\tilde{l}}_{jm}(\theta,\phi), \tag{38}$$

$$(\vec{\sigma}.\vec{L})Y^{l}_{jm}(\theta,\phi) = -(k-1)Y^{l}_{jm}(\theta,\phi), \tag{39}$$

$$(\vec{\sigma}.\hat{r})Y^{\tilde{l}}_{jm}(\theta,\phi) = -Y^{l}_{jm}(\theta,\phi), \tag{40}$$

$$(\vec{\sigma}.\hat{r})Y^{l}_{jm}(\theta,\phi) = -Y^{\tilde{l}}_{jm}(\theta,\phi), \tag{41}$$

yields two coupled differential equations as follows:

$$\left(\frac{d}{dr} + \frac{k}{r} - U(r)\right)F_{nk}(r) = \left[E_{nk} + M - \Delta(r)\right]G_{nk}(r), \tag{42}$$

$$\left(\frac{d}{dr} - \frac{k}{r} + U(r)\right)G_{nk}(r) = \left[M - E_{nk} + \Sigma(r)\right]F_{nk}(r), \tag{43}$$

where Δ and Σ have been assumed to be radial functions, that is, $\Delta(r) = V_v(r) - V_s(r)$ and $\Sigma(r) = V_v(r) + V_s(r)$.

By substituting $G_{nk}(r)$ from Equation 42 into Equation 43 and $F_{nk}(r)$ from Equation 43 into Equation 42, we have obtained the following two second-order differential equations for the upper and lower components:

$$\Bigg\{ \frac{d^2}{dr^2} - \frac{k(k+1)}{r^2} + \frac{2k}{r}U(r) - \frac{dU(r)}{dr} - U^2(r) + (E_{nk} + M - \Delta(r))(E_{nk} - M - \Sigma(r)) + \frac{-\frac{d\Delta(r)}{dr}}{\left(M + E_{nk} - \Delta(r)\right)}\left(\frac{d}{dr} + \frac{k}{r}\right) \Bigg\} F_{nk}(r) = 0, \tag{44}$$

$$\Bigg\{ \frac{d^2}{dr^2} - \frac{k(k-1)}{r^2} + \frac{2k}{r}U(r) + \frac{dU(r)}{dr} - U^2(r) + (E_{nk} + M - \Delta(r))(E_{nk} - M - \Sigma(r)) + \frac{\frac{d\Sigma(r)}{dr}}{\left(M - E_{nk} + \Sigma(r)\right)}\left(\frac{d}{dr} - \frac{k}{r}\right) \Bigg\} G_{nk}(r) = 0. \tag{45}$$

In the aforementioned equations $k(k+1) = l(l+1)$ and $k(k-1) = \tilde{l}(\tilde{l}+1)$.

Substituting Equations 1 and 2 into 45, considering pseudo-spin symmetry, taking $\Delta(r)$ as the Manning-Rosen potential and $\Sigma(r) = C_{ps} = const.$ $(d\Sigma(r)/dr = 0)$, (Meng et al., 1999, 1998), that is, the equation obtained for the upper component of the Dirac spinor $F_{nk}(r)$ becomes:

$$\Bigg\{ \frac{d^2}{dr^2} - \frac{(k+H)(k+H-1)}{r^2} + (M + E_{nk})(M + E_{nk} - C_{ps}) - (E_{nk} - M - C_{ps})\left[\frac{V_1}{(e^{\alpha r} - q)^2} + \frac{V_2}{e^{\alpha r} - q}\right] \Bigg\} G_{nk}(r) = 0. \tag{46}$$

This equation describes a particle of spin-1/2, such as the electron in the Dirac theory with Manning-Rosen potential including a tensor coupling that cannot be solved analytically; because of $(k+H)(k+H-1)/r^2$ term, we take the following approximation (Wei and Dong, 2010; Ikhdiar et al., 2010):

$$\frac{1}{r^2}\approx\frac{1}{r_0^2}\left[C_0+\frac{C_1}{e^{\alpha r}-q}+\frac{C_2}{\left(e^{\alpha r}-q\right)^2}\right], \qquad (47)$$

where C_0, C_1 and C_2 are real constant, $r_0=\beta\gamma$, where β and γ are also constant (Wei et al., 2010). By using a transformation of the form $s=e^{\alpha r}$, we rewrite it as follows:

$$\left\{\frac{d^2}{ds^2}+\frac{s-q}{s(s-q)}\frac{d}{ds}+\frac{1}{\left[s(s-q)\right]^2}\left[\xi_1 s^2+\xi_2 s+\xi_3\right]\right\}F_{nk}(s)=0, \qquad (48)$$

where

$$\xi_1=\frac{C_0\beta^2\left(k+H\right)\left(k+H-1\right)}{r_0^2}-\left(E_{nk}-M-C_{ps}\right)\left(E_{nk}+M\right)\beta^2,$$

$$\xi_2=\frac{\beta^2\left(k+H\right)\left(k+H-1\right)(2qC_0-C_1)}{r_0^2}-\left(E_{nk}-M-C_{ps}\right)\left(2q\left(E_{nk}+M\right)+V_2\right)\beta^2,$$

$$\xi_3=\frac{\beta^2\left(k+H\right)\left(k+H-1\right)(q^2C_0+C_1q+C_2)}{r_0^2}+\beta^2\left(E_{nk}-M-C_{ps}\right)\left(-\left(E_{nk}+M\right)q^2+V_1-V_2q\right) \qquad (49)$$

By comparing Equation 48 with Equation 12, we obtained the parameter set as:

$$\alpha_1=-q,\ \alpha_2=1,\ \alpha_3=1,\ \alpha_4=(1+q)/2,\ \alpha_5=-1/2,$$
$$\alpha_6=1/4+\xi_1,$$
$$\alpha_7=-(1+q)/2-\xi_2,\alpha_8=(1+q)^2/4+\xi_3,$$
$$\alpha_9=\xi_1-\xi_2+\xi_3+q^2/4,\alpha_{10}=1+2\sqrt{(1+q)^2/4+\xi_3},$$
$$\alpha_{11}=2+2\left(\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}+\sqrt{(1+q)^2/4+\xi_3}\right),$$
$$\alpha_{12}=(1+q)/2+\sqrt{(1+q)^2/4+\xi_3},$$
$$\alpha_{13}=-\frac{1}{2}-\left(\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}+\sqrt{(1+q)^2/4+\xi_3}\right). \qquad (50)$$

Using Equations 14, 16 and 50, we calculate the parameters required for the method as:

$$\pi(s)=\frac{1+q-s}{2}\pm\left[\left(\frac{1}{4}+\xi_1-k\right)s^2+\left(-\frac{1+q}{2}-\xi_2+k\right)s+\frac{(1+q)^2}{4}+\xi_3\right]^{\frac{1}{2}}, \qquad (51)$$

Where

$$k_{1,2}=\left[\frac{-q-q^2}{2}+2\xi_3-\xi_2\right]\pm2\left[\left(\frac{(1+q)^2}{4}+\xi_3\right)\left(\xi_1-\xi_2+\xi_3+\frac{q^2}{4}\right)\right]^{\frac{1}{2}}. \qquad (52)$$

Different *k's* lead to the different π *'s*. For

$$k=\left[\frac{-q-q^2}{2}+2\xi_3-\xi_2\right]-2\left[\left(\frac{(1+q)^2}{4}+\xi_3\right)\left(\xi_1-\xi_2+\xi_3+\frac{q^2}{4}\right)\right]^{\frac{1}{2}}, \qquad (53)$$

$\pi(s)$ becomes:

$$\pi(s)=\frac{1+q-s}{2}-\left(\frac{q^2}{4}+\xi_1-\xi_2+\xi_3\right)^{\frac{1}{2}}s-\left(\frac{(1+q)^2}{4}+\xi_3\right)^{\frac{1}{2}}s+\left(\frac{(1+q)^2}{4}+\xi_3\right)^{\frac{1}{2}}, \qquad (54)$$

and by using Equation 20, we obtain:

$$\tau(s)=1-2s-2\left(\frac{q^2}{4}+\xi_1-\xi_2+\xi_3\right)^{\frac{1}{2}}s-2\left(\frac{(1+q)^2}{4}+\xi_3\right)^{\frac{1}{2}}s$$

Table 1. The bound state energy eigenvalues $E_{n,k}$ in unit of fm^{-1} of the pseudo-spin symmetry Manning-Rosen potential for several values of *n* and *k* with $H=5$ for $q=1$.

$\tilde{l}$	$n, k<0$	l, j	$E_{n,k<0}(H=0)$ (Wei et al., 2010).	$E_{n,k<0}(H=0)$ Present work	$E_{n,k<0}(H=5)$ Present work	$n-1, k>0$	$l+2, j+1$	$E_{n-1,k>0}(H=0)$ Present work
1	1, -1	$2s_{1/2}$	-4.99866/-1.03238	-4.99266/-0.99343	-4.97279/-1.01382	0, 2	$4d_{3/2}$	-4.99266/-0.99343
2	1, -2	$3p_{3/2}$	-4.99772/-1.01494	-4.98425/-1.00218	-4.98425/-1.00218	0, 3	$5f_{5/2}$	-4.98425/-1.00218
3	1, -3	$4d_{5/2}$	-4.99656/-1.00561	-4.97279/-1.01382	-4.99266/-0.99343	0, 4	$6g_{7/2}$	-4.97279/-1.01382
4	1, -4	$5f_{7/2}$	-4.99517/-1.00080	-4.95822/-1.02847	-4.99834/-0.98760	0, 5	$7h_{9/2}$	-4.95822/-1.02847

The parameters $C_{ps}=-6$, $A=30.52$, $\alpha'=1.5$, $M=1$, $\beta=20$, $C_0=0.000208178$, $C_1=0.002500011408$, $C_2=0.002499999716$, $V_1=0.0009375$ and $V_2=-0.03815$ and $r_0=\beta.\log[1+2\alpha'(\alpha'-1)/A]$ are taken (Wei et al., 2010).

$$+2\left(\frac{(1+q)^2}{4}+\xi_3\right)^{\frac{1}{2}}, \qquad (55)$$

where $\tau'(s)<0$. Therefore, using Equations 22 and 50, we write the eigenvalues equation as:

$$\begin{aligned} &n[(n-1)+2] \\ &+(2n+1)\left(\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}+\sqrt{(1+q)^2/4+\xi_3}\right) \\ &+1/2-(1+q)/2-\xi_2+2\left((1+q)^2/4+\xi_3\right) \\ &+2\sqrt{\left(\xi_1-\xi_2+\xi_3+q^2/4\right)\left((1+q)^2/4+\xi_3\right)}=0. \end{aligned} \qquad (56)$$

Numerical results have calculated for this case in Table 1.

Now, let us give the corresponding upper Dirac spinor. Using Equations 23, 24 and 27, we write the corresponding unnormalized eigenfunctions obtained in terms of the function ns:

$$\rho(s)=s^{2\sqrt{(1+q)^2/4+\xi_3}}(1+s)^{2\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}}, \qquad (57)$$

and

$$y_n(s)=P_n^{\left(2\sqrt{(1+q)^2/4+\xi_3}\,,\;2\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}\right)}(1+2s), \qquad (58)$$

and

$$\varphi(s)=s^{\frac{1+q}{2}+\sqrt{(1+q)^2/4+\xi_3}}(1+s)^{-\frac{q}{2}+\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}}, \qquad (59)$$

and using Equation 28, the corresponding wave functions will be:

$$G_{nk}(s)=B_{nk}s^{\frac{(1+q)}{2}+\sqrt{(1+q)^2/4+\xi_3}}(1+s)^{-\frac{q}{2}+\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}} \times P_n^{\left(2\sqrt{(1+q)^2/4+\xi_3}\,,\;2\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}\right)}(1+2s). \qquad (60)$$

where B_{nk} is the normalization factor to be determined from the normalization condition

$$\int_0^\infty |G_{nk}(r)|^2 dr=1=b\int_0^1 s^{-1}|G_{nk}(s)|^2 ds, \qquad (61)$$

This can further be written as:

$$1=bB_{nk}^2\int_0^1 s^{2b'-1}(1-s)^{2b''+2}\left[P_n^{(2b',2b''+1)}(1-2s)\right]^2 ds. \qquad (62)$$

From which we obtain (Ikhdiar et al., 2008):

$$B_{nk}=\frac{1}{\sqrt{z(n)}}, \qquad (63)$$

$$\begin{aligned} z(n)&=b(-1)^n\frac{\Gamma(n+2b''+2)\Gamma(n+2b'+1)^2}{\Gamma(n+2b'+2b''+2)} \\ &\times\sum_{p,r=0}^{n}\left(\frac{(-1)^{p+r}\Gamma(n+2b'+r-p+1)}{p!r!(n-p)!(n-r)!\Gamma(n+2b'-p+1)}\right. \\ &\left.\times\frac{(p+2b''+2)}{\Gamma(n+2b'+r+2b''+2)}\right) \end{aligned} \qquad (64)$$

where

$$b'=\sqrt{(1+q)^2/q+\xi_3},$$
$$b''=\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}-1/2,$$

Table 2. The bound state energy eigenvalues $E_{n,k}$ in unit of fm^{-1} of the pseudo-spin symmetry Four-parameter diatomic potential for several values of *n* and *k* with $H=5$ for $q=1$, $V_1=0$, $V_2=-D=0.03815$.

$\tilde{l}$	$n,k<0$	l,j	$E_{n,k<0}(H=0)$	$E_{n,k<0}(H=5)$	$n-1,k>0$	$l+2,j+1$	$E_{n-1,k>0}(H=5)$
1	1, -1	$2s_{1/2}$	-4.992616	-4.97261/-1.03905	0, 2	$4d_{3/2}$	-4.992616
2	1, -2	$3p_{3/2}$	-4.984145	-4.984145	0, 3	$5f_{5/2}$	-4.984145
3	1, -3	$4d_{5/2}$	-4.97261/-1.03905	-4.992616	0, 4	$6g_{7/2}$	-4.97261/-1.03905
4	1, -4	$5f_{7/2}$	-4.95794/-1.05475	-4.99833	0, 5	$7h_{9/2}$	-4.95794/-1.05475

The parameters $C_{ps}=-6$, $A=30.52$, $\alpha'=1.5$, $M=1$, $\beta=20$, $C_0=0.000208178$, $C_1=0.002500011408$, $C_2=0.002499999716$ and $r_0=\beta.\log\left[1+2\alpha'(\alpha'-1)/A\right]$ are taken (Wei et al., 2010).

$$\xi_1-\xi_2+\xi_3=\frac{\beta^2(k+H)(k+H-1)[C_0(1-q)^2+C_1(1+q)+C_2]}{r_0^2}+\beta^2(E_{nk}-M-C_{ps})[-(M+E)(1-q)^2+V_2(1-q)+V_1]. \quad (65)$$

DISCUSSION

Here, in the framework of the Dirac equation with Manning-Rosen potential, the relativistic energy eigenvalues equation and the corresponding wave functions for the number of potentials that have been briefly discussed and obtained.

Choosing $\alpha=a$, $V_1=0$ and $V_2=-D$, then, potential in Equation 1 becomes the four-parameter diatomic potential (Gang, 2004), that is:

$$V(r)=-\frac{D}{e^{2ar}-q}, \quad (66)$$

With these substitutions, it follows the relativistic energy eigenvalues equation and the corresponding wave functions have been, respectively obtained as:

$$n[(n-1)+2]+(2n+1)\left(\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}+\sqrt{(1+q)^2/4+\xi_3}\right)+1/2-(1+q)/2-\xi_2+2\left((1+q)^2/4+\xi_3\right)+2\sqrt{\left(\xi_1-\xi_2+\xi_3+q^2/4\right)\left((1+q)^2/4+\xi_3\right)}=0, \quad (67)$$

and

$$G_{nk}(s)=B_{nk}s^{\frac{(1+q)}{2}+\sqrt{(1+q)^2/4+\xi_3}}(1+s)^{-\frac{q}{2}+\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}}\times P_n^{\left(2\sqrt{(1+q)^2/4+\xi_3}\ ,\ 2\sqrt{\xi_1-\xi_2+\xi_3+q^2/4}\right)}(1+2s), \quad (68)$$

where

$$\xi_1-\xi_2+\xi_3=\frac{\beta^2(k+H)(k+H-1)[C_0(1-q)^2+C_1(1+q)+C_2]}{r_0^2}+\beta^2(E_{nk}-M-C_{ps})[-(M+E)(1-q)^2-D(1-q)].$$

$$\xi_3=\frac{\beta^2(k+H)(k+H-1)(q^2C_0+C_1q+C_2)}{r_0^2}+\beta^2(E_{nk}-M-C_{ps})\left(-(E_{nk}+M)q^2+Dq\right). \quad (69)$$

Numerical results calculated for this case in Table 2.

Choosing $q=-1$, $V_1=0$ and $V_2=-1$ in Equation 1, we have (Guo and Sheng, 2005):

$$V(r)=-\frac{1}{e^{\alpha r}+1}. \quad (70)$$

Again, the relativistic energy eigenvalues equation and eigenfunctions for the Dirac equation with Woods-Saxon potential including a Coulomb-like tensor potential which have been, respectively obtained as:

$$n[(n-1)+2]+(2n+1)\left(\sqrt{\xi_1-\xi_2+\xi_3+1/4}+\sqrt{\xi_3}\right)+1/2-\xi_2+2\xi_3+2\sqrt{\left(\xi_1-\xi_2+\xi_3+1/4\right)\xi_3}=0. \quad (71)$$

And

$$G_{nk}(s)=B_{nk}s^{\sqrt{\xi_3}}(1+s)^{\frac{1}{2}+\sqrt{\xi_1-\xi_2+\xi_3+1/4}}\times P_n^{\left(2\sqrt{\xi_3}\ ,\ 2\sqrt{\xi_1-\xi_2+\xi_3+1/4}\right)}(1+2s), \quad (72)$$

where

Table 3. The bound state energy eigenvalues $E_{n,k}$ in unit of fm^{-1} of the pseudo-spin symmetry Woods-Saxon potential for several values of *n* and *k* with $H = 5$ for $q = -1$, $V_1 = 0$, $V_2 = -1$.

$\tilde{l}$	$n, k < 0$	l, j	$E_{n,k<0}(H = 0)$	$E_{n,k<0}(H = 5)$	$n-1, k > 0$	$l+2, j+1$	$E_{n-1,k>0}(H = 0)$
1	1, -1	$2s_{1/2}$	-2.207899	-2.060138	0, 2	$4d_{3/2}$	-2.207899
2	1, -2	$3p_{3/2}$	-2.144787	-2.144787	0, 3	$5f_{5/2}$	-2.144787
3	1, -3	$4d_{5/2}$	-2.060138	-2.207899	0, 4	$6g_{7/2}$	-2.060138
4	1, -4	$5f_{7/2}$	-1.962477	-2.24179	0, 5	$7h_{9/2}$	-1.962477

The parameters $C_{ps} = -6$, $A = 30.52$, $\alpha' = 1.5$, $M = 1$, $\beta = 20$, $C_0 = 0.000208178$, $C_1 = 0.002500011408$, $C_2 = 0.002499999716$ and $r_0 = \beta.\log[1 + 2\alpha'(\alpha'-1)/A]$ are taken (Wei et al., 2010).

$$\xi_1 - \xi_2 + \xi_3 = \frac{\beta^2(k+H)(k+H-1)[4C_0 + C_2]}{r_0^2} + \beta^2(E_{nk} - M - C_{ps})[-(M+E)4 + 2V_2],$$

$$\xi_3 = \frac{\beta^2(k+H)(k+H-1)(-C_0 - C_1 + C_2)}{r_0^2} + \beta^2(E_{nk} - M - C_{ps})(-(E_{nk} + M) + V_2). \quad (73)$$

Numerical results calculated for this case is as shown in Table 3.

On putting $q = 1$, $V_1 = 0$ and $V_2 = -V_0$ in the Manning-Rosen potential, then, Equation 1 reduces to the Hulthen potential (Ikhdiar et al., 2010) as:

$$V(r) = -\frac{V_0}{e^{\alpha r} - 1}. \quad (74)$$

The wave eigenfunctions and the relativistic energy eigenvalues equation for the Dirac equation with the Hulthen potential including a Coulomb-like tensor potential have been, respectively obtained as:

$$n[(n-1)+2] + (2n+1)\left(\sqrt{\xi_1 - \xi_2 + \xi_3 + 1/4} + \sqrt{1+\xi_3}\right) - 1/2 - \xi_2 + 2(1+\xi_3) + 2\sqrt{(\xi_1 - \xi_2 + \xi_3 + 1/4)(1+\xi_3)} = 0. \quad (75)$$

and

$$G_{nk}(s) = B_{nk} s^{1+\sqrt{1+\xi_3}} (1+s)^{-\frac{1}{2}+\sqrt{\xi_1-\xi_2+\xi_3+1/4}} \times P_n^{\left(2\sqrt{1+\xi_3},\; 2\sqrt{\xi_1-\xi_2+\xi_3+1/4}\right)}(1+2s), \quad (76)$$

where

$$\xi_1 - \xi_2 + \xi_3 = \frac{\beta^2(k+H)(k+H-1)[2C_1 + C_2]}{r_0^2}$$

$$\xi_3 = \frac{\beta^2(k+H)(k+H-1)(C_0 + C_1 + C_2)}{r_0^2} + \beta^2(E_{nk} - M - C_{ps})(-(E_{nk} + M) - V_0). \quad (77)$$

Numerical results calculated for this case is as shown in Table 4.

Choosing $q = 0$ and $V_2 = -V_2$ in the Manning-Rosen potential, then, Equation 1 reduces to the generalized Morse potential (Morse, 1929) as:

$$V(r) = V_1 e^{-2\alpha r} - V_2 e^{-\alpha r}, \quad (78)$$

with the following solutions:

$$n[(n-1)+2] + (2n+1)\left(\sqrt{\xi_1 - \xi_2 + \xi_3} + \sqrt{1/4 + \xi_3}\right) - \xi_2 + 2(1/4 + \xi_3) + 2\sqrt{(\xi_1 - \xi_2 + \xi_3)(1/4 + \xi_3)} = 0, \quad (79)$$

and

$$G_{nk}(s) = B_{nk} s^{\frac{1}{2}+\sqrt{1/4+\xi_3}} (1+s)^{\sqrt{\xi_1-\xi_2+\xi_3}} \times P_n^{\left(2\sqrt{1/4+\xi_3},\; 2\sqrt{\xi_1-\xi_2+\xi_3}\right)}(1+2s), \quad (80)$$

where

$$\xi_1 - \xi_2 + \xi_3 = \frac{\beta^2(k+H)(k+H-1)[C_0 + C_1 + C_2]}{r_0^2} + \beta^2(E_{nk} - M - C_{ps})[-(M+E) - V_2 + V_1].$$

$$\xi_3 = \frac{\beta^2 C_2(k+H)(k+H-1)}{r_0^2} + \beta^2(E_{nk} - M - C_{ps})V_1. \quad (81)$$

Table 4. The bound state energy eigenvalues $E_{n,k}$ in unit of fm^{-1} of the pseudo-spin symmetry Hulthen potential for several values of *n* and *k* with $H=5$ for $q=1$, $V_1=0$, $V_2=-V_0=0.03815$.

$\tilde{l}$	$n,k<0$	l,j	$E_{n,k<0}(H=0)$	$E_{n,k<0}(H=5)$	$n-1,k>0$	$l+2,j+1$	$E_{n-1,k>0}(H=0)$
1	1, -1	$2s_{1/2}$	-4.99262	-4.97261/-1.03905	0, 2	$4d_{3/2}$	-4.99262
2	1, -2	$3p_{3/2}$	-4.984144	-4.984144	0, 3	$5f_{5/2}$	-4.984144
3	1, -3	$4d_{5/2}$	-4.97261/-1.03905	-4.99262	0, 4	$6g_{7/2}$	-4.97261/-1.03905
4	1, -4	$5f_{7/2}$	-4.95794/-1.05475	-4.99833	0, 5	$7h_{9/2}$	-4.95794/-1.05475

The parameters $C_{ps}=-6$, $A=30.52$, $\alpha'=1.5$, $M=1$, $\beta=20$, $C_0=0.000208178$, $C_1=0.002500011408$, $C_2=0.002499999716$ and $r_0=\beta.\log\left[1+2\alpha'(\alpha'-1)/A\right]$ are taken (Wei et al., 2010).

Table 5. The bound state energy eigenvalues $E_{n,k}$ in unit of fm^{-1} of the pseudo-spin symmetry generalized Morse potential for several values of *n* and *k* with $H=5$ for $q=0$, $V_1=0.0009375$, $V_2=V_0=-0.03815$.

$\tilde{l}$	$n,k<0$	l,j	$E_{n,k<0}(H=0)$	$E_{n,k<0}(H=5)$	$n-1,k>0$	$l+2,j+1$	$E_{n-1,k>0}(H=0)$
1	1, -1	$2s_{1/2}$	-1.18268	-1.056234	0, 2	$4d_{3/2}$	-1.18268
2	1, -2	$3p_{3/2}$	-1.104782	-1.104782	0, 3	$5f_{5/2}$	-1.104782
3	1, -3	$4d_{5/2}$	-1.056234	-1.18268	0, 4	$6g_{7/2}$	-1.056234
4	1, -4	$5f_{7/2}$	-1.023485	-1.288454	0, 5	$7h_{9/2}$	-1.023485

The parameters $C_{ps}=-6$, $A=30.52$, $\alpha'=1.5$, $M=1$, $\beta=20$, $C_0=0.000208178$, $C_1=0.002500011408$, $C_2=0.002499999716$ and $r_0=\beta.\log\left[1+2\alpha'(\alpha'-1)/A\right]$ are taken (Wei et al., 2010).

$$+\beta^2\left(E_{nk}-M-C_{ps}\right)[-(M+E)-V_2+V_1].$$

$$\xi_3=\frac{\beta^2 C_2\left(k+H\right)\left(k+H-1\right)}{r_0^2}+\beta^2\left(E_{nk}-M-C_{ps}\right)V_1. \quad (81)$$

Numerical results calculated for this case is as shown in Table 5.

Choosing $\alpha=2\alpha$, $q=1$, $V_1=4D$ and $V_2=0$ in the Manning-Rosen potential, then, Equation 1 becomes (Ikhdair et al., 2009):

$$V(r)=\frac{4De^{-4\alpha r}}{\left(e^{2\alpha r}-1\right)^2}, \quad (82)$$

with the following solutions:

$$n[(n-1)+2]+(2n+1)\left(\sqrt{\xi_1-\xi_2+\xi_3+1/4}+\sqrt{1+\xi_3}\right)$$
$$-\xi_2+2\left(1+\xi_3\right)+2\sqrt{\left(\xi_1-\xi_2+\xi_3+1/4\right)\left(1+\xi_3\right)}=0, \quad (83)$$

and

$$G_{nk}(s)=B_{nk}s^{1+\sqrt{1+\xi_3}}(1+s)^{-\frac{1}{2}+\sqrt{\xi_1-\xi_2+\xi_3+1/4}}$$
$$\times P_n^{\left(2\sqrt{1+\xi_3}\ ,\ 2\sqrt{\xi_1-\xi_2+\xi_3+1/4}\right)}(1+2s), \quad (84)$$

where

$$\xi_1-\xi_2+\xi_3=\frac{\beta^2\left(k+H\right)\left(k+H-1\right)[2C_1+C_2]}{r_0^2}$$
$$+4D\beta^2\left(E_{nk}-M-C_{ps}\right),$$

$$\xi_3=\frac{\beta^2\left(k+H\right)\left(k+H-1\right)(C_0+C_1+C_2)}{r_0^2}$$
$$+\beta^2\left(E_{nk}-M-C_{ps}\right)\left(-\left(E_{nk}+M\right)+4D\right). \quad (85)$$

Numerical results calculated for this case is as shown in Table 6.

Conclusion

In this research, we have obtained analytically the approximate energy eigenvalues equation and the corresponding normalized wave functions of the Dirac equation for the Manning-Rosen potential coupled with a Coulomb-like tensor under the condition of the pseudo-spin symmetry using the parametric generalization of the Nikiforov-Uvarov method. The energy eigenvalues equation and corresponding wave functions of the number of potentials have been obtained and briefly discussed as

Table 6. The bound state energy eigenvalues $E_{n,k}$ in unit of fm^{-1} of the pseudo-spin symmetry spatial case potential for several values of *n* and *k* with $H=5$ for $q=1$, $V_1=4D=0.0036$, $V_2=0$.

$\tilde{l}$	$n, k<0$	l, j	$E_{n,k<0}(H=0)$	$E_{n,k<0}(H=5)$	$n-1, k>0$	$l+2, j+1$	$E_{n-1,k>0}(H=0)$
1	1, -1	$2s_{1/2}$	-4.992632/-1.010916	-4.972681/-1.030226	0, 2	$4d_{3/2}$	-4.992632/-1.010916
2	1, -2	$3p_{3/2}$	-4.984184/-1.018937	-4.984184/-1.018937	0, 3	$5f_{5/2}$	-4.984184/-1.018937
3	1, -3	$4d_{5/2}$	-4.972681/-1.030226	-4.992632/-1.010916	0, 4	$6g_{7/2}$	-4.972681/-1.030226
4	1, -4	$5f_{7/2}$	-4.958052/-1.044729	-4.99833/-1.006503	0, 5	$7h_{9/2}$	-4.958052/-1.044729

The parameters $C_{ps}=-6$, $A=30.52$, $\alpha'=1.5$, $M=1$, $\beta=20$, $C_0=0.000208178$, $C_1=0.002500011408$, $C_2=0.002499999716$ and $r_0=\beta.\log\left[1+2\alpha'(\alpha'-1)/A\right]$ are taken (Wei et al., 2010).

cases of the Manning-Rosen potential in the Dirac equation. Numerical results calculated for per cases.

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