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Gravitational effect on surface waves in a homogeneous fibre-reinforced anisotropic general viscoelastic media of higher and fractional order with voids

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In this paper, gravitational effects on propagation of surface waves in a homogeneous fibre-reinforced anisotropic general viscoelastic media of higher order with voids is investigated. The general surface wave speed is derived to study the effects of gravity on surface waves. Particular cases for Stoneley and Rayleigh waves are discussed. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. In the absence of voids our results for viscoelastic of order zero are well agreement to fibre-reinforced materials. Also by neglecting the reinforced elastic parameters, the results reduce to well known isotropic medium. Numerical results for particular materials are given and illustrated graphically. The results indicate that the effect of the gravitational, voids and the reinforced elastic parameters on surface waves are very pronounced.

Key words. Fibre-reinforced, viscoelastic, surface waves, gravity, anisotropic, voids.

INTRODUCTION

It is of great interest to study the propagation of surface waves in a homogeneous fibre-reinforced anisotropic general viscoelastic media of higher order with voids as it plays an importent role in material fracture and failure. Such problems have attracted much attention and have undergone a certain development (Bullen, 1965; Ewing and Jardetzky, 1957; Rayleigh, 1885; Stoneley, 1924). Surface waves have been well recognized in the study of earthquake, seismology, geophysics and geodynamics. These waves usually have greater amplitudes as compared with body waves and travel more slowly than body waves. There are many types of surface waves but we only discussed Stoneley and Rayleigh waves. In earthquake the movement is due to the surface waves.

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These are also used for detecting cracks and other defects in materials. Lord Rayleigh (1885) was the first to observe such kind of waves in 1885. That is why we called it Rayleigh waves. Sengupta and Nath (2001) investigated surface waves in fibre-reinforced anisotropic elastic media but their decomposition of displacement vector was not correct due to which some errors are found in their investigations (Sarvajit, 2002).

The idea of continuous self-reinforcement at every point of an elastic solid was introduced by Belfield et al. (1983). The superiority of fibre-reinforced composite materials over other structural materials attracted many authors to study different type of problems in this field. Fibre-reinforced composite structures are used due to their low weight and high strength. Two important components namely concrete and steel of a reinforced medium are bound together as a single unit so that there can be no relative displacement between them, that is, they act together as a single anisotropic unit. The artificial structures on the surface of the earth are excited during an earthquake, which give rise to violent vibrations in some cases. Engineers and architects are in search of such reinforced elastic materials for the structures that resist the oscillatory vibration. The propagation of waves depends upon the ground vibration and the physical properties of the structure material. Surface wave propagation in fiber reinforced media was discussed by various authors.

In classical theory of elasticity, the voids is an important generalization. Nunziato and Cowin (1979) and Cowin and Nunziato (1983) discussed the theory in elastic media with voids. Puri and Cowin (1985) studied the effects of voids on plane waves in linear elastic media and it is evident that pure shear waves remain unaffected by the presence of pores. Theory of thermoelastic material with voids is investigated by Lesan (1986). Good amount of literature on surface wave propagation in a generalized thermoelastic material with voids, is available in Singh and Pal (2011) and references therein. Chandrasekharaiah (1987a, b) discussed the effects of voids on propagation of plane and surface waves. Abo-Dahab (2010) investigated the propagation of P waves from stress-free surface elastic half-space with voids.

The effect of gravity on wave propagation in an elastic solid medium was first considered by Bromwich (1898). Later on gravity effects on wave propagation were discussed by various authors (Abd-Alla et al., 2013; Abd-Alla and Ahmed, 2003; De and Sengupta, 1974; Sengupta and Acharya, 1979)

Surface waves in fiber-reinforced,general viscoelastic media of higher order under gravity is discussed by kakar et. al. (2013) whereas Pal and Sengupta (1987) studied the gravitational effects in viscoelastic media. Ren et al. (2012) investigated the coupling effects of void shape and void size on the growth of an elliptic void in a fiber-reinforced hyper-elastic thin plate. Vishwakarma et al. (2013) discussed the influence of rigid boundary on the love wave propagation in elastic layer with void pores. Tvergaard (2011) studied the elastic–plastic void expansion in near-self-similar shapes. Fonseca et al. (2011) expressed the material voids in elastic solids with anisotropic surface energies. The extensive literature on the topic is now available and we can only mention a few recent interesting investigations in Abo-Dahab and Abd-Alla (2014), Abd-Alla et al. (2011), Abd-Alla and Ahmed (2003), Abd-Alla (1999), Abd-Alla and Ahmed (1999), Abd-Alla et al. (2004), Elnaggar and Abd-Alla (1989), Abd-Alla and Ahmed (1996) Abd-Alla et al. (2012) and Abd-Alla et al. (2013). Aim of this paper is to investigate the gravitational effects on propagation of surface waves in fibre-reinforced viscoelastic anisotropic media of higher order with voids. The general surface wave speed is derived to study the effect of gravity and voids on surface waves. Particular cases for Stonely and Rayleigh waves are discussed. The results obtained in this investigation are more general in the sense that some earlier published results are obtained from our result as special cases. Numerical results are given and illustrated graphically.

FORMULATION OF THE PROBLEM

The constitutive relation of an anisotropic and elastic solid is expressed by the generalized Hooke's law, which can be written as:

$$\tau_{ij} = C_{ijkl}\,\varepsilon_{kl} \qquad i, j, k, l = 1, 2, 3.$$

where, τ_{ij} are the Cartesian components of the stress and ε_{ij} is the strain tensor which is related with the displacement vector, u_i ; C_{ijkl} are the components of a fourth-order tensor called the elasticities of the medium. The Einstein convention for repeated indices is used.

In the absence of body forces, the field equations in the presence of voids may be taken as follows:

$$\tau_{ij,j} = \rho \ddot{u}_i, \tag{1}$$

$$\alpha\phi_{,ii} - \omega_0\phi - \varpi\dot{\phi} - \beta u_{i,i} = \rho\kappa\ddot{\phi} \tag{2}$$

$$\tau_{ij} = C_{ijkl}\varepsilon_{kl} + \beta\delta_{ij}\phi \tag{3}$$

In these equations, ϕ is the so-called volume fraction field. $\alpha, \beta, \omega_0, \varpi$ and κ are new material constants characterizing the presence of voids. ρ is the mass density. Comma followed by index shows partial derivative with respect to coordinate. The Einstein convention for repeated indices is used. Thus Above equation under gravity force G becomes:

$$C_{ijkl}u_{k,jl} + G_i + \beta\phi_{,i} = \rho\ddot{u}_i \tag{4}$$

Medium is consisting of two homogeneous anisotropic fibre-reinforced semi-infinite elastic solid media M_1 and M_2 with different elastic and reinforcement parameters. The two media are perfectly welded in contact at a plane interface. Let us take orthogonal Cartesian axes $Ox_1x_2x_3$ with the origin at O. Ox_2 is pointing vertically upwards into the medium, M_1 ($x_2 > 0$). Each of the media M_1 ($x_2 > 0$) and M_2 ($x_2 < 0$) separated at $x_2 = 0$.

It is assumed that the waves travel in the positive direction of the x_1-axis and at any instant, all particles have equal displacements in any direction parallel to Ox_3. In view of those assumptions, the propagation of waves will be independent of x_3. Therefore all derivatives with respect to x_3 will be zero.

The general equation for a fibre-reinforced linearly elastic anisotropic media with respect to a direction $\bar{a} = (a_1, a_2, a_3)$ is as follows (Sengupta and Nath, 2001):

$$C_{ijkl}\varepsilon_{kl} = D_\lambda \varepsilon_{kk}\delta_{ij} + 2D_{\mu_T}\varepsilon_{ij} + D_\alpha(a_k a_m \varepsilon_{km}\delta_{ij} + \varepsilon_{kk}a_i a_j) + 2(D_{\mu_L} - D_{\mu_T})(a_i a_k \varepsilon_{kj} + a_j a_k \varepsilon_{ki}) + D_\beta(a_k a_m \varepsilon_{km} a_i a_j),$$

Strain tensor is $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ and D_λ, D_{μ_T} are elastic parameters. D_α, D_β and $(D_{\mu_L} - D_{\mu_T})$ are reinforced anisotropic viscoelastic parameters of higher order, s, defined as:

$$D_\lambda = \lambda_k \left(\frac{\partial}{\partial t}\right)^k \qquad D_\mu = \mu_k \left(\frac{\partial}{\partial t}\right)^k$$

$$D_\alpha = \alpha_k \left(\frac{\partial}{\partial t}\right)^k \qquad D_{\mu_L} = \mu_{L_k} \left(\frac{\partial}{\partial t}\right)^k$$

$$D_\beta = \beta_k \left(\frac{\partial}{\partial t}\right)^k \qquad D_{\mu_T} = \mu_{T_k} \left(\frac{\partial}{\partial t}\right)^k$$

$$k = 0, 1, 2 \ldots s.$$

An Einstein summation convention for repeated indices over "*k*" is used and comma followed by an index denotes the derivative with respect to coordinate.

u_i are the displacement vectors components. By choosing the fibre direction as $\bar{a} = (1, 0, 0)$, the components of stress becomes as follows:

$$\tau_{11} = (D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)\varepsilon_{11} + (D_\lambda + D_\alpha)\varepsilon_{22} + (D_\lambda + D_\alpha)\varepsilon_{33} + \beta\phi,$$

$$\tau_{22} = (D_\lambda + D_\alpha)\varepsilon_{11} + (D_\lambda + 2D_{\mu_T})\varepsilon_{22} + D_\lambda\varepsilon_{33} + \beta\phi,$$

$$\tau_{33} = (D_\lambda + D_\alpha)\varepsilon_{11} + D_\lambda\varepsilon_{22} + (D_\lambda + 2D_{\mu_T})\varepsilon_{33} + \beta\phi,$$

$$\tau_{13} = 2D_{\mu_L}\varepsilon_{13},$$

$$\tau_{12} = 2D_{\mu_L}\varepsilon_{12},$$

$$\tau_{23} = 2D_{\mu_T}\varepsilon_{23}.$$

By choosing the fibre direction as $\bar{a} = (1, 0, 0)$; also by taking all derivatives w.r.t. x_3 zero. The Equation (4) of motion takes the following form:

$$(D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta)u_{1,11} + (D_\alpha + D_\lambda + D_{\mu_L})u_{2,21} + D_{\mu_L}u_{1,22} + \rho g u_{3,1} = \rho\ddot{u}_1 - \beta\phi_{,1} \tag{5a}$$

$$(D_\alpha + D_{\lambda_k} + D_{\mu_L})u_{1,12} + D_{\mu_L}u_{2,11} + (D_{\lambda_k} + 2D_{\mu_T})u_{2,22} + \rho g u_{3,2} = \rho\ddot{u}_2 - \beta\phi_{,2} \tag{5b}$$

$$D_{\mu_L} u_{3,11} + D_{\mu_T} u_{3,22} - \rho g(u_{1,1} + u_{2,2}) = \rho\ddot{u}_3, \tag{5c}$$

From Equation (2), we have:

$$\alpha\left(\phi_{,11} + \phi_{,22}\right) - \omega_0\phi - \varpi\dot{\phi} - \beta\left(u_{1,1} + u_{2,2}\right) = \rho\kappa\ddot{\phi} \tag{5d}$$

Similarly, we can get similar relations in M_2 with $\rho, \alpha, \beta, \kappa, c_v$ D_α, D_λ, D_{μ_L}, D_{μ_T} and D_β are replaced by $\rho', \alpha', \beta', \kappa', c'_v$ $D_{\alpha'}, D_{\lambda'}$, $D_{\mu'_L}$ $D_{\mu'_T}$ and $D_{\beta'}$, that is, all the parameters in medium M_1 are denoted by super script "dash".

Equations (5) in simplified form can be written as:

$$h_3u_{1,11} + h_2u_{2,21} + h_1u_{1,22} + \rho g u_{3,1} = \rho\ddot{u}_1 - \beta\phi_{,1} \tag{6a}$$

$$h_4u_{2,22} + h_2u_{1,12} + h_1u_{2,11} + \rho g u_{3,2} = \rho\ddot{u}_2 - \beta\phi_{,2} \tag{6b}$$

$$h_1 u_{3,11} + h_5 u_{3,22} - \rho g(u_{1,1} + u_{2,2}) = \rho \ddot{u}_3 \quad (6c)$$

$$\alpha\left(\phi_{,11} + \phi_{,22}\right) - \omega_0 \phi - \varpi \dot{\phi} - \beta\left(u_{1,1} + u_{2,2}\right) = \rho \kappa \ddot{\phi} \ , \quad (6d)$$

where

$$h_1 = D_{\mu_L}, \ ,$$
$$h_2 = D_\alpha + D_\lambda + D_{\mu_L},\ h_3 = D_\lambda + 2D_\alpha + 4D_{\mu_L} - 2D_{\mu_T} + D_\beta$$
$$h_4 = D_{\lambda_k} + 2D_{\mu_T} \text{ and } h_5 = D_{\mu_T}$$

SOLUTION OF THE PROBLEM

To solve the coupled thermoelastic equations, we make the assumptions:

$$u_1, u_2, u_3 = \hat{u}_1(x_2), \hat{u}_2(x_2), \hat{u}_3(x_2)\, exp\left\{i\omega(x_1 - ct)\right\}$$
$$\phi = \ \hat{\phi}(x_2)\ exp\left\{i\omega(x_1 - ct)\right\} \quad (7)$$

Thus coupled equations (6a, b and c)) becomes:

$$(\hbar_1 D^2 - \omega^2 \hbar_3 + \omega^2 \rho c^2)\hat{u}_1 + i\omega \hbar_2 D\hat{u}_2 + i\omega \rho g \hat{u}_3 + i\omega\beta\hat{\phi} = 0$$

$$(\hbar_5 D^2 - \hbar_1 \omega^2 + \rho \omega^2 c^2)\hat{u}_3 - \rho g(i\omega \hat{u}_1 + D\hat{u}_2) = 0$$

and

$$\left\{\alpha\left(D^2 - \omega^2\right) - \omega_0 + i\omega c\varpi + \omega^2 c^2 \rho\kappa\right\}\hat{\phi} - \beta\left(i\omega\hat{u}_1 + D\hat{u}_2\right) = 0$$

where

$$\hbar_1 = \mu_{Lk}(-i\,\omega c)^k,\ \hbar_2 = (\alpha_k + \lambda_k + \mu_{Lk})(-i\,\omega c)^k,$$
$$\hbar_3 = (\lambda_k + 2\alpha_k + 4\mu_{Lk} - 2\mu_{Tk} + \beta_k)(-i\,\omega c)^k,$$
$$\hbar_4 = (\lambda_k + 2\mu_{Tk})(-i\,\omega c)^k,\ \hbar_5 = \mu_{Tk}(-i\,\omega c)^k.$$

Above set of equation can be written as

$$\left.\begin{aligned}
&(\hbar_1 D^2 - A_1)\hat{u}_1 + i\omega\hbar_2 D\hat{u}_2 + i\omega\rho g\hat{u}_3 + i\omega\beta\hat{\phi} = 0,\\
&(\hbar_4 D^2 - A_2)\hat{u}_2 + i\omega\hbar_2 D\hat{u}_1 + \rho g D\hat{u}_3 + \beta D\hat{\phi} = 0,\\
&\left(h_5 D^2 - A_2\right)\hat{u}_3 - \rho g(i\omega\hat{u}_1 + D\hat{u}_2) = 0\\
&(D^2 - A_3)\hat{\phi} - \beta(i\omega\hat{u}_1 + D\hat{u}_2) = 0
\end{aligned}\right\} \quad (8)$$

where

$$A_1 = \omega^2\left(\hbar_3 - \rho c^2\right)$$
$$A_2 = \omega^2\left(\hbar_1 - \rho c^2\right)$$
$$A_3 = \omega^2 + \frac{\omega_0 - i\omega c\varpi - \omega^2 c^2 \rho\kappa}{\alpha}$$

From above set of equations, for non-trival solution, we have:

$$\begin{vmatrix}
(\hbar_1 D^2 - A_1) & i\omega\hbar_2 D & i\omega\rho g & i\omega\beta\\
i\omega\hbar_2 D & (\hbar_4 D^2 - A_2) & \rho g D & \beta D\\
-i\omega\rho g & -\rho g D & (\hbar_5 D^2 - A_2) & 0\\
-i\omega\beta & -\beta D & 0 & (D^2 - A_3)
\end{vmatrix}(\hat{u}_1, \hat{u}_2, \hat{\phi}) = 0$$

This implies

$$(D^8 - ED^6 + FD^4 - GD^2 + H)(\hat{u}_1, \hat{u}_2, \hat{\phi}) = 0$$

where

$$E = \frac{1}{\hbar_1\hbar_4\hbar_5}\left\{\left(\hbar_1 A + \hbar_4 A_2 - \rho^2 g^2\right)\right\}$$
$$F = \frac{1}{\hbar_1\hbar_4\hbar_5}\left\{\rho^2\omega^2 g^2\left(A_1 + 2\hbar_2 - \hbar_4\right) + \hbar_1 B + A_2 A\right\}$$
$$G = \frac{1}{\hbar_1\hbar_4\hbar_5}\left\{\rho^2\omega^2 g^2\left(\left(A_1 + 2\hbar_2 - \hbar_4\right)A_3 - A_2\right) + A_2 B + \hbar_1 C\right\}$$
$$H = \frac{1}{\hbar_1\hbar_4\hbar_5}\left(\rho^2\omega^2 g^2\left(A_2 A_3\right) + A_2 C\right)$$
$$A = \left(\hbar_4 A_1 + \hbar_1(A_2 + \hbar_4 A_3 - \beta^2) + \omega^2\hbar_2^2\right)$$
$$B = \left\{\left(A_1 A_2 + \hbar_4 A_1 A_3 + \hbar_1 A_2 A_3 - \omega^2\hbar_2^2 A_3\right) + \beta A_4(A_1 + 2\omega^2\hbar_2 + \hbar_4\omega^2)\right\}$$
$$C = \left(A_1 A_2 A_3 - \omega^2 A_2 \beta^2\right)$$
$$Let\ \ D^2 = m$$

Auxiliary equation becomes:

$$m^4 - Em^3 + Fm^2 - Gm + H = 0 \quad (9)$$

E,*F*,*G* and *H* must be positive for real positive roots (*m*). In the absence of gravity the above equation is cubic and if there are no voids then the above equation is quadratic in *m* and it is easy to solve.

Let mi (i=1,2,3,4) be four positive real roots, then solution by normal mode method has the following form:

$$\hat{u}_1 = \sum_{n=1}^{4} M_n\ e^{-m_n x_2}, \quad (10a)$$

$$\hat{u}_2 = \sum_{n=1}^{4} M_{1n}\ e^{-m_n x_2}, \quad (10b)$$

$$\hat{u}_3 = \sum_{n=1}^{4} M_{2n}\ e^{-m_n x_2}, \quad (10c)$$

$$\hat{\phi}=\sum_{n=1}^{4}M_{3n}\ e^{-m_n x_2}, \qquad (10d)$$

where M_n, M_{1n}, M_{2n} and M_{3n} are some parameters. By using Equations (10a to d) into Equations (8), we get the following relations:

$$M_{1n}=H_{1n}M_n\ ,$$
$$M_{2n}=H_{2n}M_n\ ,$$
$$M_{3n}=H_{3n}M_n\ ,$$

where

$$H_{1n}=\frac{i\,\omega(A_2+\hbar_2 m_n^2-\hbar_4 m_n^2)}{A_1 m_n-\hbar_2\omega^2 m_n-\hbar_1 m_n^3},$$
$$H_{2n}=\frac{\hbar_1 m_n^2-A_2}{\rho g\,(i\,\omega-m_n H_{1n})},$$
$$H_{3n}=\frac{A_3-m_n^2}{\beta(i\,\omega-m_n H_{1n})}.$$

Hence we obtain the expressions of the displacement components, volume fraction field and stresses as follows

$$u_1=\sum_{n=1}^{4}M_n\ e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (11a)$$

$$u_2=\sum_{n=1}^{4}H_{1n}M_n\ e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (11b)$$

$$u_3=\sum_{n=1}^{4}H_{2n}M_n\ e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (11c)$$

$$\phi=\sum_{n=1}^{4}H_{3n}M_n\ e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (11d)$$

Also it is found that

$$\tau_{12}=\sum_{n=1}^{4}\hbar_1\left(-m_n+i\omega H_{1n}\right)M_n e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}$$
$$\tau_{22}=\sum_{n=1}^{4}\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_n H_{1n}-\beta H_{3n}\}M_n e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}$$
$$\tau_{23}=\sum_{n=1}^{4}\hbar_5\left(-m_n H_{2n}\right)M_n e^{-m_n x_2}\ exp\{i\omega(x_1-ct)\}, \qquad where\ \hbar_5=\mu_{Tk}(-i\omega c)^k$$

Similar expressions can be obtained for second mediun and present them with super script dashes as follows:

$$u_1'=\sum_{n=1}^{4}M_n'\ e^{-m_n' x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (12a)$$

$$u_2'=\sum_{n=1}^{4}H_{1n}'M_n'\ e^{-m_n' x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (12b)$$

$$u_3'=\sum_{n=1}^{4}H_{2n}'M_n'\ e^{-m_n' x_2}\ exp\{i\omega(x_1-ct)\}, \qquad (12c)$$

$$\phi'=\sum_{n=1}^{4}H_{3n}'M_n'\ e^{-m_n' x_2}\ exp\{i\,\omega(x_1-ct)\}. \qquad (12d)$$

Also it is found that:

$$\tau_{12}'=\sum_{n=1}^{4}\hbar_1'\left(-m_n'+i\,\omega H_{1n}'\right)M_n' e^{-m_n' x_2}\ exp\{i\,\omega(x_1-ct)\},$$
$$\tau_{22}'=\sum_{n=1}^{4}\{i\,\omega(\hbar_2'-\hbar_1')-\hbar_4 m_n' H_{1n}'-\beta' H_{3n}'\}M_n' e^{-m_n' x_2}\ exp\{i\,\omega(x_1-ct)\},$$
$$\tau_{23}'=\sum_{n=1}^{4}\hbar_5\left(-m_n' H_{2n}'\right)M_n' e^{-m_n' x_2}\ exp\{i\,\omega(x_1-ct)\}.$$

In order to determine the secular equations, we have the following boundary conditions.

BOUNDARY CONDITIONS

1. The displacement components and volume fraction field between the mediums are continuous, that is, $u_1=u_1'$, $u_2=u_2'$, $u_3=u_3'$ and $\phi=\phi'$ on $x_2=0$, for all x_1 and *t.*
2. Stress continuity exists, i.e. $\tau_{12}=\tau_{12}'$, $\tau_{22}=\tau_{22}'$, $\tau_{23}=\tau_{23}'$ on $x_2=0$, for all x_1 and *t.*
3. It is assumed that the following relation hold:

$$\left(\frac{\partial\phi}{\partial x_2}+h\phi\right)_{medium\,M_1}=\left(\frac{\partial\phi'}{\partial x_2}+h\phi'\right)_{medium\ M_2},\ \text{on the plane}\ x_2=0\ ,\forall\,x_1\ and\ t,$$

where h is a constant.
Boundary conditions implies the following equations:

$$\left.\begin{array}{l}M_1+M_2+M_3+M_4=M_1'+M_2'+M_3'+M_4'\\ H_{11}M_1+H_{12}M_2+H_{13}M_3+H_{14}M_4=H_{11}'M_1'+H_{12}'M_2'+H_{13}'M_3'+H_{14}'M_4'\\ H_{21}M_1+H_{22}M_2+H_{23}M_3+H_{24}M_4=H_{21}'M_1'+H_{22}'M_2'+H_{23}'M_3'+H_{24}'M_4'\\ H_{31}M_1+H_{32}M_2+H_{33}M_3+H_{34}M_4=H_{31}'M_1'+H_{32}'M_2'+H_{33}'M_3'+H_{34}'M_4'\end{array}\right) \qquad (13a)$$

$$\left.\begin{aligned}
&\sum_{n=1}^{4} \hbar_1\left(-m_n+i\omega H_{1n}\right)M_n=\sum_{n=1}^{4} \hbar_1'\left(-m_n'+i\omega H_{1n}'\right)M_n' ,\\
&\sum_{n=1}^{4}\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_n H_{1n}-\beta H_{2n}\right\}M_n=\\
&\qquad\sum_{n=1}^{4}\left\{i\omega(\hbar_2'-\hbar_1')-\hbar_4' m_n' H_{1n}'-\beta' H_{2n}'\right\}M_n' ,\\
&\sum_{n=1}^{4}\hbar_5\left(-i\omega c\right)^k\left(-m_n H_{2n}\right)M_n=\sum_{n=1}^{4}\hbar_5'\left(-m_n' H_{2n}'\right)M_n'\\
&\sum_{n=1}^{4}(h-m_n)H_{3n}M_n=\sum_{n=1}^{4}(h'-m_n')H_{3n}'M_n'
\end{aligned}\right\} \quad (13b)$$

Elimination of constants $M_n \; and \; M_n', \; (n=1,2,3,4)$ from above set of relation, gives the following secular equation for surface wave in a fibre reinforced viscoelastic material of higher order *s* under gravity with voids.

$$\det(a_{pq})=0; \quad p=q=1,2,3,4,5,6,7,8. \quad (14)$$

where

$$\begin{aligned}
&a_{11}=1,\; a_{12}=1,\; a_{13}=1, a_{14}=1,\; a_{15}=-1,\; a_{16}=-1,\; a_{17}=-1,\; a_{18}=-1\\
&a_{21}=H_{11},\; a_{22}=H_{12},\; a_{23}=H_{13},\; a_{24}=H_{14},\; a_{25}=-H_{11}',\; a_{26}=-H_{12}',\; a_{27}=-H_{13}',\; a_{28}=-H_{14}',\\
&a_{31}=H_{21},\; a_{32}=H_{22},\; a_{33}=H_{23},\; a_{34}=H_{24},\; a_{35}=-H_{21}',\; a_{36}=-H_{22}',\; a_{37}=-H_{23}',\; a_{38}=-H_{24}',\\
&a_{41}=H_{31},\; a_{42}=H_{32},\; a_{43}=H_{33},\; a_{44}=H_{34},\; a_{45}=-H_{31}',\; a_{46}=-H_{32}',\; a_{47}=-H_{33}',\; a_{48}=-H_{34}',
\end{aligned}$$

$$\begin{aligned}
&a_{51}=\hbar_1\left(-m_1+i\omega H_{11}\right), \qquad a_{52}=\hbar_1\left(-m_2+i\omega H_{12}\right),\\
&a_{53}=\hbar_1\left(-m_3+i\omega H_{13}\right), \qquad a_{54}=\hbar_1\left(-m_3+i\omega H_{14}\right),\\
&a_{55}=-\hbar_1'\left(-m_1'+i\omega H_{11}'\right), \qquad a_{56}=-\hbar'\left(-m_2'+i\omega H_{12}'\right),\\
&a_{57}=-\hbar'\left(-m_3'+i\omega H_{13}'\right), \qquad a_{58}=-\hbar'\left(-m_4'+i\omega H_{14}'\right)\\
&a_{61}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_1 H_{11}-\beta H_{21}\right\},\\
&a_{62}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_2 H_{12}-\beta H_{22}\right\},\\
&a_{63}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_3 H_{13}-\beta H_{23}\right\},\\
&a_{64}=\left\{i\omega(\hbar_2-\hbar_1)-\hbar_4 m_3 H_{14}-\beta H_{24}\right\}\\
&a_{65}=-\left\{i\omega(\hbar_2'-\hbar_1')-\hbar_4' m_1' H_{11}'-\beta' H_{21}'\right\},\\
&a_{66}=-\left\{i\omega(\hbar_2'-\hbar_1')-\hbar_4' m_2' H_{12}'-\beta' H_{22}'\right\},\\
&a_{67}=-\left\{i\omega(\hbar_2'-\hbar_1')-\hbar_4' m_3' H_{13}'-\beta' H_{23}'\right\},\\
&a_{68}=-\left\{i\omega(\hbar_2'-\hbar_1')-\hbar_4' m_4' H_{14}'-\beta' H_{24}'\right\}\\
&a_{71}=\left\{\hbar_5 m_1 H_{21}\right\},\; a_{72}=\left\{\hbar_5 m_2 H_{22}\right\},\\
&a_{73}=\left\{\hbar_5 m_3 H_{23}\right\},\; a_{74}=\left\{\hbar_5 m_4 H_{24}\right\},\\
&a_{75}=-\left\{\hbar_5' m_1' H_{21}'\right\},\; a_{76}=-\left\{\hbar_5' m_2' H_{22}'\right\},\\
&a_{77}=-\left\{\hbar_5' m_3' H_{23}'\right\},\; a_{78}=-\left\{\hbar_5' m_4' H_{24}'\right\},\\
&a_{81}=(h-m_1)H_{31},\; a_{82}=(h-m_2)H_{32},\; a_{83}=(h-m_3)H_{33},\; a_{84}=(h-m_4)H_{34},\\
&a_{85}=-(h'-m_1')H_{31}',\; a_{86}=-(h'-m_2')H_{32}',\; a_{87}=-(h'-m_3')H_{33}',\; a_{88}=-(h'-m_4')H_{34}'.
\end{aligned}$$

PARTICULAR CASES

Stoneley waves

Equation (14) is the secular equation for Stonely waves in a fibre reinforced viscoelastic media of higher order. For k = 0, results are similar to Abd-Alla (2003). If rotational, voids and fiber-reinforced parameters are ignored, then for k = 0, the results are same as Stoneley (1924).

Rayleigh waves

Rayleigh wave is a special case of the above general surface wave. In this case we consider a model where the medium, M_1 is replaced by vacuum. Since the boundary, $x_2=0$ is adjacent to vacuum. It is free from surface traction. So the stress boundary condition in this case may be expressed as:

$\tau_{12}=0$, $\tau_{22}=0$ on $x_2=0$, for all x_1 and *t*.

$$\frac{\partial\phi}{\partial x_2}+h\phi=0, \text{ on the plane } x_2=0 , \forall\, x_1 \; and \; t,$$

It is assumed that gravitational field produces a hydrostatic initial stress. It produced by a slow process of creep where the shearing stresses tend to small or vanish after a long period of time. Equilibroim conditions of initial stress are:

$$\frac{\partial\tau_{11}}{\partial x_1}=0, \; \frac{\partial\tau_{11}}{\partial x_2}+\rho g=0$$

Thus above set of equations reduces to:

$$\sum_{n=1}^{4} \hbar_1\left(-m_n + i\omega H_{1n}\right)M_n = 0 ,$$

$$\sum_{n=1}^{4}\left\{i\omega(\hbar_2-\hbar_1) - \hbar_4\left(-i\omega c\right)^k m_n H_{1n} - \beta H_{2n}\right\}M_n = 0,$$

$$\sum_{n=1}^{4}(m_n - h)H_{3n}M_n = 0,$$

$$\sum_{n=1}^{4}\left\{i\omega\hbar_3 - (\hbar_2-\hbar_1)m_n H_{1n}) - \beta H_{3n}\right\}M_n = 0$$

Eliminating the constants M_1, M_2, M_3 and M_4 we get the wave velocity equation for Rayleigh waves in the fibre-reinforced viscoelastic media of order *s* under the influence of gravity as follows:

$$\det(b_{lm}) = 0; \quad l = m = 1,2,3,4. \tag{15}$$

where

$$b_{11} = \hbar_1\left(-m_1 + i\omega H_{11}\right), \quad b_{12} = \hbar_1\left(-m_2 + i\omega H_{12}\right),$$

$$b_{13} = \hbar_1\left(-m_3 + i\omega H_{13}\right), \quad b_{14} = \hbar_1\left(-m_3 + i\omega H_{14}\right),$$

$$b_{21} = \left\{i\omega(\hbar_2-\hbar_1) - \hbar_4 m_1 H_{11} - \beta H_{21}\right\},$$

$$b_{22} = \left\{i\omega(\hbar_2-\hbar_1) - \hbar_4 m_2 H_{12} - \beta H_{22}\right\},$$

$$b_{23} = \left\{i\omega(\hbar_2-\hbar_1) - \hbar_4 m_3 H_{13} - \beta H_{23}\right\},$$

$$b_{24} = \left\{i\omega(\hbar_2-\hbar_1) - \hbar_4\left(-i\omega c\right)^k m_3 H_{14} - \beta H_{24}\right\}$$

$$b_{31} = (m_1 - h)H_{31}, \quad b_{32} = (m_2 - h)H_{32},$$

$$b_{33} = (m_3 - h)H_{33}, \; b_{34} = (m_4 - h)H_{34},$$

$$b_{41} = \left\{i\omega\hbar_3 - (\hbar_2-\hbar_1)m_1 H_{11}) - \beta H_{31}\right\},$$

$$b_{42} = \left\{i\omega\hbar_3 - (\hbar_2-\hbar_1)m_2 H_{12}) - \beta H_{32}\right\},$$

$$b_{43} = \left\{i\omega\hbar_3 - (\hbar_2-\hbar_1)m_3 H_{13}) - \beta H_{33}\right\},$$

$$b_{44} = \left\{i\omega\hbar_3 - (\hbar_2-\hbar_1)m_4 H_{14}) - \beta H_{34}\right\}.$$

Equation (15) is the secular equation for Rayleigh wave for the medium M_1. For k = 0 and by ignoring the voids and gravitational effects our results are same as that of Sengupta and Nath (2001). If one ignores the fibre-reinforced parameters also then results are same as Rayleigh (1885).

NUMERICAL SIMULATION AND DISCUSSION

The following values of elastic constants are considered Chattopadhyay et al. (1987) for mediums M and M_1 respectively.

$\rho = 2660 Kg/m^3$, $\lambda = 5.65\times10^{10}\,Nm^{-2}$, $\mu_T = 2.46\times10^{9}\,Nm^{-2}$, $\mu_L = 5.66\times10^{9}\,Nm^{-2}$,
$\alpha = -1.28\times10^{9}\,Nm^{-2}$, $\beta = 220.90\times10^{9}\,Nm^{-2,}$

$\rho = 7800 Kg/m^3$, $\lambda = 5.65\times10^{9}\,Nm^{-2}$, $\mu_T = 2.46\times10^{10}\,Nm^{-2}$, $\mu_L = 5.66\times10^{10}\,Nm^{-2}$,
$\alpha = -1.28\times10^{10}\,Nm^{-2}$, $\beta = 220.90\times10^{10}\,Nm^{-2,}$

The numerical technique outlined above was used to obtain secular equation, surface waves velocity and attenuation coefficients under the effects of rotation in two models with voids.

For the sake of brevity some computational results are being presented here. The variations are shown in Figures 1 and 2, respectively.

Figure 1a to i show the variation of the magnitude of the frequency equation $|\Delta|$, Stoneley wave velocity $\mathrm{Re}(|\Delta|)$ and attenuation coefficient $\mathrm{Im}(|\Delta|)$ with respect to the frequency ω for different values of order k, gravity field g and phase velocity c. The magnitude of the frequency equation increases with increasing of frequency, while it decreases with increasing of order and gravity field and when effect of phase velocity it increases with increasing of phase velocity, as well, Stoneley wave velocity decreases with increasing of frequency, while it increases with increasing of order and gravity field and when effect of phase velocity, it decreases with increasing of phase velocity and the attenuation coefficient increases with increasing of frequency, except when effect of phase velocity it decreases with increasing of frequency, while it increases with increasing of order , as well it decreases with increasing of gravity field and phase velocity.

Figures 2a to i show the variation of the magnitude of the frequency equation $|\Delta|$, Stoneley wave velocity $\mathrm{Re}(|\Delta|)$ and attenuation coefficient $\mathrm{Im}(|\Delta|)$ with respect to the frequency ω for different values of order k, gravity field g and phase velocity c. The magnitude of the frequency equation increases with increasing of frequency, while it decreases with increasing of order and gravity field and when effect of phase velocity it increases with increasing of phase velocity, as well, Stoneley wave velocity decreases with increasing of frequency, while it increases with increasing of order and gravity field and when effect of phase velocity, it decreases with increasing of phase velocity and the attenuation coefficient increases with increasing of frequency and when effect of phase velocity it increases and decreases gradually with increasing of frequency, while it decreases with increasing of phase velocity.

Finally, one can see that there is a similarity between

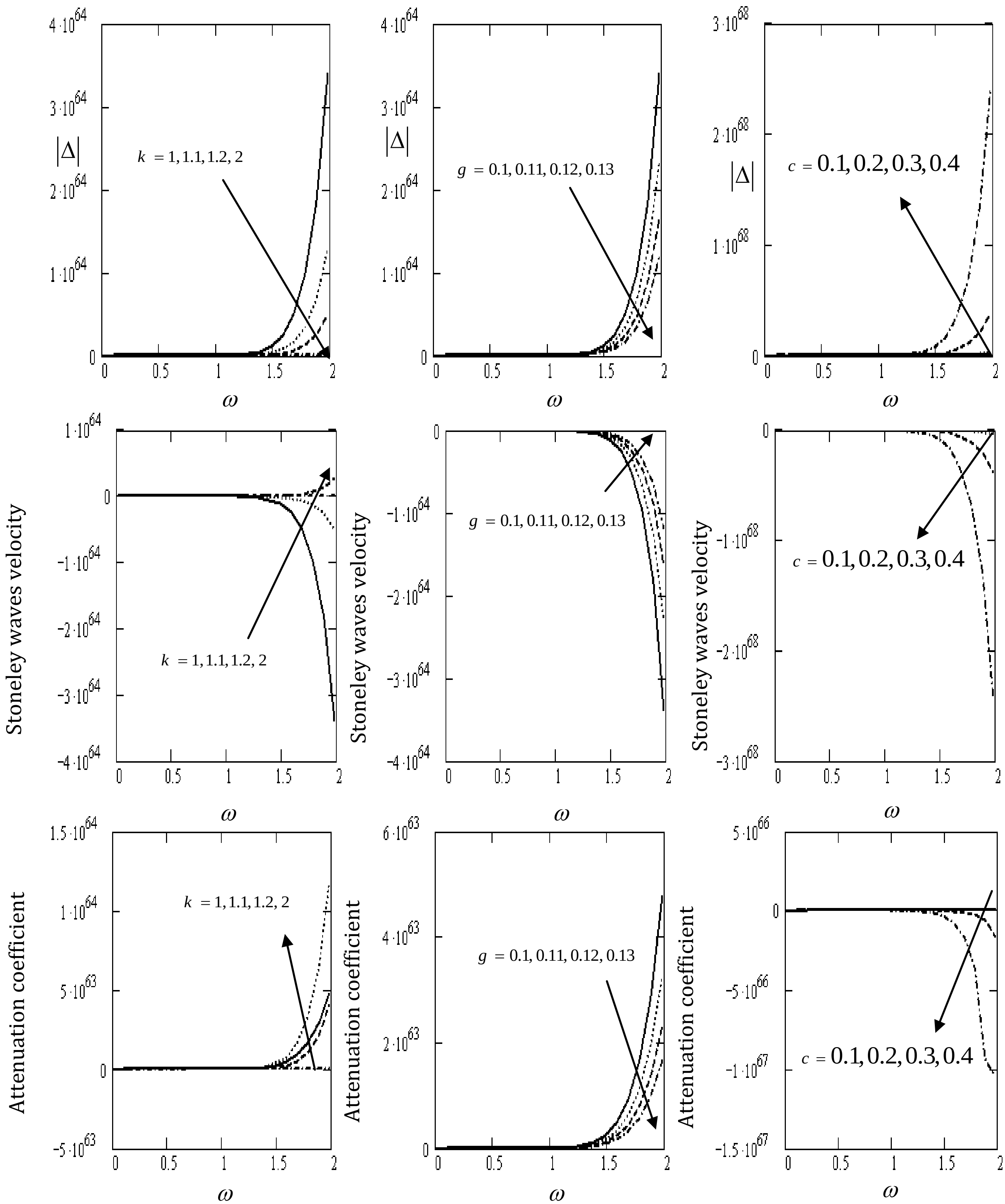

Figure 1. Variation of $|\Delta|$, velocity ($\mathrm{Re}(|\Delta|)$) and attenuation coefficient ($\mathrm{Im}(|\Delta|)$) for Stoneley waves with respect to ω with variation of k, g and c.

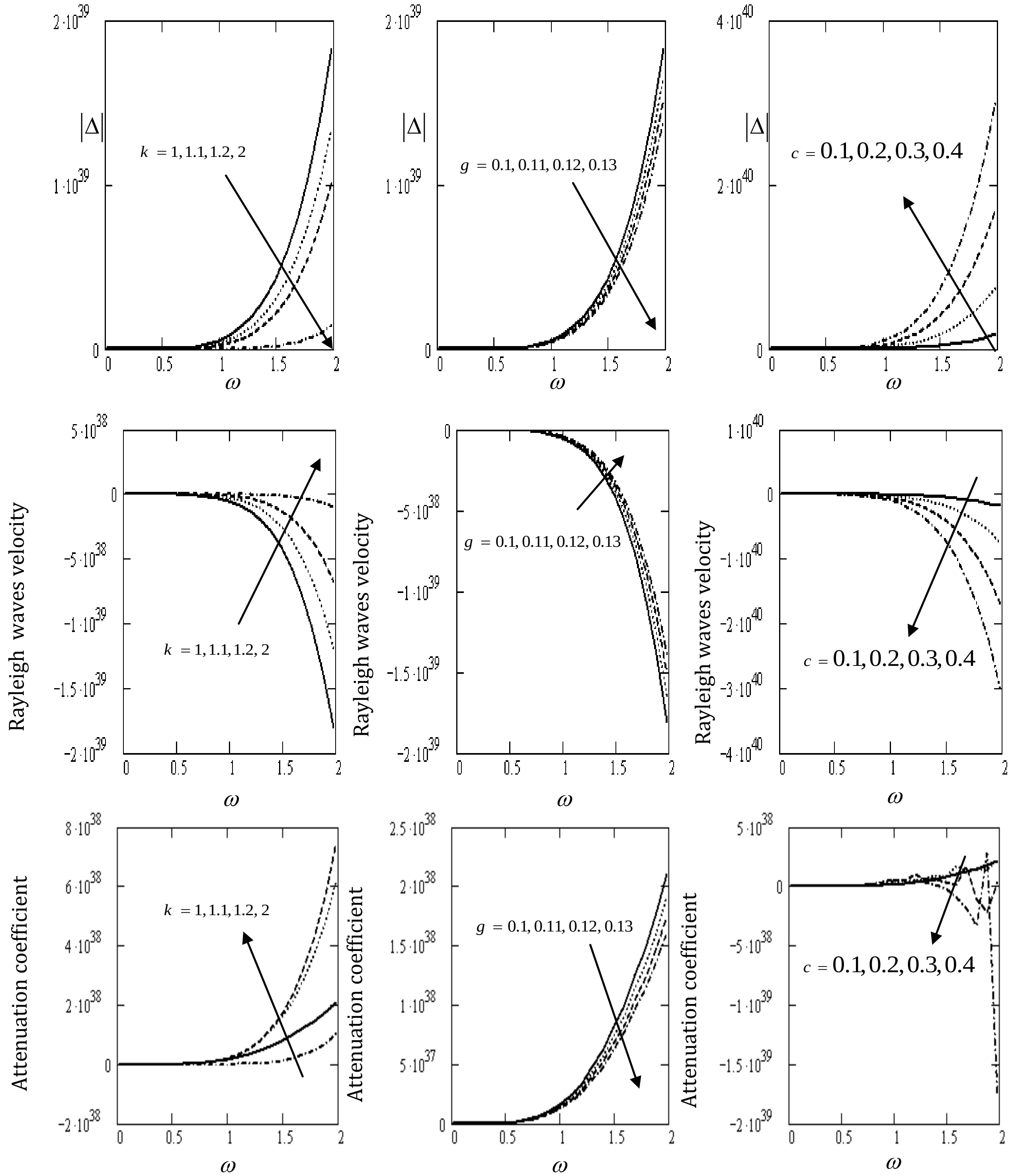

Figure 2. Variation of $|\Delta|$, velocity ($\mathrm{Re}(|\Delta|)$) and attenuation coefficient ($\mathrm{Im}(|\Delta|)$) for Rayleigh waves with respect to ω with variation of k, g and c.

the graphs of two waves types (that is, Stoneley and Rayleigh) in the behavior but there are differences between the values and part of their behavior.

CONCLUSION

Due to the complicated nature of the governing equations of the fibre-reinforced anisotropic general viscoelastic media of higher order with voids, the work done in this field is unfortunately limited in number. The method used in this study provides a quite successful in dealing with such problems. This method gives exact solutions in the fibre-reinforced anisotropic elastic media without any assumed restrictions on the actual physical quantities that appear in the governing equations of the problem considered. Important phenomena are observed in all these computations:

1. It was found that the solutions obtained in the context of the fibre-reinforced anisotropic general viscoelastic media of higher integer and fractional order with voids, however, exhibit the behavior of speeds of wave propagation.
2. By comparing Figures 1 and 2, it is found that the wave velocity has the same behavior in both media. But with the passage of gravity field, numerical values of wave velocity in the viscoelastic media are large in comparison due to the viscoelastic fiber-reinforced.
3. Special cases are considered as Stoneley and Rayleigh waves only.
4. The results presented in this paper should prove useful for researchers in material science, designers of new materials.
5. Study of the phenomenon of rgravity field is also used to improve the conditions of oil extractions. Finally, if the rotation is neglected, the relevant results obtained are deduced to the results obtained by Sengupta and Nath (2001).

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