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Full Length Research Paper

Equations of unsteady flow in curved trapezoidal channels

Alireza Mosalman[1]*, Mohammadreza Mosalman[2] and H. Mosalman Yazdi[2]

[1]Faculty of Engineering, Islamic Azad University, Mehriz Branch, Iran.
[2]Faculty of Engineering, Islamic Azad University, Maybod Branch, Iran.

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Investigation of unsteady flows in curved channels and solving the related equations are one of the main issues in hydraulics. In this study, the numerical model proposed by 'beam and warming' using the finite difference method is presented to solve unsteady flow equations in curved-trapezoidal channels and it has been used by taking advantage of modified K ε Model (regarding some modification upon K and ε adjacent to the wall) and resistance control by satisfaction of Courant–Friedrichs conditions. In this model the effect of modified frictional slope has been simulated. This model was applied to the two hydraulics engineering related issues (flows created as a result of dam failure and flow in a curvature channel of 180 degree). The results of numerical model are compared with the Miller laboratory data, which show a high level of agreement.

Key words: Unsteady flow, hydraulic, trapezoidal channel.

INTRODUCTION

Nowadays, water resources have a significant role on the development of countries, and very important on the economical progresses. In this regard, many researches have made on the open channels, water planning and the serviceability of these resources to improve the water distribution, flood controlling, energy production and controlling sediments.

Studies of unsteady flows were done at first by Saint-Venant while considering prismatic channels. Two or three dimensional models are used in curved and non prismatic channels. Copper and Verden Hill proposed a preliminary mathematical model for solving two dimensional equations of depth average (Molls et al., 1998). The model of depth average was proposed in 1978 by Rodi and Rastogi. Moreover, in most of the proposed models by researchers for investigation of the hydraulic flows, the standard frictional slope is used (Rodi, 1980; Younus and Chaudhry, 1994; Jiang and Li, 2010).

*Corresponding author: E-mail: mosalmanyazdi@yahoo.com.

Turbulence models investigation

Turbulence models are categorized based on the Eddy-viscosity concept. In the simple turbulence models which the Eddy-viscosity are used in, the transformation of production rate and turbulence loss from a point to another one are not considered and thus it will not be a correct way to show it. To consider the significance of turbulence transformation, some models are introduced where the transport equations are used for explanation of turbulence qualities. In these models, all transport equations are used in the differential form for the Reynolds stress $\overline{u_i u_j}$ and scalar qualities such as K and ε . However in some models, in addition to the transport equation for speed scale, another equation is used for the length scale. However, different models based on the used equation can be categorized to different models such as the zero, one and two models (Hanjalic, 1994; Nagano et al., 1997; Ferrey and Aupoix, 2006).

Zero equation model is a model that does not have transport equation for turbulence flow qualities. This turbulent model is based on the Eddy-viscosity concept that was obtained from experiments by trail and error method and some empirical formulas and therefore according to the Eddy-viscosity, this model is completely different with the fixed Eddy-viscosity model, Prandtl free-shear-larger and mixing- length models (Prandtl, 1925; Ferro and Baiamonte, 1994; Jiang and Li, 2010).

One equation model was proposed by Prandtl and Kolmogorov to overcome the limitation of Prandtl mixing length in which the Eddy-viscosity turbulence can be expressed by some algebra equations better than the zero equation model. In this model considering a transport equation for velocity oscillation scale and using mixing length assumptions are so important. Based on their assumptions, Eddy-viscosity is proportional to the velocity oscillation movement and the length scale (Prandtl, 1925; Huai et al., 2009; Samanta et al., 2009).

$$\upsilon_t = \dot{V} \cdot L = C\sqrt{K \cdot L} \quad (1)$$

Where, L is a length scale, the same as mixing which affects the transportation process, it is similar to kinetic energy K. The most important physical scale for oscillation velocities is $\sqrt{K}$ where K is the energy of turbulence flow and states the intensity of turbulence in three directions. Thereby, Prandtl and Kolmogorov obtained the K equation with consideration of aforementioned suppositions (Prandtl, 1925; Hanjalic, 1994; Moryossef and Levy, 2006).

Two equation model does not have the difficulty of one equation model in determining a relationship between υ_t and L as the length scale which can be used for many different flows. In the other words, this model some of equations are used for velocity oscillation scale and length scale. Hence, there are two equation models. As mentioned before, turbulent displacement is not taken into account in the zero models. Consequently, physical influence of flow of previous history is not considered in the simple algebra models. In order to take this physical effect, the transport equation can be written based on Navier-stokes. The most common method for turbulence flow is turbulent flow (K ε). This model introduces two transportation equations for K and ε. Where, K is the turbulence kinetic energy and ε is its relative losses rate of it (Rodi, 1980; Wilson, 2004; Tang and Knight, 2008).

$$K = \frac{1}{2}(\overline{u_i' u_i'}) = \frac{1}{2}[\overline{u'^2} + \overline{v'^2} + \overline{w'^2}] \quad (2)$$

$$\varepsilon = \upsilon_t [\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}] \quad (3)$$

These equations are generally similar to Navier-stokes equations and can be solved with the same method. With some modification using the Reynolds stress and Anishtain sum rule along with continuity equations, the general equation form of the K ε model for the unsteady flows of semi practical transport will be as (Fabian et al., 1975):

$$\frac{\partial K}{\partial t} + \frac{\partial (uK)}{\partial x} + \frac{\partial (vK)}{\partial y} = \frac{\partial}{\partial y}(\frac{\upsilon_t}{\sigma_K}\frac{\partial K}{\partial y}) + \frac{\partial}{\partial x}(\frac{\upsilon_t}{\sigma_K}\frac{\partial K}{\partial x}) + G - \varepsilon \quad (4)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (u\varepsilon)}{\partial x} + \frac{\partial (v\varepsilon)}{\partial y} = \frac{\partial}{\partial y}(\frac{\upsilon_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}) + \frac{\partial}{\partial x}(\frac{\upsilon_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}) + C_{\varepsilon 1}\frac{\varepsilon}{K}G - C_{\varepsilon 2}\frac{\varepsilon^2}{K} \quad (5)$$

$$G = \upsilon_t [2(\frac{\partial v}{\partial y})^2 + 2(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})^2] \quad (6)$$

$$\upsilon_t = C_\mu \cdot \frac{K^2}{\varepsilon} \quad (7)$$

Turbulence method introduces five constant proposed by Launder Spalding as shown in Table 1 (Ferro and Baiamonte, 1994; Hanjalic, 1994; Moryossef and Levy, 2006).

Modified frictional slope equations (S_f)

For solving two dimensional equations, the channel cross section is divided to many small rectangular (Figure 1), then, depth average velocity, cross section area, the wet perimeter of each element is calculated. Following that, the modified frictional slope is calculated by using the equations below. The frictional slope equation in direction s (Molls et al. 1998):

$$S_{fs} = \frac{n^2 v_s \sqrt{v_s^2 + v_n^2}}{C_0^2 (h - ltg\,\alpha)^{4/3}} \cdot \phi_j (1 + \frac{h_1 + h_N}{B_j (N-1)})^{4/3} \quad (8)$$

The frictional slope equation in direction n:

$$S_{fn} = \frac{n^2 v_n \sqrt{v_s^2 + v_n^2}}{C_0^2 (h - ltg\,\alpha)^{4/3}} \cdot \phi_j (1 + \frac{h_1 + h_N}{B_j (N-1)})^{4/3} \quad (9)$$

Effective stress investigation

In the depth average process, some estimations for channel bottom stress, stress due to wind blowing and effective stresses which excludes Reynolds stress, stresses due to the turbulence and momentum dissipation are considered. Effective stresses are imposed to the vertical sides of element as a tangent. These stresses are calculated in accordance with the turbulence viscosity concept. The effective stress is obtained by consideration of the Boussinesy turbulence viscosity concept and taking into account the depth average velocity in it. Now, using the assumptions of Boussinesy, effective stresses equations, continues equation, momentum, average depth also K equation (the distribution of the depth average turbulence kinetic energy) and ε equation (kinetic energy rate of dissipation) are as follows:

Depth averaged continuity equation:

$$\frac{\partial h'}{\partial t} + \frac{1}{(1+\frac{n}{R})}\frac{\partial (v_s h')}{\partial s} + \frac{\partial (v_n h')}{\partial n} + \frac{v_n h'}{R(1+\frac{n}{R})} = 0 \quad (10)$$

Depth averaged momentum equation in s-direction:

$$\frac{\partial}{\partial t}[v_s h'] + \frac{1}{(1+\frac{n}{R})}\frac{\partial}{\partial s}[h' v_s^2] + \frac{\partial}{\partial n}[h' v_n v_s] + \frac{2}{R(1+\frac{n}{R})}[h' v_n v_s] + \frac{g}{(1+\frac{n}{R})}\frac{\partial}{\partial s}[\frac{h'^2}{2}] = \frac{g h'}{(1+\frac{n}{R})}(S_{0s} - S_{fs}) + \frac{1}{\rho}\frac{1}{(1+\frac{n}{R})}\frac{\partial [h'\tau_{ss}]}{\partial s} + \frac{1}{\rho}\frac{\partial [h'\tau_{sn}]}{\partial n} + \frac{1}{\rho}\frac{2}{R(1+\frac{n}{R})}[h'\tau_{sn}] \quad (11)$$

Table 1. Constant values of K ε model.

C_μ	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	σ_K	σ_ε
0.09	1.43	1.92	1.0	1.3

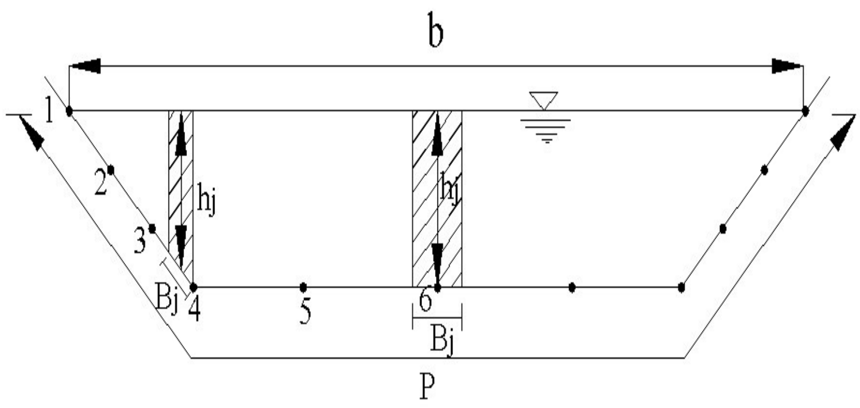

Figure 1. Trapezoidal channel cross section for frictional slope calculation.

Depth averaged momentum equation in n-direction:

$$\frac{\partial}{\partial t}[v_n h' + \frac{1}{(1+\frac{n}{R})}\frac{\partial}{\partial s}[v_s v_n h'] + \frac{\partial}{\partial n}[v_n^2 h'] + \frac{1}{R(1+\frac{n}{R})}[v_n^2 h' - v_s^2 h'] + g\frac{\partial}{\partial n}[\frac{h'^2}{2}] = gh'(S_{0n} - S_{fn}) + \frac{1}{\rho}\frac{1}{(1+\frac{n}{R})}\frac{\partial[h'\tau_{sn}]}{\partial s} + \frac{1}{\rho}\frac{\partial[h'\tau_{nn}]}{\partial n} + \frac{1}{\rho}\frac{1}{R(1+\frac{n}{R})}h'(\tau_{nn} - \tau_{sn}) \quad (12)$$

K equation is:

$$\frac{\partial[h'K]}{\partial t} + \frac{\partial[h'v_s K]}{\partial s} + \frac{\partial[h'v_n K]}{\partial n} = \frac{\partial}{\partial s}[\frac{\upsilon_t}{\sigma_K}\frac{\partial[h'K]}{\partial s}] + \frac{\partial}{\partial n}[\frac{\upsilon_t}{\sigma_K}\frac{\partial[h'K]}{\partial n}] + \frac{\upsilon_t}{h'}\{2[\frac{\partial[h'v_S]}{\partial s}]^2 + 2[\frac{\partial[h'v_n]}{\partial n}]^2 + [\frac{\partial[h'v_s]}{\partial n} + \frac{\partial[h'v_n]}{\partial s}]^2\} + \frac{g}{c^2}(\sqrt{v_s^2 + v_n^2})^3 - \varepsilon h' \quad (13)$$

ε equation is:

$$\frac{\partial[h'\varepsilon]}{\partial t} + \frac{\partial[h'v_s\varepsilon]}{\partial s} + \frac{\partial[h'v_n\varepsilon]}{\partial n} = \frac{\partial}{\partial s}[\frac{\upsilon_t}{\sigma_\varepsilon}\frac{\partial[h'\varepsilon]}{\partial s}] + \frac{\partial}{\partial n}[\frac{\upsilon_t}{\sigma_\varepsilon}\frac{\partial[h'\varepsilon]}{\partial n}] + \frac{\varepsilon C_1}{K}\frac{\upsilon_t}{h'}\{2[\frac{\partial[h'v_s]}{\partial s}]^2 + 2[\frac{\partial[h'v_n]}{\partial n}]^2 + [\frac{\partial[h'v_s]}{\partial n} + \frac{\partial[h'v_n]}{\partial s}]^2\} - \frac{C_2}{K}\varepsilon^2 h' + \frac{C_2 C_\mu^{1/2} g^{5/4}(\sqrt{\bar{v}_n^{2} + \bar{v}_s^{2}})}{h' D^{1/2} C^{5/2}} \quad (14)$$

$$f_h = \frac{h'}{1 + 0.57\frac{K^2}{\varepsilon^2}(\frac{\partial v_s}{\partial n} + \frac{v_s}{R})\frac{v_s}{R}} \quad (15)$$

Where, $C_\mu = 0.09$, $C_1 = 1.44$, $C_2 = 1.92$, $D = 0.1$, $\sigma_K = 1$, $\sigma_\varepsilon = 1.3$ and $h' = h - l \times tg\alpha$. Turbulence viscosity which is calculated by K ε model, cannot be used directly for curved channels because this model does not consider flow line curves. Leschziner and Rodi correction coefficient is used to modify this default. It is noticeable that these coefficients are introduced while considering the flow lines curved to be in the horizontal plane (Cheng and Farokhi, 1992; Younus and Chaudhry, 1994; Ye and Mccorquodale, 1998).

RESULTS AND DISCUSSION

Numerical calculations and experimental models

In this study, the recent model of sensitivity is

Table 2. Test condition at the channel entrance (Miller and Chaudhry, 1989).

Test no.	Depth (cm)	Velocity (m/s)	Froude no.
1	12.19	2.01	1.84
2	10.36	1.70	1.68
3	8.23	1.31	1.46
4	6.7	1.03	1.27
5	5.18	0.75	1.05

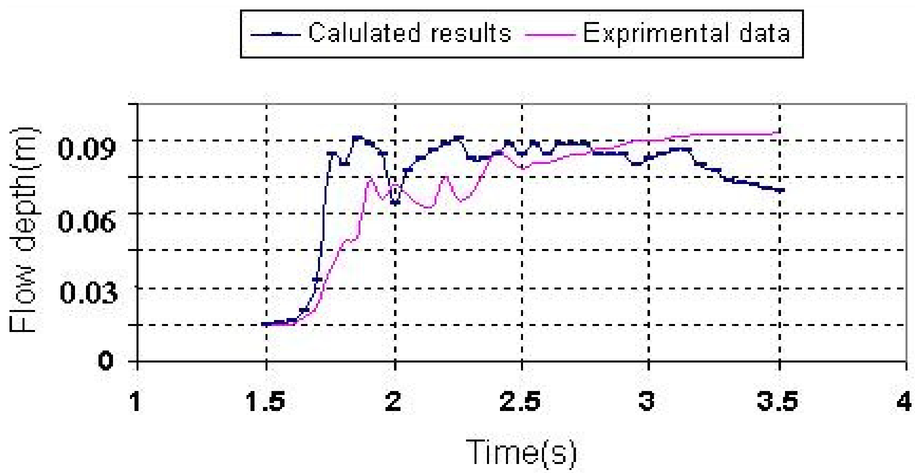

Figure 2. Experimental and calculational results comparison, test 1, station 1.

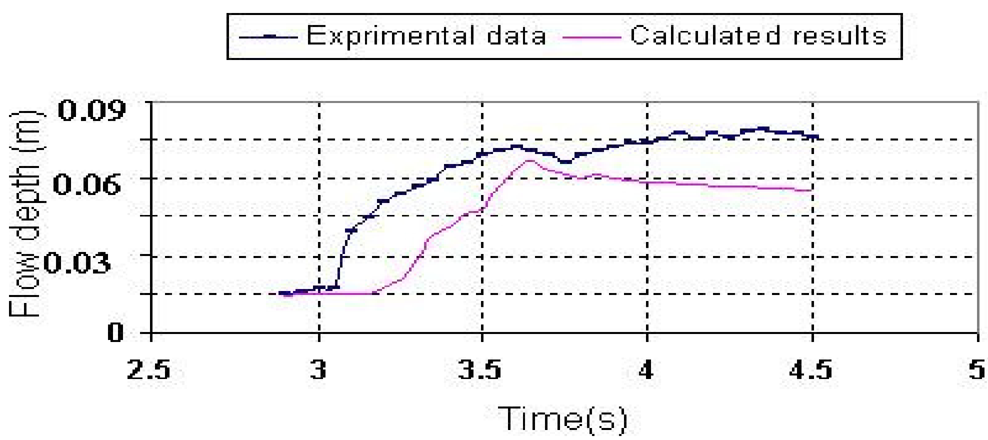

Figure 3. Experimental and calculational results comparison at internal beach, test 1, station 2.

investigated to compute and investigate the flow after dam fracture with experimental data stated by the Miller model. Unsteady conditions with variation of values at upstream are simulated. As shown in Table 2, the flow created at the down stream channel is super critical at most of the times.

The results calculations for one case of the aforementioned cases are shown in Table 2. These results are compared with the experimental data. The calculated wave's heights and their arrival times to three stations which are located at distances of 2.74, 5.16 and 6 m in downstream are compared with the experimental values. The results comparisons of first test at depth of 12.19 cm are shown in Figures 2 to 6. At this station the wave head was one dimensional and the surface of water did not have any variation. As indicated in figures, the

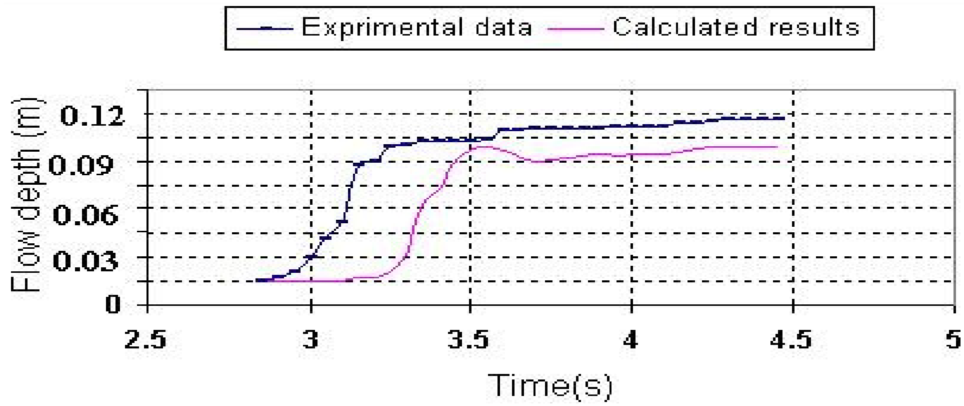

Figure 4. Experimental and calculational results comparison at external beach, test 1, station 2.

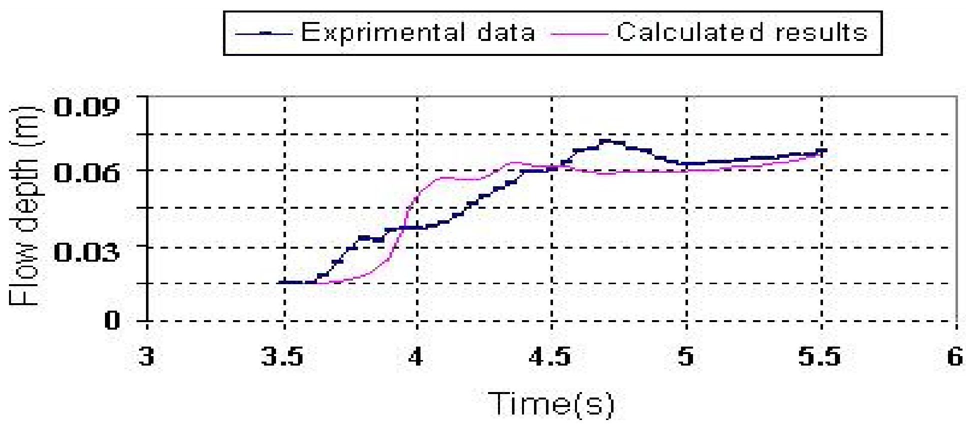

Figure 5. Experimental and calculational results comparison at internal beach, test 1, station 3.

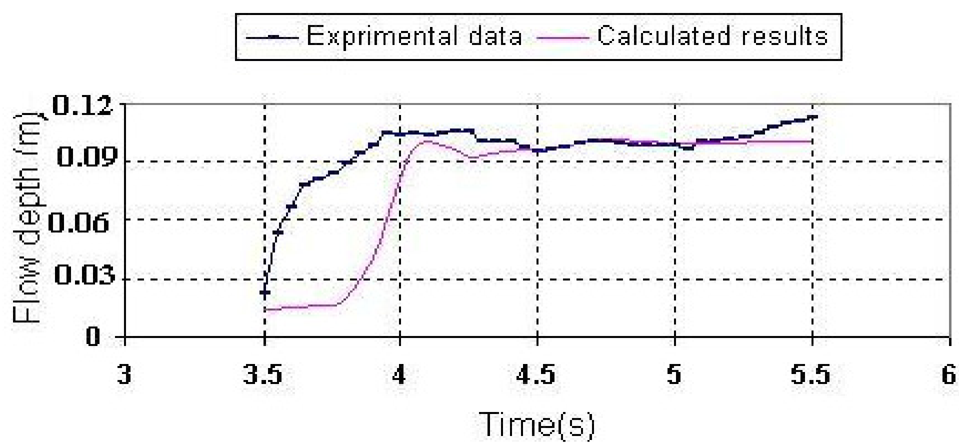

Figure 6. Experimental and calculational results comparison at external beach, test 1, station 3.

calculation data, similar to the experimental values, get to the summit at the same times. Hence, it confirms the mathematical results. In the third station, which is located at a 90 degrees curve, the water height difference between the internal and external curves is more than the second station.

Conclusions

In this paper, a numerical model for investigation of unsteady flow in the curve channel and examining the influence of effective stresses due to turbulence by using modified frictional slope is proposed. Two dimensional equations of depth average are obtained in the channel which the coordinates are fitted based on the laws of motion and conservation of mass. In this model turbulence stresses are modeled based on the Boussinesy assumptions and by the introduction of the $K \ \varepsilon$ model which presented by Chapman and Kuo. In addition the modified frictional slope equation of Thomas Molls is used for frictional slope calculations. The Leschziner and Rodi correction coefficient is used to consider the effect of flow lines on the turbulence viscosity of the $K \ \varepsilon$ standard model. Results comparison of numerical model with experimental data indicates a good correspondence and it also indicates that the turbulence energy variations and losses rate are affiliated to the velocity variations.

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