# Full Length Research Paper

# The modeling and optimization method of bi-level CARP-based express logistics system

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Express logistics problem is a typical optimization problem. Considering the macro-allocation optimization problem and the micro routing scheduling problem, a bi-level CARP-based optimization modeling which the objective is to determine a set of feasible vehicle trips of minimum total cost has been constructed to describe the express logistics problem. Furthermore, an optimization method has been proposed based on the ant colony algorithm (ACA). Case studies shows effectiveness of methods.

**Key words:** Express logistics, bi-level carp, optimization, ant colony algorithm.

# INTRODUCTION

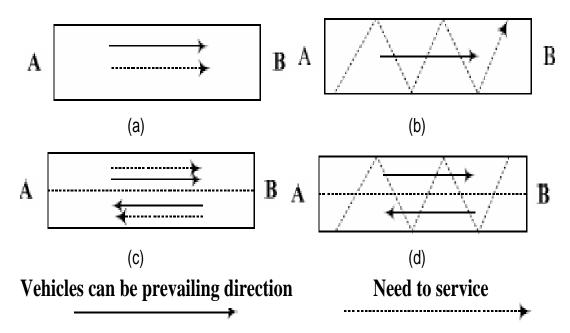
Economic globalization gets express logistics industries quickly develop. As clients are often located separately and they usually change their demand frequently, express logistic companies have to spend lots of resources in goods receiving and sending with low efficiency. This became the weak point of the whole supply chain. Therefore, to get highly efficient management of express logistics, it is crucial to optimize the allocation route, vehicles and depots. Express logistics problem is a typical problem that people know how to optimize their transportation vehicles and routes because express logistics usually faces depots in different locations which are owed by different people, different sorts of goods with varied prices and different sorts of vehicles whose numbers and costs are different. Currently, there are two modeling and optimization methods: one is vehicle routing problem (VRP), such as the goods allocated to supermarkets by allocation center, another one is arc routing problem (ARP), for example, the city council know how to arrange the daily street cleaning route. The CARP has raised a growing interest in the last two decades because of important applications. As mentioned in the

This paper conducts an extended bi-level optimization model based on standard CARP. This model takes two steps to get things done: first, arrange the place and the number of depots and the number of vehicles; secondly, decide the optimized driving route for those vehicles so that all demands can be met in time while keeping transportation costs as low as possible. Recently, more and more scholars have studied applications of the interaction between evolution and learning (Xing et al., 2008a, b, 2010a). These approaches keep useful features of previous individuals to improve the performance of current individuals (Xing et al., 2009).

In fact, such approaches outperform traditional evolutionary algorithms on several benchmarks (for example, flexible job shop scheduling problem, traveling salesman problem, capacitated arc routing problem) (Xing et al., 2010b; Ho et al., 2007; Louis and McDonnell, 2004). In this paper, an improved ant colony algorithm was proposed to the bi-level CARP-based express logistics problems.

literature, CARP arises naturally in various industrial settings such as the planning of mail delivery or school bus services, the routing of street sweepers, waste collection vehicles, gritting trucks or snow plows, and the inspection of gas pipelines, oil pipelines or electric power lines, etc.

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**Figure 1.** Express logistics service mode. a) One way road, one-way service; b) one-way road Z service; c) two way road, two way service and d) two way road, two-way Z service.

# THE METHOD OF BI-LEVEL EXPRESS LOGISTICS MODEL

# Related description

Express company provides lodging service for customers. Suppose that customers are all living at either sides of the streets (roads), considering the traffic rules and different places of those customers, express company can provide its service through four measures:

- (a) If the street (road) is one-way only and customer are all living in one side of the street (road), then the driver can drive one-way to finish the task (Figure 1a);
- (b) If the street (road) is one-way only and customer are living in both sides of the street (road), then the driver can drive one-way but keep crossing the street to finish the task (Figure 1b);
- (c) If the street (road) is two-way only and customers are living in both sides of the street (road), then the driver can drive the street back and forth to finish the task (Figure 1c);
- (d) If the street (road) is two-way only and customers are living in both sides of the street (road), then the driver can drive at either way but keep crossing the street to finish the task (Figure 1d).

The optimization problem for express logistics system can be described in a given directed connected graph; some (or each) arc needs to be serviced. For each arc that needs to be serviced, its demands and service costs are certain. Multiple homogenous vehicles can be used to deal with the service demand (same costs, same load and same speed). In a given period, the same task needs to be carried out repeatedly. The place and the number of storm rooms, the number of vehicles vary, the optimization problem is to keep the total costs (fixed costs and transportation costs) as low as possible by optimizing the driving routs for all vehicles.

#### Definition

G = (V,A): A directed connected graph,  $\ V$  is node set, A is arc set.

d u: deadheading cost when vehicle pass arc  $u \in A$ ,

$$d(u) = \alpha \times Dis(u) \tag{1}$$

Where,  $\alpha$  is deadheading cost conversion coefficient. s u: transportation coasts when vehicle pass  $\operatorname{arc} u \in A$ ,

$$s(u) = \beta \times Dis(u) \times q(u) \tag{2}$$

Where,  $\beta$  is transportation cost conversion coefficient. q(u) is the vehicle load when vehicle pass arc u, Dis(u)

is the length of arc u:

$$Dis(u) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
 (3)

In which,  $(x_1, y_1)$  is the starting point of arc u,  $(x_2, y_2)$  is the ending point,

De : The depot set,  $De = D_1, D_2, \cdots, D_{{\scriptscriptstyle N}1}$ 

 $D_i = (x_i, y_i)$  is the coordinate of depot i,

 $N_1$ : The number of depots,

 $C_{ii}$ : The cost of good j in depot i,

 $S_{ii}$ : The number of good j in depot i,

 $N_2$ : The type of the good,

 $K_{ii}$ : The demand for good j at point i,

 $p_i$ : Number of available vehicles in depot i, so  $P=\{p_1,\ p_2,\ L\ ,\ p_{N1}\ \}$  indicates the distribution of the vehicles in all depots.

 $q_i$ : Using cost of available vehicle in depot i, so  $Q=\{q_1,\ q_2,\ L\ ,\ q_{N1}\ \}$  indicates the using costs for available vehicles in all depots (this can be simplified as a fixed costs).

W: Maximum allowed workload for each vehicle.

# Output

$$R_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij}, f_{ij})$$
 (4)

Where,  $R_{ij}$  is the jth route for vehicle i,  $a_{ij}$  is the starting point for  $R_{ij}$ ,  $b_{ij}$  is the depot in which the goods needs to be carried for  $R_{ij}$ ,  $c_{ij}$  is the ending point for  $R_{ij}$ ,  $d_{ij}$  is the total weight of the carried goods for  $R_{ij}$ ,  $e_{ij}$  is the total costs for carried good for  $R_{ij}$ ,  $f_{ij}$  is the total time to carry good for  $R_{ij}$ .

# **Objectives**

The target is to minimize the total costs (purchasing costs and transportation costs), in other words, is to minimize the total costs for carrying all goods. The total costs can be divided into two parts: purchasing costs and transportation costs. The purchasing costs can be

described as:

$$C_a = \sum_i e_i \tag{5}$$

The transportation costs can be described as:

$$C_{B} = \sum_{q_{i}, \delta_{i}} + \sum_{i} \alpha \times \langle a_{i}b_{i} \rangle + \alpha \times \langle b_{i}c_{i} \rangle + \beta \times \langle b_{i}c_{i} \rangle \times d_{i}$$
 (6)

Where,  $d_i$  is a Boolean variable, indicates whether vehicle i is used for carrying or not.  $\langle a_i \ b_i \rangle$  和  $\langle b_i \ c_i \rangle$  indicate the distance respectively. In total, the objective function of express logistics optimization problem can be described as:

$$Min \quad C_T = C_A + C_B \tag{7}$$

## **Constraints**

There are amount of varying factors when applying express logistics to reality. It is essential to take assumptions to simplify and optimize the model. The assumptions are as follows:

(1) In a single task, the demand allocated to a vehicle should not be beyond the allowed workload of that vehicle. In any case, for each i and j, require:

$$d_{ij} \, \pounds \, W$$
 (8)

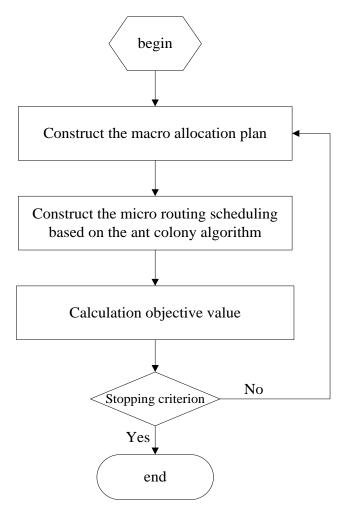
- (2) In a service circle, demand from every arc should be met.
- (3) In a single task, every vehicle takes goods from one depot.
- (4) Once finished the task is completed, every vehicle should turn back to the starting depot.
- (5) The working time for every vehicle cannot go beyond the maximum service time.

#### THE PROPOSED APPROACH

# Framework of bi-level CARP optimization solution

The bi-level express logistics optimization model introduced by this study takes one city or one area as research object. The problems from the following two fields have been taken into consideration:

i) Macro allocation optimization problem, such as how many logistics centers should we set up, where should



**Figure 2.** Solution frameworks of the express logistics optimization problem.

we set up those centers, how many vehicles should we allocate to each center, etc.

ii) Micro route optimization problem, such as, under given conditions, how to insure that express goods can be delivered to customers in time, how to optimize the delivery route to maintain the total delivery costs as low as possible.

To solve this optimization problem efficiently, the paper takes the following framework to construct detailed solution (Figure 2).

# Computation of the least number of depots

Since all required arcs should be serviced within the maximal service time T and vehicle routes must start and end at the same depot, the least number of depots is

decided by the maximum travel distance of trucks, and can be defined as:

$$\min N^* 
\begin{cases}
1 \le N^* \le N_1 \\
dis i \le B/2
\end{cases} 
dis i = \min_{1 \le j \le N_1, d_j = 1} SPL(i, j) 
B = S \times T$$
(9)

Where,  $d_j=1$  denote the node j is selected as a depot, B is the truck's driving distance in the service time T, dis i denote the distance between node i  $1 \le i \le N_1$  and depot (there may be multiple depots), SPL i, j denote the shortest distance between node i and j in the directed connected graph G. The location of depot can be chosen randomly, only need to satisfy the constraint: dis  $i \le B/2$ .

# Micro routing optimization method based on ACA

The current research shows that the standard CARP problem is actually the NP-Hard problem. To solve the micro routing optimization problem, this paper conducts optimization research based on ant colony algorithm (ACA).

# The basic principle of ACA

The ACO framework was introduced by Dorigo et al. (1991) and presents a novel nature-inspired met heuristic that has been used successfully in the past to obtain high-quality solutions to complex optimization problems in a reasonable amount of computational time. The transfer strategy in ACA as:

$$p_{ij}^{k} \ t = \begin{cases} \frac{\left[\tau_{ij} \ t \right]^{\alpha} \square \left[\eta_{ij} \ t \right]^{\beta}}{\sum_{s \subseteq allowed_{k}} \left[\tau_{is} \ t \right]^{\alpha} \square \left[\eta_{is} \ t \right]^{\beta}} & if \ j \in allowed_{k} \\ 0 & otherwise \end{cases}$$
(10)

Where,  $allowed_k = C - tabu_k$  denote the city allowed to select by k th ant in the next step,  $\alpha$  denote the relative importance of track,  $\beta$  denote the relative importance of

visibility,  $\eta_{ij}$  t is heuristic function, can be defined as:

$$\eta_{is} \quad t = \frac{1}{d_{ij}} \tag{11}$$

Where  $d_{ij}$  denote the distance between two cities, pheromone updating rule is:

$$\tau_{ii} t + n = 1 - \rho \quad \Box \tau_{ii} t + \Delta \tau_{ii} t \tag{12}$$

$$\Delta \tau_{ij} \ t = \sum_{k=1}^{m} \Delta \tau_{ij}^{k} \ t$$

Which,  $\rho$  is pheromone evaporation coefficient,  $\Delta \tau_{ij}$  t denote the pheromone incremental at routing i, j in this circulate, at the begin time  $\Delta \tau_{ij}$   $\theta = \theta$ .  $\Delta \tau_{ij}^k$  t denote the pheromone by k th ant leave over at routing i, j in this circulate:

$$\Delta \tau_{ij}^{k} \ t = \begin{cases} \frac{Q}{L_{k}} & \text{if the kth ant only leave in this loop.} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Where, Q denote pheromone strength.  $L_k$  is the travel length of k th ant in the circles.

# **Detail step**

Step 1: Iterative counter  $N_C = 1$ .

Step 2: Distribute m ant uniform to n depot set.

Step 2.1: Depot set i = 1.

Step 2.2: Ant number k=1.

Step 2.3: Select a depot randomly from depot set i.

Step 2.4: Selecting an arc has not been served from the feasible set, calculate the travel distance.

Step 2.5: If have service time longer than the maximum service time, return to step 2.3.

Step 2.6: Else, calculate the transition probability, then update the taboo set and feasible set.

Step 2.7: If feasible set is not null, return to step 2.3.

Step 2.8: Let k = k + 1.

Step 2.9: If k < m, return to step 2.3.

Step 2.10: Else, let i = i + 1.

Step 2.11: If i < n, return to step 2.2.

Step 3: Record the shortest routing in this iteration. Update the pheromone strength.

Step 4: If  $Nc < N \max$ , return to step 2.

Step 5: End.

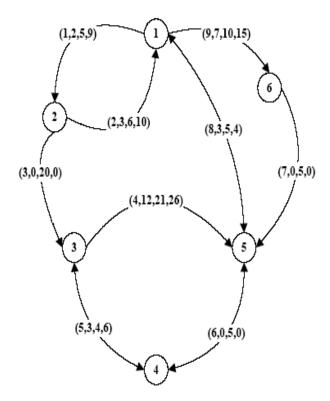


Figure 3. Network of bi-level CARP problem.

# **Case studies**

A simply case of express logistics bi-level optimization problem is shown in Figure 3. There are six nodes and nine arcs in this case. The symbol in every arc denote as arc serial number, deadheading cost, transportation cost, demand respectively. If symbol of demand in one arc equal to zero, it is meaning that the arc has no demand. In this case, it is supposed that the construct cost of one depot is 10,000. The purchase cost, workload and speed of vehicle are equal to 2000, 15 kg and 100 km/h respectively. The maximum service time is given as 0.5 h. All tasks will be carried out 100 times repeatedly. The transportation cost conversion coefficient  $\beta$  is given as 1.6. The initial parameter selection of ACA can referred to the literature. The other initial parameters can be given: number m = 20 , the maximum iterative number  $N_{\rm max} = 50$  . According to the method in this study, we can obtain the optimization result of this case show as:

(1) The depot number is equal to 2, the location of depots is node 1 and 3 respectively. So we have De=1,0,1,0,0;

(2) The vehicle number is equal to 3, the distribution of vehicles in all depots is  $P = \{2, 1\}$ ;

Table 1. Optimization result of this case.

Vehicle routing	Service ID	Deadheading cost	Transportation cost	Driving time	Total demand
$l_1 = 1, 2$	$f_1 = 1,1$	11	50	0.176	5
$l_2 = 9,7,8$	$f_2 = 1,0,1$	20	39	0.32	10
$l_3 = 4,6,5$	$f_3 = 1,0,1$	30	62	0.48	15

- (3) The feasible routing set is shown as Table 1;
- (4) The minimum cost which is the objective of optimization problem in this case is 35700.

# **CONCLUSIONS**

The main contributions of this study may be summarized as follows: (1) describes the express logistics business process, (2) construct the bi-level CARD-based optimization modeling, (3) propose the solving methods for optimization modeling based on the ACA, (4) case study and verification. There are many aspects that we intend to address in the near future. For example, we would like to construct an improved heuristic algorithm to save the compute time, etc. We believe that it would be beneficial to apply this method to the express logistic application.

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