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An efficient Markov model for reliability analysis of predictive hybrid m-out-of-n systems

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In this paper, predictive hybrid redundancy has been extended to large-scale control systems comprising n modules. In m-out-of-n systems, if m-out-of-n modules are in agreement, the system can report consensus; otherwise the system fails. While in our new extension, if there is no agreement, a history record of previous successful result(s) is used to predict the output. In order to analyze the reliability of this system, we present a Markov model based on which the reliability has been computed and compared with m-out-of-n redundancy. The results of simulation demonstrated that the new redundancy improves overall system reliability in all examined scenarios, especially when the number m is large.

Key words: Fault-tolerant systems, redundancy, m-out-of-n systems, voting algorithm, reliability.

INTRODUCTION

Systems used in many critical applications may show erratic behavior and possible hazardous consequences due to inherent faults. Economic, social and moral pressures are driving the need to reduce such hazards in systems behavior (Latif-Shabgahi et al., 2003). This is particularly true for process and motor control (Gaeid and Ping, 2011) in industries where poisonous, flammable or explosive materials are used, in transport (for example, railway systems, avionics, automobile and X-by-wire systems), nuclear power plants and military applications (Woo and Kook, 2000; Crosby, 2007; Xiao-Jun et al., 2007; Yamasaki and Shibata, 2007; Changki and Medioni, 2008; Risser et al., 2008; Harangi et al., 2010; Narayanaswamy et al., 2010). In such applications, redundancy is a major approach for improving or sustaining the normal behavior of a system in an environment where it may not be possible to eliminate the faults entirely (Johnson, 1989; Ezhilchelvan et al., 2004; Kim et al., 2008; Mu and Systems QUoTSoE, 2008; Zarafshan et al., 2010).

Redundancy includes the addition of some information, time and hardware or software resources to a particular system for providing beyond what is required for its normal function. As the most appropriate form of hardware redundancy in control systems, replication of hardware modules can be in forms of passive (static), active (dynamic) and hybrid.

Hiding fault occurrence and preventing faults for leading to errors is the aim in passive techniques in order to achieve fault tolerance. In the most basic form, there is no need for fault detection and system reconfiguration for such techniques and the occurrence of faults is simply masked from the output of the system. In order to mask the occurrence of faults, passive hardware redundancy hinges on voting mechanisms (Johnson, 1989). N-modular redundancy (NMR), recommended as a method of providing fault masking in hardware systems, is the commonest form of passive redundancy. In this method, $n > 2$ redundant hardware modules are employed satisfying a common specification (Goseva-Popstojanova and Grnarov, 1991) and the occurrence of fault(s) is hidden from the system output by a voting unit. The redundant modules perform a same function on the same input data in parallel and deliver their results to the voter. Subsequently, the voter arbitrates between the achieved results and yields a single output. A voting algorithm performs this arbitration process in which the result of the

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Abbreviations: NMR, N-modular redundancy; MTTF, mean time to failure; TMR, triple modular redundancy.

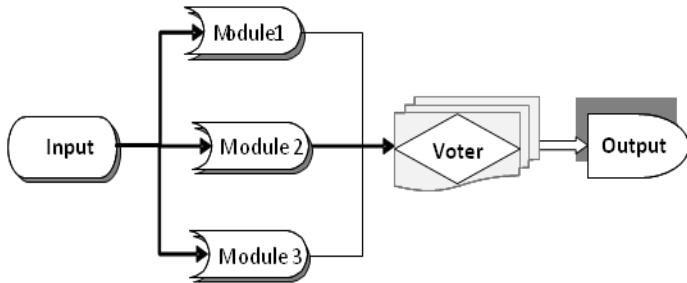


Figure 1. TMR System.

majority is usually believed to be correct output used by the system (Johnson, 1989). The general form of these systems is known as m-out-of-n systems in which m out of n modules should produce same values so that the voter makes an output. In Figure 1, the architecture of triple modular redundancy (TMR), in other words a 2-out-of-3 system, is illustrated.

The second form of redundancy is active approach, which tries to identify faulty module(s) and carry out a number of actions to remove faulty element(s) from the system. In other words, this technique needs system reconfiguration for tolerating faults.

The last form of hardware redundancy is hybrid approach where the attractive benefits of both passive and active techniques are tried to be employed for tolerating faults. Such method is definitely more expensive than the other approaches and is required to avoid momentary errors particularly in safety-critical control systems where obtaining high reliability is necessary (Johnson, 1989). Whereas fault masking is executed to avoid the generation of erroneous results in the system, the mechanisms for fault detection, fault location and fault recovery are performed in hybrid approach to reconfigure the system in case of fault occurrence (Johnson, 1989).

Predictive hybrid redundancy (Kim et al., 2008) is of the several hybrid redundancy techniques presented in literature. In this paper, this technique is extended to m-out-of-n control systems and a performance model of the systems implemented by this approach is presented. The new extension is called as Predictive Hybrid m-out-of-n system and henceforth will be briefly referred to as PHmn.

To investigate the reliability, one famous approach is Markov modeling in which the probabilistic behavior of one system state depends on the previous one. This type of behavior is nonhereditary or memoryless. Problems are significantly simplified through the inclusion of the Markovian property regarding the fact that the past knowledge is decoupled from the future by the knowledge of the present (Ramakumar, 1996; Stabelberg, 2009). Based on this approach, behavior of some physical systems and the system used in this paper are categorized. In this research, we investigate the reliability

of PHmn based on Markov modeling and present the results of the analysis in different scenarios by the use of simulations.

RELATED WORKS

Hybrid voting

Hybrid voting algorithms which incorporate Prediction (Acton, 1970; Bass, 1995; Latif-Shabgahi et al., 2004; Karimi et al., 2009; Karimi et al., 2010; Zarafshan et al., 2010) and Smoothing (Latif-Shabgahi et al., 2003; Karimi et al., 2009) have been presented for cyclic control systems where there are some relationship between the result in one cycle and the result in the next (Latif-Shabgahi et al., 2004). They are actually the extended forms of majority voter (Lorcak et al., 1989; Karimi et al., 2009; 2010) and their function is based on a two-phase algorithm; if during the first phase, majority result cannot be attained, in the second phase, an acceptance test is consequently performed to find a probably correct voting result. If both phases are unable to yield an output, the voter will break down. The activity of finding proper voter output in second phase in both approaches is based on some calculations on voter history records.

If there is no agreement among the results of redundant variants in the smoothing voter (Latif-Shabgahi et al., 2003), the credible output of current voting cycle will be chosen from the closest value to the latest voter successful result. However, if the measured distance is less than smoothing threshold which is a pre-defined value, that probable result is subsequently chosen as the voter output; or else, the voter fails to produce answer.

To produce the expected result of the current voting cycle in predictive voters (Acton, 1970; Bass, 1995; Latif-Shabgahi et al., 2004), a history record of preceding results are utilized in case of detecting no agreement among the voter inputs. To do so, this result is compared with each voter inputs. Then, the input of each voter having a distance from the expected result less than a predictive threshold, which is actually a predefined value, will be chosen as the voter output; otherwise, the decision making will be failed by the voter. In order to implement this voter, various prediction methods were employed including the first order, second order and third order predictive voter. Figure 2 demonstrates the general flowchart of hybrid voting algorithms incorporating Prediction and Smoothing.

Hybrid voting algorithms can be implemented in both hardware and software systems; for example predictive hybrid redundancy in Kim et al. (2008) for X-by-wire systems is a hardware implementation of predictive hybrid voters, whereas the software implementation and Analysis of predictive Hybrid voters can be found in Latif-Shabgahi et al. (2003) and Latif-Shabgahi et al. (2001).

Reliability analysis

Reliability implies the probability that the system functions correctly during a complete interval of time (Johnson, 1989; Blischke and Murthy, 2000; Kuo and Zuo, 2003; Stabelberg, 2009). In addition to availability, safety, and maintainability, reliability is conceivably the most significant characteristic of a system and is considered as a proper measure for comparing the performance of different control systems. Combinatorial modeling and Markov modeling as the commonest analytical approaches are the most popular techniques for reliability analysis.

Combinatorial approaches (Johnson, 1989; Stabelberg, 2009; Radwan et al., 2011) are employed to calculate the probability of a system remaining in the operational mode(s) using different

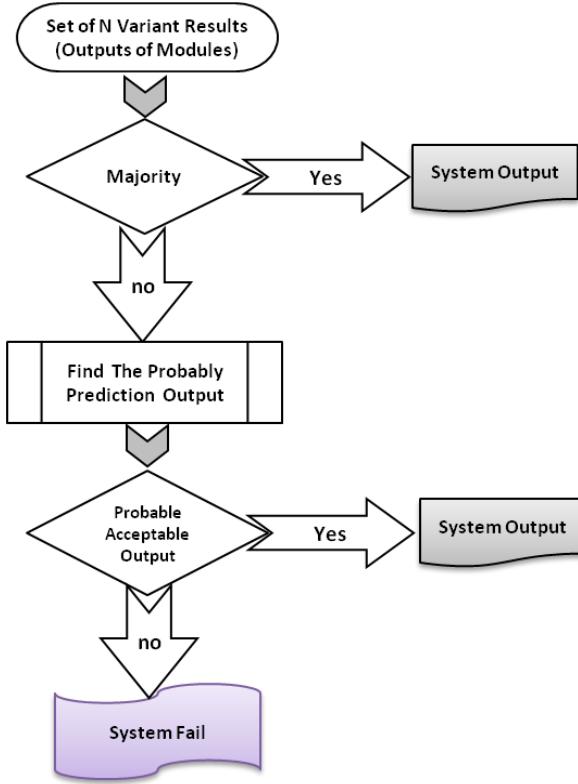


Figure 2. Flowchart of hybrid predictive voting algorithms.

methods of probabilistic techniques. The probabilities of the events making a system operate, are enumerated to obtain an estimate of system reliability. However, not many systems can be modeled simply by combinatorial methods, which is a major problem by itself. In addition, reliability expressions are very complicated and modeling the repair process occurring in many systems is not easy. Hence, we often use Markov modeling (Johnson, 1989; Mu and Systems QUoTSoE, 2008).

A special type of stochastic process whose future probabilistic behavior is uniquely determined by its present state has been firstly introduced by Russian mathematician A. Markov in 1907 (Ramakumar 1996). Markov processes are stochastic because they develop in time in a manner controlled by probabilistic laws, so they can be either discrete in time space (which is known as Markov chain) or continuous Markov model (Ramakumar, 1996; Stapelberg, 2009).

For computing the reliability of m-out-of-n systems either with repair or without repair, a variety of approaches have been proposed in literature. While some studies are focused on new algorithms or formulas for computing the reliability (Rai et al., 1987; Behr and Camarinopoulos, 1997; Demir, 2009; Stapelberg, 2009; Habib et al., 2010; Karimi et al., 2010; Moghaddass and Zuo, 2011) others have focused on reducing time complexity of earlier algorithms (IB, 1995; Min-Sheng, 2004) or have considered more aspects in their reliability analysis (Jer-Shyan and Rong-Jaye, 1994; Krishnamoorthy and Rekha, 2001).

In this paper, predictive hybrid redundancy has been extended to large-scale control systems comprising n modules. Some preliminaries on the reliability analysis of predictive hybrid redundancy have already been presented in (Karimi et al., 2010). In order to analyze the reliability behavior of PHmn, we utilize Markov model with continuous states.

System description

Our control system comprises n units working in parallel and a voter, as presented in Figure 3. All units are identical and fault of a unit cannot have an effect on the others. Once a unit fails, it is sent for repair.

A two-phase decision is performed by the voter generating outputs of identical hardware modules. In first phase, m-out-of-n voting is performed which operates successfully if and only if at least m units are good and functioning. This voter simply does the majority voting if $m \geq \lfloor (n+1)/2 \rfloor$. If the number of faulty modules exceeds n-m, the switch selects the suitable module output based on the result of process in prediction control unit in which an input value is selected as the voter output based on the voter history records and pre-defined thresholds. However, decision-making is not possible and system fails if the prediction phase does not succeed.

System reliability analysis

By definition, reliability of a system is a function of time. While $R(t)$ is described as the conditional probability that the system performs correctly during the interval time, $[t_0, t]$, unreliability is the conditional probability that the system begins to perform incorrectly for the period of the same interval time, as long as the system was performing correctly at time t_0 (Johnson, 1989). One popular way to compute the reliability is the Markov modeling a system into several discrete states, s_0, s_1, \dots, s_n , in which the probabilistic behavior of state j depends on state i , ($i \neq j$ and $i < j$).

Definition 1: P_{ij} is the rate of departure from state i to state j . In state i , the only possible transitions can be from state $i \rightarrow i+1$ for $i=0, 1, 2, \dots, n-m$, P_r , F with a departure rate of $P_{ij} = \lambda_i$ and from state $i \rightarrow i-1$ for $i=1, 2, \dots, n-m+1$, departure rate of $P_{ij} = \mu_i$, where:

$$\lambda_i = \begin{cases} (n-i)\lambda & \text{if } 0 \leq i \leq n-m \\ \lambda_{Pr} & \text{for } i \in Pr \text{ state} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$\mu_i = \begin{cases} i\mu & \text{if } 1 \leq i \leq n-m \\ \mu_{Pr} & \text{for } i \in Pr \text{ state} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Definition 2: $\pi_i(t)$ denotes the probability that the system is in state i at time t . Therefore, the probability of finding the system in state i at time $(t + \Delta t)$ is;

$$\pi_i(t + \Delta t) = \sum_{\substack{j=1 \\ i \neq j}}^n P_{ji} \Delta t \pi_j(t) + \left[1 - \sum_{\substack{j=1 \\ i \neq j}}^n P_{ij} \Delta t \right] \pi_i(t) \quad (3)$$

Rearranging and dividing by Δt , then taking the limit as $\Delta t \rightarrow 0$ lead to

$$\pi'_i(t) = \sum_{\substack{j=1 \\ i \neq j}}^n P_{ji} \pi_j(t) - \pi_i(t) \sum_{\substack{j=1 \\ i \neq j}}^n P_{ij}. \quad (4)$$

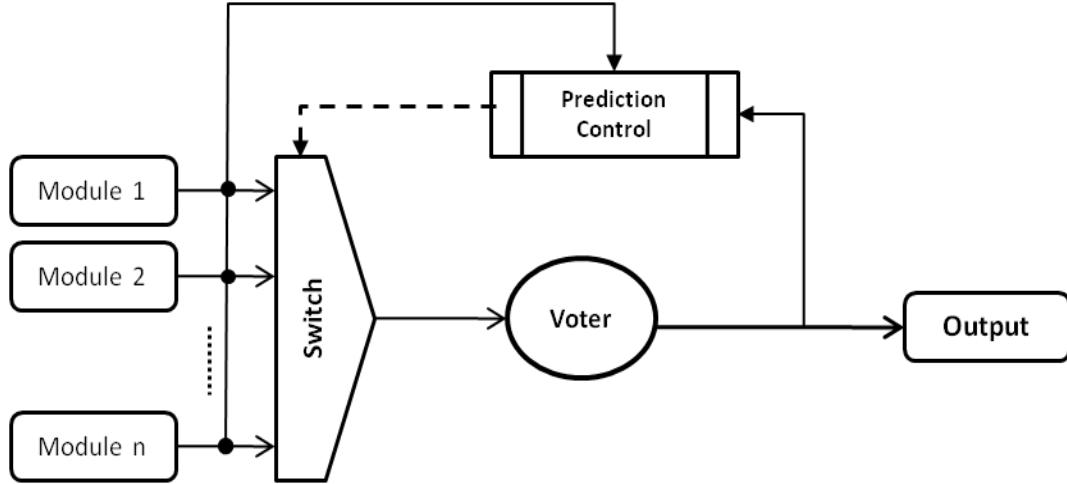


Figure 3. Structure of PHmn.

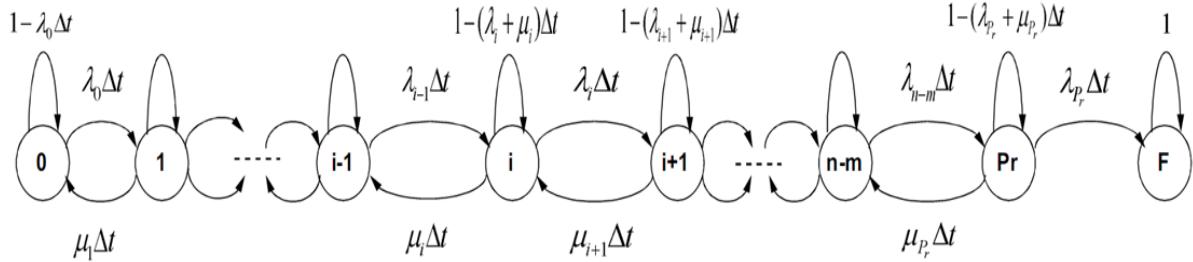


Figure 4. Markov reliability model of predictive hybrid m-out of-n systems.

Based on the system description, the system state transition diagram given in Figure 4 displays the Markov reliability model of PHmn. The model (Figure 4) consists of two types of states including operating (the states 0..n-m and Pr) and failed (the state F). The system function is performed in operating states (Villemeur, 1992; Moghaddass and Zuo, 2011) and its output is considered to be correct. However, there may be some faulty units among the states labeled with zero to m. If exactly i modules are down, the system is considered to be in state i . Thus, state zero refers to the fully operational state. It is assumed that the system is initially in this state.

Considering the system in state 0 and a failure occurs, system switches from state 0 to state 1 with failure rate λ_0 (label 1 denotes the occurrence of one failure in the system). If this fault is repaired, the system returns to the previous state with repair rate μ_1 . This process similarly happens for next states so far as state $n-m$. If $n-m+1$ failure occur, that is, the first phase fails, system goes to state Pr with failure rate λ_{n-m} (state Pr refers to the second phase or the prediction phase of the algorithm). If prediction is possible, the system returns to its latest safe state, that is, state $n-m$, with repair rate μ_{pr} ; otherwise the system will fail. It is assumed that units have exponential failure rates and repair rates. Moreover, two transitions are not allowed at the same time.

Based upon the relations of state i with its neighbors we have:

$$\pi'_i(t) = -(\lambda_i + \mu_i)\pi_i(t) + \lambda_{i-1}\pi_{i-1}(t) + \mu_{i+1}\pi_{i+1}(t). \quad (5)$$

For $1 \leq i \leq n-m$. In the other words, we can extract the following set of equations:

$$\pi'_0(t) = -n\lambda\pi_0(t) + \mu\pi_1(t), \quad (6)$$

$$\begin{aligned} \pi'_i(t) = & -((n-i)\lambda + i\mu)\pi_i(t) + (n-i+1)\lambda\pi_{i-1}(t) \\ & + (i+1)\mu\pi_{i+1}(t) \quad : \text{for } 1 \leq i \leq n-m, \end{aligned} \quad (7)$$

$$\pi'_F(t) = \lambda_{pr}\pi_{pr}(t), \quad (8)$$

$$\pi_F(t) = \lambda_{pr} \int_0^t \pi_{pr}(z) dz \quad (9)$$

$$\pi_0(t) + \pi_1(t) + \dots + \pi_{n-m}(t) + \pi_{pr}(t) + \pi_F(t) = 1; \text{ for } t \geq 0. \quad (10)$$

Since the system is working in all states except F , the reliability is

defined as:

$$R(t) = 1 - \pi_F(t) \quad (11)$$

From definition 2, $\pi'_F(t)$ is calculated as follows:

$$\pi'_F(t) = \lambda_{Pr} \pi_{Pr}(t). \quad (12)$$

$\pi_F(t)$ can be calculated by two ways, either by using Equation (13):

$$\pi_F(t) = \lambda_{Pr} \int_0^t \pi_{Pr}(z) dz. \quad (13)$$

or taking Laplace transform of Equation (12):

$$\pi_F(t) = \mathcal{L}^{-1} \pi_F(s). \quad (14)$$

By taking Laplace transform of Equations (6-11),

$$(s + \lambda_0) \pi_0(s) - \mu_1 \pi_1(s) = 1. \quad (15)$$

For $1 \leq i \leq n-m$,

$$(s + (\lambda_i + \mu_i)) \pi_i(s) - \lambda_{i-1} \pi_{i-1}(s) - \mu_{i+1} \pi_{i+1}(s) = 0, \quad (16)$$

And for predictive state it is obtained that:

$$(s + (\lambda_{Pr} + \mu_{Pr})) \pi_{Pr}(s) - \lambda_{n-m} \pi_{n-m}(s) = 0, \quad (17)$$

Where π_{n-m} is extracted by replacing $i=n-m$ in Equation (16).

From Figure 4, it is simply concluded that:

$$s \pi_F(s) - \lambda_{Pr} \pi_{Pr}(s) = 0$$

and from Equation (10) for all states:

$$s(\pi_0(s) + \pi_1(s) + \dots + \pi_{n-m}(s) + \pi_{Pr}(s) + \pi_F(s)) = 1. \quad (19)$$

Afore mentioned Laplace equations are easily solved if matrix notion is utilized. Equation (20) is the rewritten form of these equations,

$$CP(s) = P0$$

Where,

$$C = \begin{bmatrix} s+\lambda_0 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_0 & s+\lambda_1+\mu_1 & \mu_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 & s+\lambda_2+\mu_2 & \mu_3 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \lambda_{n-m} & s+\lambda_{Pr}+\mu_{Pr} \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & \lambda_{Pr} \end{bmatrix}_{l \times l}, \quad \text{for } l=n-m+2 \quad (20)$$

$$P(s) = \begin{bmatrix} \pi_0(s) \\ \pi_1(s) \\ \vdots \\ \pi_{n-m}(s) \\ \pi_{Pr}(s) \\ \pi_F(s) \end{bmatrix}_{(n-m+2) \times 1} \quad P0 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}_{(n-m+2) \times 1} \quad (21)$$

Matrix C obtained from transition matrix of Markov state diagram (matrix D). For more details please see Example 1.

To obtain the reliability of the system, Laplace transform of Equation (11) is presented in Equation (22).

$$R(s) = 1 - \pi_F(s). \quad (22)$$

$R(s)$ depends on the value $\pi_F(s)$ obtained from Equation (18), however $\pi_{Pr}(s)$ should be afore obtained. It is possible to calculate $\pi_{Pr}(s)$ by Equation (23):

$$\pi_{Pr}(s) = \frac{\Delta_{Pr}(s)}{\Delta_F(s)} \quad (23)$$

$\Delta_F(s)$ and $\Delta_{Pr}(s)$ respectively denote the determinant of the matrix C and the determinant of the matrix obtained from C by replacing the last column of C with the vector $P0$. It can be easily shown that:

$$\Delta_{Pr}(s) = \prod_{i=0}^{n-m} \lambda_i \quad (24)$$

Since $\Delta_F(s)$ is a polynomial in s of order $(n-m+2)$ and with a leading coefficient of 1,

$$(\Delta_{Pr}(s))^{-1} = \left(\prod_{i=1}^{n-m+2} (s - s_i) \right)^{-1} \quad (18) \quad (25)$$

This equation might be rewritten in for m of Equation (26), once partial fraction technique was applied.

$$(\Delta_{Pr}(s))^{-1} = \sum_{j=1}^{n-m+2} \left[\prod_{\substack{i=1 \\ i \neq j}}^{n-m+2} (s_j - s_i) \right]^{-1} \frac{1}{s - s_j} \quad (26)$$

Substituting (24) and (26) in (23) and taking inverse Laplace transform yield to:

$$\pi_{Pr}(t) = \prod_{i=0}^{n-m} \lambda_i \sum_{j=1}^{n-m+2} \left[\prod_{\substack{i=1 \\ i \neq j}}^{n-m+2} (s_j - s_i) \right]^{-1} e^{s_j t} \quad (27)$$

Based on Equations (13) and (27), $R(t)$ is considered as;

$$\begin{aligned}
R(t) &= 1 - \lambda_{Pr} \prod_{i=0}^{n-m} \lambda_i \sum_{j=1}^{n-m+2} \frac{e^{s_j t} - 1}{s_j \prod_{\substack{i=1 \\ i \neq j}}^{n-m+2} (s_j - s_i)} \\
&= 1 - \frac{n! \lambda_{Pr} \lambda^{n-m+1}}{m!} \sum_{j=1}^{n-m+2} \frac{e^{s_j t} - 1}{s_j \prod_{\substack{i=1 \\ i \neq j}}^{n-m+2} (s_j - s_i)}
\end{aligned} \tag{28}$$

In order to compute the system's mean time to failure (MTTF), as a solution, matrix M is needed to be achieved from Equation (29) in which matrix Q is the result of eliminating the rows and the columns corresponding to the absorbing state, that is, state F, from matrix D (the transition matrix of the Markov state diagram).

$$M = [I - Q]^{-1} \tag{29}$$

However, in this paper another solution is preferred in which:

$$MTTF = \int_0^{\infty} R(t) dt = \sum_{i=0}^{Pr} \int_0^{\infty} P_i(t) dt. \tag{30}$$

The selection of the second solution arises from the difficulty of solving an inverse form of differential equation solver with large number of variables. As a result, Laplace technique is used simply. Since,

$$\int_0^{\infty} P_i(t) dt = P_i(s) \Big|_{s=0} \tag{31}$$

MTTF is obtained as follows.

$$MTTF = \sum_{i=0}^{Pr} P_i(s) \Big|_{s=0} = \sum_{i=0}^{Pr} \frac{\Delta_i(0)}{\Delta_F(0)} \tag{32}$$

Example 1: For more clarity, an example is presented to find the reliability for a 1-out-of-2 system and for a Predictive Hybrid 1-out-of-2 system. The state transition matrix for the traditional system is as follows.

$$D = \begin{bmatrix} 1-2\lambda & 2\lambda & 0 \\ \mu & 1-(\lambda+\mu) & \lambda \\ 0 & 0 & 1 \end{bmatrix} \tag{33}$$

The State transition Matrix, P, is delineated in (34).

$$P = \begin{bmatrix} P_0(t) \\ P_1(T) \\ P_F(T) \end{bmatrix}$$

The equation $B = [D - I]^{Tr}$ determines Matrix B based on Matrix

D. Then, Laplace technique is applied to Equation (11), and then Matrix C is calculated by replacing $C = s.I - B$ in Equation (20).

$$C = \begin{bmatrix} s+2\lambda & \mu & 0 \\ 2\lambda & s+(\lambda+\mu) & 0 \\ 0 & \lambda & s \end{bmatrix}, P(s) = \begin{bmatrix} P_0(s) \\ P_1(s) \\ P_F(s) \end{bmatrix}, P0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \tag{35}$$

$$P_0(s) + P_1(s) + P_F(s) = 1 \quad \text{and} \quad C.P(S) = P0$$

After replacing Equation (35) in (2), solving the ordinary Laplace equations and Laplace inverse transform, the value of reliability, R(t), is determined based on Equation (22).

The mathematical solution to calculate reliability of Predictive Hybrid 1-out-of-2 system, that is, when prediction is used, is similar to 1-out-of-2 system. The related equations are presented in (36).

$$C = \begin{bmatrix} s+2\lambda & \mu & 0 & 0 \\ 2\lambda & s+(\lambda+\mu) & \mu_{Pr} & 0 \\ 0 & \lambda & s+(\lambda_{Pr}+\mu_{Pr}) & 0 \\ 0 & 0 & \lambda_{Pr} & s \end{bmatrix}, P(s) = \begin{bmatrix} P_0(s) \\ P_1(s) \\ P_{Pr}(s) \\ P_F(s) \end{bmatrix}, P0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{36}$$

$$P_0(s) + P_1(s) + P_{Pr}(s) + P_F(s) = 1$$

Similarly, R(t) can be calculated by taking Laplace inverse form of $\pi_F(s)$ in Equation (22).

In Figure 5 the comparative plots of the reliability for both systems are delineated by using MATLAB 7.11.0(R2010b), and assuming $\lambda = 0.2$, $\mu = 0.4$, $\lambda_{Pr} = 0.2$, $\mu_{Pr} = 0.4$, and $t=[0.1\dots 1]$ with step $+0.1$. As it is perceived, the reliability of Predictive Hybrid 1-out-of-2 system is higher than the reliability of 1-out-of-2 system (see Figure 5).

The results of simulation for large scale systems by using numerical calculations are discussed in next section, when different repair rates and failure rates are assumed.

EXPERIMENTAL RESULTS

In this section the reliability of the systems is compared in different situations based on what we have obtained for the reliability of PHmn from mathematics and probabilistic calculations and the reliability of traditional m-out-of-n systems mentioned in Misra (1992); Eryilmaz and Zuo (2010); Moghaddass and Zuo (2011) and Radwan et al. (2011). As an assumption for state Pr, the rate of failure, λ_{Pr} , and the rate of repair, μ_{Pr} , are respectively $n\lambda$ and $n\mu$.

In order to establish a comprehensive comparison, we examined different scenarios by changing the values of n, m, λ and μ . Based on our results the reliability behaviors of some scenarios are essentially similar. For instance, the reliability behavior⁽³⁴⁾ of the systems when n=10, 1<m<10, $\lambda=0.5$ and $\mu=0.5$ is similar to when n=128, 1<m<128, $\lambda=0.5$ and $\mu=0.5$. There are similar results

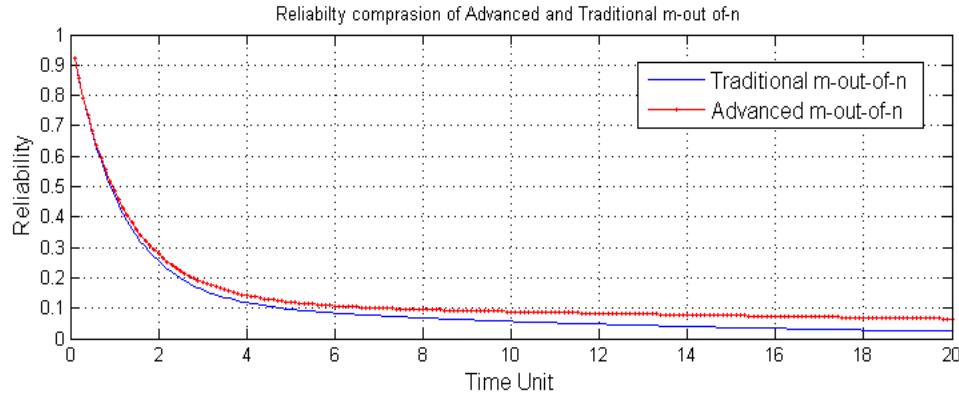


Figure 5. Behavior of the system reliability for traditional and advanced 1-out-of-2.

Table 1. Mean reliability of PHmn Vs. m-out-of-n system when $(\lambda/\mu)<1$, $n=128$, and $1<m\leq 33$.

Parameter	$\lambda=0.1, \mu=0.9$	$\lambda=0.2, \mu=0.8$	$\lambda=0.3, \mu=0.7$	$\lambda=0.4, \mu=0.6$
PH_{mn}	0.99999987933294	0.99999987933294	1	0.999999999999999
$m\text{-out-of-}n$	1	1	1	0.999999999999998
Improvement %	-1.2067e-006	-3.663735981263017e-13	0	1.110223024625159e-13

when we consider certain values of m and n , but changing the values of λ and μ . In order to summarize, we present the choicest results of such similar conditions.

As an assumption, the rates of failure of each component are the same λ . Similarly, the rates of repair of each component are the same μ .

In this section, the reliability of $m\text{-out-of-}n$ system and PH_{mn} are compared where the number of redundant modules is supposed to be 128 and the condition of agreement (m) is varied from 2 to $\lfloor(n/2)+1\rfloor$ (in this case 65). This assumption covers major types of voters with different ranges of agreement; from 2-out-of- n voter to Majority voter that is the severest voter in terms of achieving the consensus.

The results are perceived in following scenarios:

The effect of m, λ , and μ variation on the reliability

As shown in Figure 4, once the system has gone from state i to state j by failure rate of λ_i , it might be repaired and return to the previous working state by repair rate μ_j except for the fail state. It can be obviously claimed that the larger the rate of failure, the more susceptible the current state is to fail, because the probability of failure is more than the probability of repair. It is also expected that a state with a larger rate of repair, is likely to be repaired rather than going to next state, which perceptibly is closer to fail state. In this subsection, we analyze the results of

simulations based on the values of repair rate and of failure rate used in the system, so the results are shown when:

- $(\lambda/\mu)<1$, that is, failure rate is smaller than repair rate,
- $(\lambda/\mu)=1$, that is, failure rate is equal to repair rate,
- $(\lambda/\mu)>1$, that is, failure rate is larger than repair rate.

We also classify the agreement into hard and soft. If m is negligible in comparison with n , then the agreement is defined as soft ($m\ll n/4$); otherwise, we call it as hard ($n/4 < m \leq n/2+1$) (these bands are arbitrary and only used to analyze the results). To clarify this definition, consider voting among a certain number of people as an example. So to the extent the number of required yes votes is smaller than the population ($m \ll n$), definitely reaching an agreement is very easier than with larger required votes. As it is perceived in following subsections, this truth influenced all presented results.

Experiment 1 ($\lambda/\mu)<1$

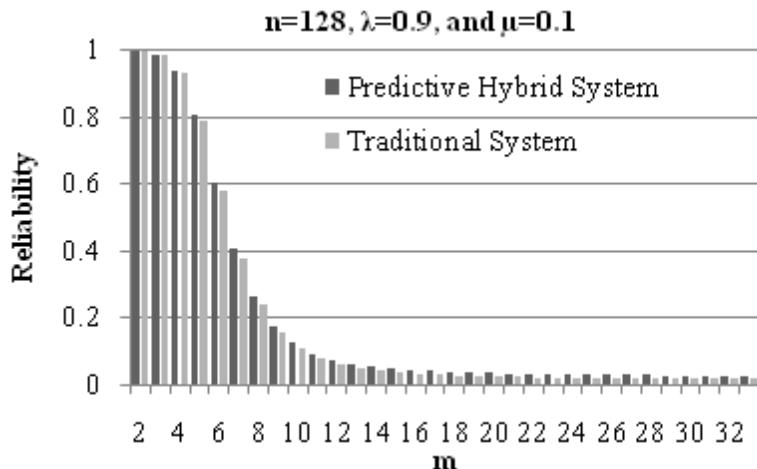
Tables 1-3 show the numerical values of reliability for PH_{mn} and $m\text{-out-of-}n$ systems by using the obtained relations gotten earlier and the reliability improvement of predictive hybrid system in comparison with traditional $m\text{-out-of-}n$ system. In these tables, four cases are displayed in all of which $n=128$ and $(\lambda/\mu)<1$, that is, failure rate is

Table 2. Mean reliability of PHmn Vs. m-out-of-n system when $(\lambda/\mu) < 1$, $n=128$, and $33 < m \leq 65$.

Parameter	$\lambda=0.1, \mu=0.9$	$\lambda=0.2, \mu=0.8$	$\lambda=0.3, \mu=0.7$	$\lambda=0.4, \mu=0.6$
PH_{mn}	0.999999987933294	0.999999987933294	0.999999210839797	0.992320571268929
$m\text{-out-of-}n$	1	0.999999999999964	0.999998280010314	0.986931762237581
Improvement %	-1.2067e-006	2.065014825802866e-12	9.308310840677657e-05	0.546016374944738

Table 3. Mean Reliability of PHmn Vs. m-out-of-n system when $(\lambda/\mu) < 1$, $n=128$, and $1 < m \leq 65$.

Parameter	$\lambda=0.1, \mu=0.9$	$\lambda=0.2, \mu=0.8$	$\lambda=0.3, \mu=0.7$	$\lambda=0.4, \mu=0.6$
PH_{mn}	0.999999987933293	0.999999999999989	0.999999999999989	0.996160285634464
$m\text{-out-of-}n$	1	0.999999999999982	0.99999914005157	0.993465881118790
Improvement %	-1.2067e-006	6.994405055138613e-13	4.654151420006174e-05	0.271212586852043

**Figure 6.** Behavior of system reliability for $n=128$, $1 < m \leq 33$, $\lambda=0.9$ and $\mu=0.1$.

smaller than repair rate. For more accuracy and better judgment, the values are shown with 15 digits.

Tables 1-3 are different from the point of types of agreement. The results for soft agreement are shown in Table 1, whereas in Table 2, hard agreement is presented. Table 3 displays the overall view of reliability when m covers both soft and hard agreement. When λ is significantly smaller than μ , it can be translated that the reliability of independent modules is high. Thus, in most cases, the reliability of a system comprises high reliable modules expected to be high. That is the main reason for the reliability of both systems being very close to 1. The tables also demonstrate that PHmn is more successful than m-out-of-n to improve the system reliability especially when the failure rate of the modules increases; however, it shows the use of PHmn is not economic when agreement is soft and when λ is very small

(negative or zero improvements are signs of suitably of m-out-of-n system). In other cases, the PHmn has a good reliability performance and based on application, it may be a replacement for an m-out-of-n system.

Experiment 2 ($\lambda/\mu > 1$)

$(\lambda/\mu) > 1$ refers to the time when the probability of a system failure is more than the repair ability of the system. Relevant plots of this situation are shown in Figures 6-7.

Figures 6-7 give us three major results:

- They demonstrate the improvement in the reliability of PHmn compared to m-out-of-n system; whether the agreement (that is, value of m) is soft or hard. Note that a well designed fault tolerant system should be able to

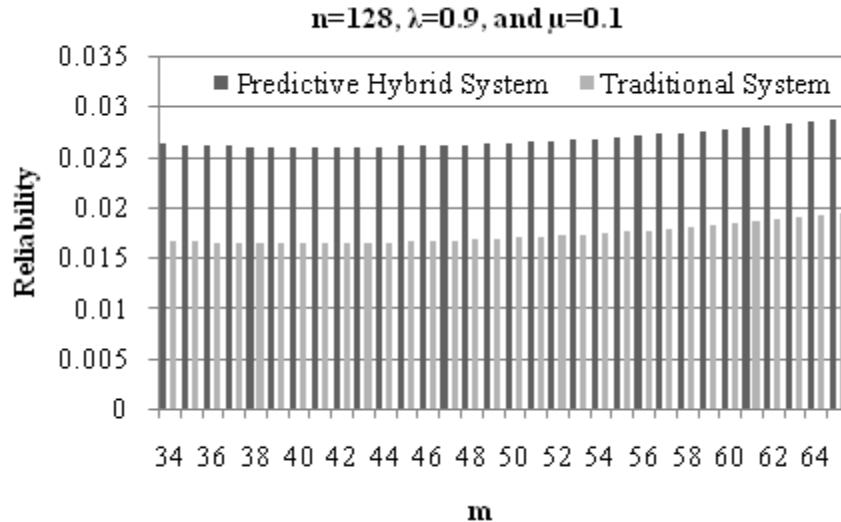


Figure 7. Behavior of system reliability for $n=128$, $33 \leq m \leq 65$, $\lambda=0.9$ and $\mu=0.1$.

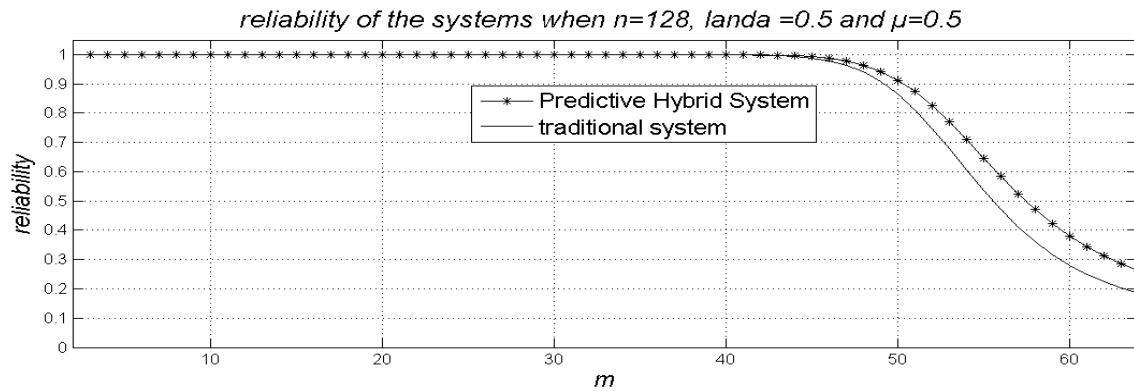


Figure 8. Behavior of the system reliability for $n=128$, $1 < m < 65$ and $\lambda= \mu=0.5$.

increase the overall reliability especially when the components are unreliable; accordingly, these plots present how valuable PHmn is. The calculation shows that the reliability of PHmn is maximum 10.80% better than traditional m-out-of-n system.

2. The overall reliability of both systems is less than the results presented in experiments 1(5.1.1) and 3(5.1.3). Theoretically, it was expected because a system with high probability of failure and low probability of repair, that is, $(\lambda/\mu)>1$, is much susceptible to fail, consequently it would gain the worst reliability in comparison with $(\lambda/\mu)\leq 1$.

3. Moreover, the agreement gets harder (see the variation from $m=2$ to 65) when the reliability of the systems decreases. This behavior was also expected, as the effect of population (m) increase to make consensus hard. Nevertheless, PHmn works better than m-out-of-n systems as a result of 6.60% (Figure 6) and 54.79% (Figure 7) improvement in the reliability for soft agreement and hard agreement, respectively. It is

another significant achievement; especially when the agreement gets hard.

Experiment 3. $(\lambda/\mu)=1$

This experiment covers a moderate system in view of failure occurrence, which is neither very close to fault free conditions in which the probability of failure and of repair are respectively very low and very high (as discussed in 5.1.1), nor very susceptible to failure (as discussed in 5.1.2).

In Figure 8, the reliability of the systems when $\mu=\lambda=0.5$ has been presented demonstrating that the improvement in the reliability of PHmn in comparison with m-out-of-n systems is 2.62% in overall, and respectively 6.20e-07% and 6.17% for soft and hard agreements. In this experiment, decrease in the reliability for large m 's was also predictable (similar to previous experiments). Furthermore, agreement is achieved simply when m

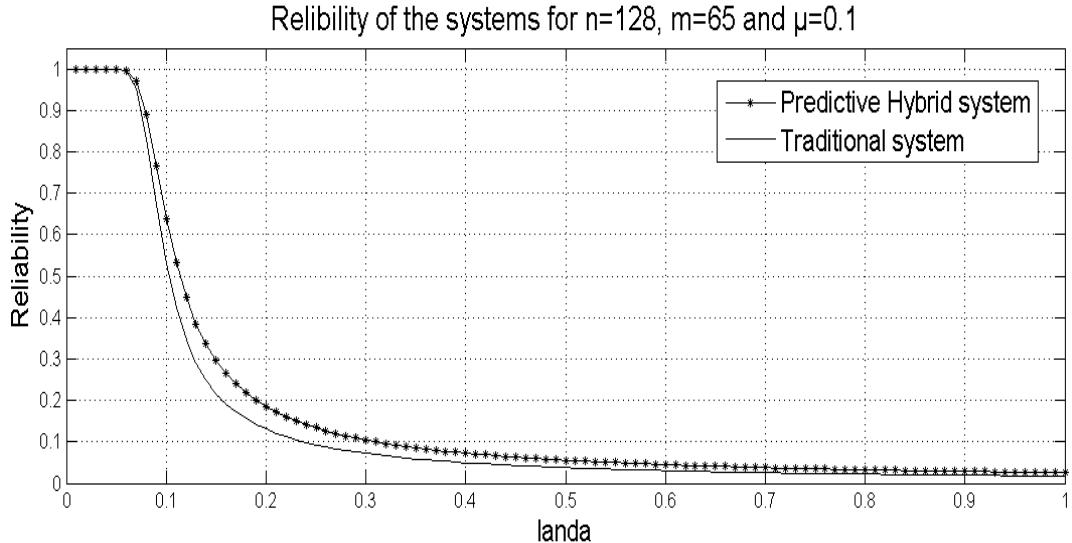


Figure 9. Behavior of the system reliability for $n=128$, $m=65$, $0<\lambda<1$ and $\mu=0.1$.

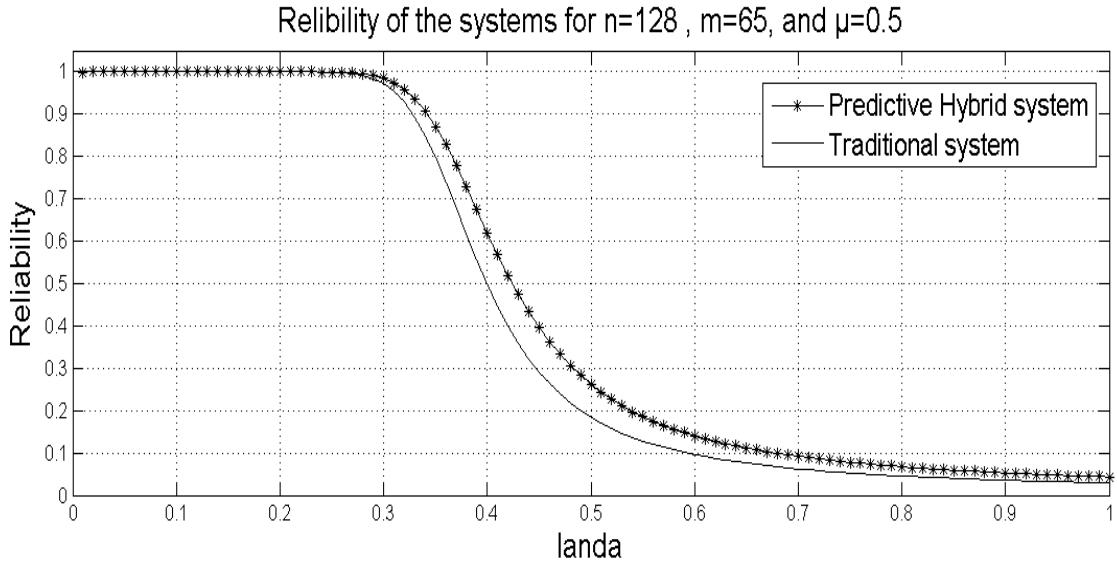


Figure 10. Behavior of the system reliability for $n=128$, $m=65$, $0<\lambda<1$ and $\mu=0.5$.

posses a small value in comparison with n . Hence, the reliability of both systems for small m 's is close to 1.

Based on the plots in Figure 8, in the applications where rate of failure and repair are equal and the agreement is soft, the use of m -out-of- n systems is most probably preferred to avoid cost overheads.

The effect of λ and μ variation on the reliability

In this section, the effect of λ variation on a 65-out-of-128 system is investigated, because the plots for this value of

n are clearer than the smaller values of n , and n is large enough to investigate the reliability of large-scale systems. Furthermore, $m=65$ is the band of hard agreement which is predicted to have the severe condition of a system reliability (we have investigated other values of m where all showed the similar behavior).

In Figures (9-11) the repair rate is fixed and the failure rate is varied from 0 and 1, whereas in Figures (12-14) the failure rate is fixed and repair rate is $0<\mu<1$. As the most significant result, the reliability of PHmn in all these figures is obviously more than m -out-of- n systems.

Theoretically, it is expected that by increasing the

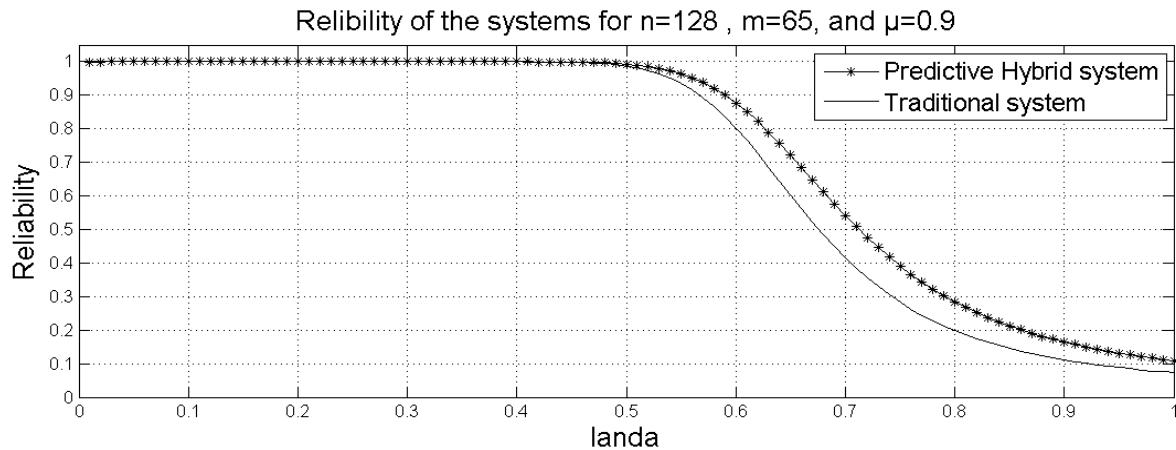


Figure 11. Behavior of the system reliability for $n=128$, $m=65$, $0<\lambda<1$ and $\mu=0.9$.

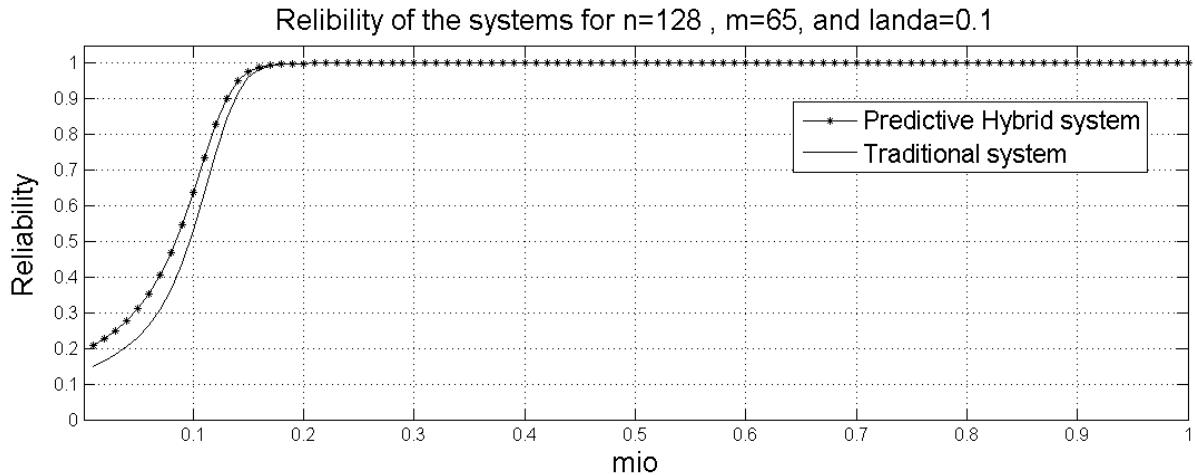


Figure 12. Behavior of the system reliability for $n=128$, $m=65$, $0<\mu<1$ and $\lambda=0.1$.

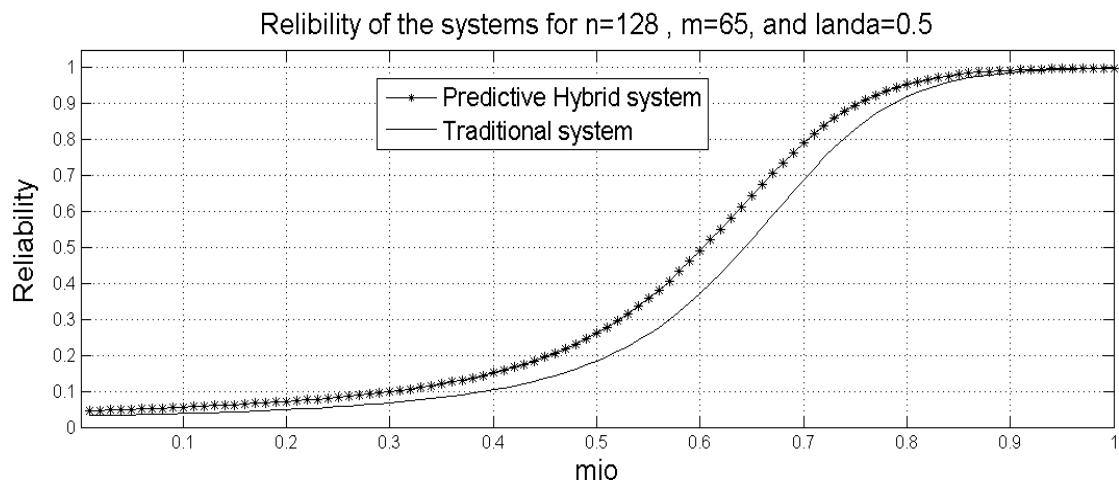
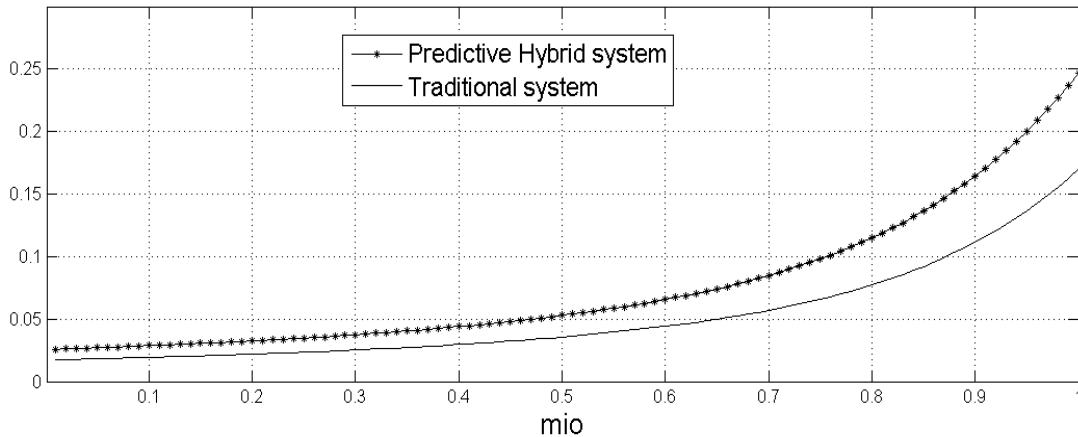


Figure 13. Behavior of the system reliability for $n=128$, $m=65$, $0<\mu<1$ and $\lambda=0.5$.

Reliability of the systems for $n=128$, $m=65$, and $\lambda=0.9$ **Figure 14.** Behavior of the system reliability for $n=128$, $m=65$, $0<\mu<1$ and $\lambda=0.9$.**Table 4.** Mean Reliability of PHmn Vs. m-out-of-n system.

Parameter	$0 < \lambda < 1$, $\mu = 0.1$	$0 < \lambda < 1$, $\mu = 0.5$	$0 < \lambda < 1$, $\mu = 0.9$	$0 < \mu < 1$, $\lambda = 0.1$	$0 < \mu < 1$, $\lambda = 0.5$	$0 < \mu < 1$, $\lambda = 0.9$
PHmn	0.1700	0.4697	0.7372	0.9307	0.4436	0.0764
m-out-of-n	0.1447	0.4363	0.7006	0.9192	0.3972	0.0517

failure rate when repair rate is constant, the probability of failure increases and the reliability of systems decreases (Figures 9-11). Similarly, increase in the reliability by increasing the repair rate is predicted; meanwhile, the failure rate is a fixed value (Figures 12-14).

Figures 9-11 show the results of simulations when the failure rate is varied, $0 < \lambda < 1$, and repair rate is 0.1 in Figure 9; 0.5 in Figure 10; and 0.9 in Figure 11. The average reliability of PHmn and m-out-of-n system for Figures 9-11 is delineated in Table 4. The average reliability of where failure rate is constant and repair rate is varied from 0 to 1 is also perceived in this table.

As it is perceived clearly in Figure 9, the reliability of both systems decrease while λ increases (since the probability of failure has been increased). In this figure, $\mu=0.1$, that is, the probability of system repair is insignificant. Consequently, the average reliability of this case is the smallest value.

Plots in Figure 11 refer to the highest repair rate ($\mu=0.9$). So the higher reliability is expected in comparison with Figures 9 and 10. This expectation comes true based on the results of Table 2.

Accordingly, the reliability of the systems in Figure 10 in which the repair rate is moderate ($\mu=0.5$), is respectively less and more than the plots in Figure 11 and in Figure 9 (see Table 4).

The percentage of the reliability improvement in PHmn versus the traditional system is in average 17.49% when

$\mu=0.1$; 7.63% when $\mu=0.5$ and 5.21% when $\mu=0.9$ (other values of μ have been also investigated in which positive improvement in the reliability of PHmn was obtained).

If the failure rate is constant and the repair rate is varied ($0 < \mu < 1$), the probability that faulty modules being repaired increases by rising the repair rate; subsequently, the reliability increases. This rising behavior is noticeably significant in Figures 12-14. Moreover, for small values of the failure rate, the probability of the failure and the reliability will become small and large, respectively. As a result, the average reliability for $\lambda=0.5$ is more than $\lambda=0.9$.

The results of the simulations demonstrate the improvement of the average reliability of PHmn to 1.25, 11.69 and 47.81% when λ is respectively 0.1, 0.5, and 0.9 in comparison with m-out-of-n system (other values of λ have also been investigated in which increasing in the reliability of PHmn was obtained).

As the other conclusion from Figures 9-14, with less failure and more repair rates, the reliability of both approaches is closed to 1. Although the small μ and large λ lead to the worst reliability behavior of the system, large λ has higher negative effect on the overall reliability in comparison with small μ . The reliability has reached to maximum 1 when $\mu=0.1$ and $0 < \lambda < 1$ (Figure 9), whereas none of the values of μ has attained even a value near 1 in Figure 14. The average reliability in the first case (Figure 9) is 0.7372 for PHmn while it is 0.0764 for Figure 14. These scenarios are highlighted in Table 4.

Therefore, in a large scale highly reliable application, the most important assumption is utilizing the modules which are as much as possible up, because as they fail, their repair and restore in operational manner are not likely possible.

Conclusion and future works

In this paper, predictive hybrid redundancy has been extended to large scale control systems that comprise n redundant hardware modules. If m-out-of-n modules are in agreement, the system can make an output; otherwise, a history record of previous successful result(s) is used to predict the result of current cycle; while in traditional m-out-of-n redundancy, system fails if it can not find consensus. In order to investigate the reliability of the new extension of predictive hybrid redundancy which is called as predictive hybrid m-out-of-n system, a Markov reliability model has been presented in this paper, then the reliability of the new system has been computed and simulated in different scenarios of repair rate, μ , failure rate, λ , and m (minimum requirement for consensus) all of which are the effective parameters on the system reliability based on the computed reliability equation. The effect of these parameters has been examined and discussed based on which the extended system has totally higher reliability than traditional m-out-of-n system. In all cases, the use of our new extension is the best choice especially when the large-scale control systems are dealt with. The exception is for the situations where the number m is very small and the use of traditional system is favoured due to the cost preferences.

In future works, the other parameters influencing the system dependability, for example, availability, safety, etc, will be investigated.

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We would like to appreciate all the reviewers.

Nomenclatures: n , Number of modules; i , number of failed components in the system ($i=0, 1, \dots, n-m+1$); λ_i , failure rate of the system when there are i failed components ($0 \leq i \leq n-m$ or $i \geq n-m$); μ_i , repair rate of the system when there are i failed components ($1 \leq i \leq n-m+1$); $\pi_i(t)$, the probability that there are i failed components in the system at time t ($0 \leq i \leq F$); m , number of agreed modules; $\pi'_i(t)$, first derivation of $\pi_i(t)$ ($0 \leq i \leq F$); $L_i(s)$, laplace transform of $\pi_i(t)$ ($0 \leq i \leq F$); $L_i^{-1}(s)$, an Inverse Laplace transform of $L_i(s)$; ($0 \leq i \leq F$); $R(t)$, reliability function of system at time t.

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