International Journal of the Physical Sciences Vol. 6(25), pp. 6098-6109, 23 October, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.1128
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Free vibration of symmetric angle-ply laminated cylindrical shells of variable thickness including shear deformation theory

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Accepted 27 September, 2011

Free vibrational study of symmetric angle-ply laminated cylindrical shells of variable thickness including first order shear deformation theory using spline function approximation is studied. The equations of motion for the cylindrical shells are derived using first order shear deformation theory. The solutions of displacement functions are assumed in a separable form to obtain a system of coupled differential equations in terms of displacement and rotational functions, and these functions are approximated by Bickley-type splines of order three. The vibrations of three and five layered shells, made up of two different types of order of the layers of materials and two types of boundary conditions are considered. A generalized eigenvalue problem is obtained and solved numerically for an eigenfrequency parameter and an associated eigenvector of spline coefficients. Parametric studies are made for the frequency parameters with respect to the coefficients of thickness variations, length-to-radius ratio, length-to-thickness ratio and ply angles under different boundary conditions. In the present work, the results are expected to be more accurate and more suitable for immediate application in the areas of missiles, aviation, shipping, surface transport and a large number of industries related to the cement and chemicals.

Key words: Free vibration, angle-ply, shear deformation, cylindrical shells, variable thickness.

INTRODUCTION

Composite shell structures are widely used in many areas like aviation, ship building, chemical, industries, etc. The laminated shells structures are important in these fields since, they have high specific stiffness, better damping and shock absorbing characteristics. The study of free vibration analysis of laminated cylindrical shell is very important since the frequencies depend on ply orientation, material properties, number of layers and boundary conditions (Greenberg and Stavsky, 1980; Alam and Asnani, 1984; Sharma and Darvizeh, 1987; Naritha, 1992). Since the shell are laminated, the natural frequencies can be modified by including the variable thickness along the axial direction of the cylindrical shell (Suzuki et al., 1982; Viswanathan and Navaneethakrishnan, 2005; Viswanathan and Sheen, 2009). Sivadas and Ganesan (1993) studied the vibration of axisymmetric thick cylindrical shell of variable thickness. In their study, the thickness parameter was considered as the ratio of the radius to average thickness. Rezaee and Hassannejad (2010) analyzed the problem on damped free vibration of beam with a fatigue crack using energy balance method. Lam and Qian (2000) presented analytical solution on free vibration of symmetric angle-ply laminated cylindrical shell using first order shear deformation theory and complex method. Bayat et al. (2010) analyzed the nonlinear behavior of structure under harmonic loading. Ganapathi et al. (2004) dealt with vibration of laminated angle-ply non-circular cylindrical shells using finite element approach. The higher-order

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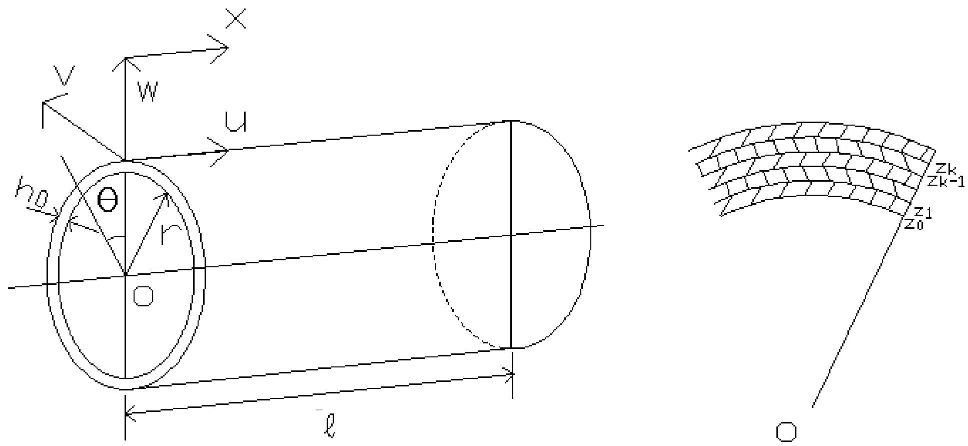

Figure 1. Layered circular cylindrical shell of constant thickness: geometry.

theories of isotropic circular cylindrical shells with effect of shear deformation and rotatory inertia were studied by Matsunaga (1998, 1999, 2007) for both vibration and buckling problems with constant thickness.

In the present study, free vibration of symmetric angle-ply laminated cylindrical shells including shear deformation with variable thickness is discussed by applying the collocation with splines. The thickness variations are assumed to be linear, exponential and sinusoidal along the longitudinal direction of the shell. The problem is formulated by including the first order shear deformation theory and then, the system of coupled differential equations are obtained in terms of displacement and rotational functions depending on the space coordinates. These functions are approximated by Bickley-type cubic splines. Collocation procedure has been adopted to obtain a set of field equations. These equations along with the set of boundary conditions reduce to a set of homogeneous equations on the assumed spline coefficients which tends to the generalized eigenvalue problem. This problem is solved for the frequency parameter using eigensolution technique to obtain many frequencies as required, starting from the least. From the eigenvectors, the spline coefficients are computed from which the mode shapes can be constructed. Parametric studies have been made for the frequency parameters with respect to the thickness of variation parameter, length ratio, circumferential node number, ply angles and boundary conditions. However, numerical results are presented and discussed in terms of graph and tables.

FORMULATION OF THE PROBLEM

Consider a composite laminated circular cylindrical shell having length ℓ, thickness h and radius r. The x coordinate of the shell is taken along the meridional direction, θ coordinate along the circumferential direction and z along the thickness direction (Figure 1). The reference surface of the shell is taken at its middle surface.

According to the first order shear deformation theory, the displacements components u, v, w can be written as:

$$u(x,\theta,z,t) = u_0(x,\theta,t) + z\psi_x(x,\theta,t)$$

$$v(x,\theta,z,t) = v_0(x,\theta,t) + z\psi_\theta(x,\theta,t)$$

$$w(x,\theta,z,t) = w_0(x,\theta,t) \tag{1}$$

where u_0, v_0, w_0 are the mid plane displacements, ψ_x, ψ_θ are the shear rotations of any point on the mid surface normal to the xz and θz plane, respectively and t is the time .

The strain-displacement relations of the cylindrical shells having the radius r are given as:

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + z\frac{\partial \psi_x}{\partial x}, \quad \varepsilon_\theta = \frac{1}{r}\frac{\partial v_0}{\partial \theta} + \frac{w_0}{r} + \frac{z}{r}\frac{\partial \psi_\theta}{\partial \theta}, \quad \gamma_{x\theta} = \frac{1}{r}\frac{\partial u_0}{\partial \theta} + \frac{\partial v_0}{\partial x} + z\left(\frac{1}{r}\frac{\partial \psi_x}{\partial \theta} + \frac{\partial \psi_\theta}{\partial x}\right)$$

$$\gamma_{xz} = \psi_x + \frac{\partial w_0}{\partial x} \quad and \quad \gamma_{\theta z} = \psi_y + \frac{1}{r}\frac{\partial w_0}{\partial \theta} - \frac{v_0}{r} \tag{2}$$

The stress-strain relations of the k -th layer by neglecting the transverse normal strain and stress, are of the form:

$$\{\sigma\} = \left[Q^{(k)}\right]\left\{\varepsilon_x^{(k)} \ \varepsilon_\theta^{(k)} \ \gamma_{x\theta}^{(k)} \ \gamma_{xz}^{(k)} \ \gamma_{\theta z}^{(k)}\right\} \tag{3}$$

When the materials are oriented at an angle α with the x -axis, the transformed stress-strain relations are:

$$\{\sigma\}=\left[\overline{Q}^{(k)}\right]\left\{\varepsilon_x^{(k)}\ \varepsilon_\theta^{(k)}\ \gamma_{x\theta}^{(k)}\ \gamma_{xz}^{(k)}\ \gamma_{\theta z}^{(k)}\right\} \tag{4}$$

where $\left[\overline{Q}^{(k)}\right]=[T]^{-1}\left[Q^{(k)}\right][T]$, and is given as shown in the Appendix.

The stress resultants and stress couples are given by:

$$\left(N_x, N_\theta, N_{x\theta}, Q_x, Q_\theta\right)=\int_z \left(\sigma_x, \sigma_\theta, \tau_{x\theta}, \tau_{xz}, \tau_{\theta z}\right) dz$$

$$\left(M_x, M_\theta, M_{x\theta}\right)=\int_z \left(\sigma_x, \sigma_\theta, \tau_{x\theta}\right) z\,dz \tag{5}$$

Applying Equation 2 into Equation 4 and then substituting into Equation 5, to obtain the equations of stress-resultants and moment resultants as:

$$\begin{pmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{pmatrix}=\begin{pmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{pmatrix}\begin{pmatrix} \frac{\partial u_0}{\partial x} \\ \frac{1}{r}\frac{\partial v_0}{\partial \theta}+\frac{w_0}{r} \\ \frac{\partial v_0}{\partial x}+\frac{1}{r}\frac{\partial u_0}{\partial \theta} \\ \frac{\partial \psi_x}{\partial x} \\ \frac{1}{r}\frac{\partial \psi_\theta}{\partial \theta} \\ \frac{\partial \psi_\theta}{\partial x}+\frac{1}{r}\frac{\partial \psi_x}{\partial \theta} \end{pmatrix} \tag{6}$$

and

$$\begin{pmatrix} Q_\theta \\ Q_x \end{pmatrix}=K\begin{pmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{pmatrix}\begin{pmatrix} \psi_\theta+\frac{1}{r}\frac{\partial w_0}{\partial \theta}-\frac{v_0}{r} \\ \psi_x+\frac{\partial w_0}{\partial x} \end{pmatrix} \tag{7}$$

in which A_{ij}, B_{ij} and D_{ij} are respectively, extensional rigidities, the bending-stretching coupling rigidities and the bending rigidities and K is the shear correction factor that depends on lamina properties and lamination scheme, and may be calculated by various static and dynamic methods (Whitney and Sun, 1973).

In this study, the thickness of the k-th layer is assumed in the form:

$$h_k(x)=h_{0k}\ g(x) \tag{8}$$

where h_{0k} is a constant thickness

In general, the thickness variation of each layer is assumed in the form:

$$h(x)=h_0\ g(x)$$

and

$$g(x)=1+C_\ell \frac{x}{\ell}+C_e \exp\left(\frac{x}{\ell}\right)+C_s \sin\left(\frac{\pi x}{\ell}\right) \tag{9}$$

The thickness becomes uniform if $g(x)=1$.

Therefore, the elastic coefficients A_{ij}, B_{ij} and D_{ij} corresponding to layers of uniform thickness with superscript 'c' can easily be obtained as:

$$A_{ij}=A_{ij}^c g(x),\quad B_{ij}=B_{ij}^c g(x),\ D_{ij}=D_{ij}^c g(x) \tag{10}$$

where

$$A_{ij}^c=\sum_k \overline{Q}_{ij}^{(k)}\left(z_k-z_{k-1}\right),$$

$$B_{ij}^c=\frac{1}{2}\sum_k \overline{Q}_{ij}^{(k)}\left(z^2_k-z^2_{k-1}\right),$$

$$D_{ij}^c=\frac{1}{3}\sum_k \overline{Q}_{ij}^{(k)}\left(z^3_k-z^3_{k-1}\right) \tag{11}$$

and z_k, z_{k-1} are boundaries of the k-th layer.

On substitution of the Equation 10 into Equations 6 and 7, and then substituting into the equations of equilibrium of a cylindrical shell and applying the condition of symmetric in angle-ply laminates (that is, $A_{16}, A_{26}, A_{45}, D_{16}, D_{26}$ and B_{ij} are identically zero), one can obtain the following differential equations as:

$$\left(A_{11}g'\frac{\partial}{\partial x}+A_{11}g\frac{\partial^2}{\partial x^2}+A_{66}g\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)u_0+\left(A_{12}g'\frac{1}{r}\frac{\partial}{\partial \theta}+\left(A_{12}+A_{66}\right)\frac{g}{r}\frac{\partial^2}{\partial x\partial \theta}\right)v_0$$
$$+\left(A_{12}g'\frac{1}{r}+A_{12}g\frac{1}{r}\frac{\partial}{\partial x}\right)w=I_1\frac{\partial^2 u_0}{\partial t^2} \tag{12}$$

$$\left(A_{66}g'\frac{1}{r}\frac{\partial}{\partial \theta}+A_{26}g\frac{1}{r}\frac{\partial^2}{\partial x\partial \theta}+A_{12}g\frac{1}{r}\frac{\partial^2}{\partial x\partial \theta}\right)u_0+\left(A_{66}g'\frac{\partial}{\partial x}+A_{66}g\frac{\partial^2}{\partial x^2}+A_{22}g\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right.$$
$$\left.-KA_{44}g\frac{1}{r^2}\right)v_0+KA_{44}g\frac{1}{r}\beta_\theta+\left(A_{22}+KA_{44}\right)g\frac{1}{r^2}\frac{\partial w}{\partial \theta}=I_1\frac{\partial^2 v_0}{\partial t^2} \tag{13}$$

$$-A_{12}g\frac{1}{r}\frac{\partial u_0}{\partial x}-\left(A_{22}+KA_{44}\right)g\frac{1}{r^2}\frac{\partial v_0}{\partial \theta}+\left(KA_{55}g'\frac{\partial}{\partial x}+KA_{55}g\frac{\partial^2}{\partial x^2}+KA_{44}g\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}-A_{22}g\frac{1}{r^2}\right)w$$

$$+\left(KA_{55}g'+KA_{55}g\frac{\partial}{\partial x}\right)\beta_x+KA_{44}g\frac{1}{r}\frac{\partial}{\partial \theta}\beta_\theta=I_1\frac{\partial^2 w}{\partial t^2} \tag{14}$$

$$KA_{55}g\frac{\partial}{\partial x}w+\left(D_{11}g'\frac{\partial}{\partial x}+D_{11}g\frac{\partial^2}{\partial x^2}+D_{66}g\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}-KA_{55}g\right)\beta_x$$

$$+\left(D_{12}g'\frac{1}{r}\frac{\partial}{\partial \theta}+\left(D_{12}+D_{66}\right)g\frac{1}{r}\frac{\partial^2}{\partial x\partial \theta}\right)\beta_\theta=I_3\frac{\partial^2 \beta_x}{\partial t^2} \tag{15}$$

$$KA_{44}g\frac{1}{r}v_0-KA_{44}g\frac{1}{r}\frac{\partial}{\partial \theta}w_0+\left(D_{66}g'\frac{1}{r}\frac{\partial}{\partial \theta}+\left(D_{12}+D_{66}\right)g\frac{1}{r}\frac{\partial^2}{\partial x\partial \theta}\right)\beta_x$$

$$+\left(D_{66}g'\frac{\partial}{\partial x}+D_{66}g\frac{\partial^2}{\partial x^2}+D_{22}g\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}-KA_{44}g\right)\beta_\theta=I_3\frac{\partial^2 \beta_\theta}{\partial t^2} \tag{16}$$

where I_1 and I_3 are the normal and rotatory inertia coefficients defined by:

$$(I_1, I_3)=\int \rho^{(k)}(1, z^2)\,dz \tag{17}$$

and

$$g=g(x),\ \ g'=\frac{dg(x)}{dx}$$

The displacement components u_0, v_0, w_0 and shear rotations ψ_x, ψ_θ are assumed in the form of:

$$u_0(x,\theta,t)=U(x)\cos n\theta\, e^{i\omega t}$$
$$v_0(x,\theta,t)=V(x)\sin n\theta\, e^{i\omega t}$$
$$w_0(x,\theta,t)=W(x)\cos n\theta\, e^{i\omega t}$$
$$\psi_x(x,\theta,t)=\Psi_x(x)\cos n\theta\, e^{i\omega t}$$
$$\psi_\theta(x,\theta,t)=\Psi_\theta(x)\sin n\theta\, e^{i\omega t} \tag{18}$$

where ω is the angular frequency of vibration, t is the time and n is the circumferential node number.

Using Equation 18 into the Equations 12 to 17, the resulting equation becomes in the matrix form as:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix}\begin{Bmatrix} U \\ V \\ W \\ \psi_x \\ \psi_\theta \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{19}$$

where

$$L_{11}=A_{11}g\frac{d^2}{dx^2}+A_{11}g'\frac{d}{dx}-A_{66}g\frac{n^2}{r^2}+I_1\omega^2,\quad L_{12}=(A_{12}+A_{66})g\frac{n}{r}\frac{d}{dx}+A_{12}g'\frac{n}{r},$$

$$L_{13}=A_{12}g\frac{1}{r}\frac{d}{dx}+A_{12}g'\frac{1}{r},\quad L_{14}=L_{15}=0,\quad L_{21}=-(A_{12}+A_{66})g\frac{n}{r}\frac{d}{dx}-A_{66}g'\frac{n}{r},$$

$$L_{22}=A_{66}g\frac{d^2}{dx^2}+A_{66}g'\frac{d}{dx}-(A_{22}n^2+KA_{44})g\frac{1}{r^2}+I_1\omega^2,\quad L_{23}=-(A_{22}+KA_{44})g\frac{n}{r^2},$$

$$L_{24}=0,\quad L_{25}=KA_{44}g\frac{1}{r},\quad L_{31}=-A_{12}g\frac{1}{r}\frac{d}{dx},\quad L_{32}=-(A_{22}+KA_{44})g\frac{n}{r^2},$$

$$L_{33}=KA_{55}g\frac{d^2}{dx^2}+KA_{55}g'\frac{d}{dx}-(KA_{44}n^2+A_{22})g\frac{1}{r^2}+I_1\omega^2,\quad L_{34}=KA_{55}g\frac{d}{dx}+KA_{55}g',$$

$$L_{35}=KA_{44}g\frac{n}{r},\quad L_{41}=0,\quad L_{42}=0,\quad L_{43}=-KA_{55}g\frac{d}{dx},$$

$$L_{44}=D_{11}g\frac{d^2}{dx^2}+D_{11}g'\frac{d}{dx}-(D_{66}\frac{n^2}{r^2}+KA_{55})g+I_3\omega^2,\quad L_{45}=(D_{12}+D_{66})g\frac{n}{r}\frac{d}{dx}+D_{12}g'\frac{n}{r},$$

$$L_{51}=0,\quad L_{52}=KA_{44}g\frac{1}{r},\quad L_{53}=KA_{44}g\frac{n}{r},\quad L_{54}=-(D_{12}+D_{66})g\frac{n}{r}\frac{d}{dx}-D_{66}g'\frac{n}{r},$$

$$L_{55}=D_{66}g\frac{d^2}{dx^2}+D_{66}g'\frac{d}{dx}-(D_{22}\frac{n^2}{r^2}+KA_{44})g+I_3\omega^2 \tag{20}$$

METHOD OF SOLUTION

Transformation

The non-dimensional parameters are introduced as follows:

$X=\dfrac{x}{\ell}$, a distance coordinate , and $X\in[0,1]$;

$\lambda=\omega\ell\sqrt{\dfrac{I_1}{A_{11}}}$, a frequency parameter;

$H=\dfrac{h_0}{r}$, a thickness ratio;

$L = \frac{\ell}{r}$, a length parameter;

$\delta_k = \frac{h_k}{h}$, relative layer thickness of the k -th layer (21)

Thickness variation

The thickness $h_k(X)$ of the k -th layer at any point X can be expressed as:

$h_k(X) = h_{0k}\ g(X)$

where

$$g(X) = 1 + C_\ell X + C_e \exp(X) + C_s \sin(\pi X) \quad (22)$$

Case 1

If $C_e = C_s = 0$, then the thickness variation becomes linear. In this case it can easily shown that :

$C_\ell = \frac{1}{\eta} - 1$, where η is the taper ratio $h_k(0)/h_k(1)$. (23)

Case 2

If $C_\ell = C_s = 0$, then the excess thickness over uniform thickness varies exponentially.

Case 3

If $C_\ell = C_e = 0$, then the excess thickness varies exponentially.

It may be noted that the thickness of any layer at the end $X = 0$ is h_{0k} for the cases 1 and 3, but is $h_{0k}(1 + C_e)$ for the case 2.

The following range of values of the thickness coefficients are considered as:

$0.5 \le \eta \le 2.1$, $-0.2 \le C_e \le 0.2$, $-0.5 \le C_s \le 0.5$. (24)

Spline collocation procedure

The displacement functions U, V, W and rotational functions Ψ_X, Ψ_Θ are approximated by cubic spline functions in the range of $X \in [0,1]$ as:

$$U^*(X) = \sum_{i=0}^{2} a_i X^i + \sum_{j=0}^{N-1} b_j (X - X_j)^3\ H(X - X_j)$$

$$V^*(X) = \sum_{i=0}^{2} c_i X^i + \sum_{j=0}^{N-1} d_j (X - X_j)^3\ H(X - X_j)$$

$$W(X) = \sum_{i=0}^{2} e_i X^i + \sum_{j=0}^{N-1} f_j (X - X_j)^3\ H(X - X_j)$$

$$\Psi_X^*(X) = \sum_{i=0}^{2} g_i X^i + \sum_{j=0}^{N-1} p_j (X - X_j)^3\ H(X - X_j)$$

$$\Psi_\Theta^*(X) = \sum_{i=0}^{2} l_i X^i + \sum_{j=0}^{N-1} q_j (X - X_j)^3\ H(X - X_j) \quad (25)$$

Here, $H(X - X_j)$ is the Heaviside step functions. The range of X is divided in to N subintervals, at the points $X = X_s$ and $s = 1, 2, 3, ..., N-1$. The width of each subinterval is $1/N$ and $X_s = s/N$ ($s = 0, 1, 2, ..., N$), since the knots X_s are chosen equally spaced.

The assumed spline functions given in Equation 25 are approximated at the nodes (coincide with the knots), and these splines satisfy the differential equations given in Equation 19, at all X_s and resulting into the homogeneous system of $(5N + 5)$ equations in the $(5N + 15)$ unknown spline coefficients.

The following boundary conditions are considered to analyze the problem:

1. Clamped-Clamped (C-C) (both the ends are clamped).
2. Clamped-Free (C-F) (one end is clamped and the other end is free).

By applying each of these boundary condition separately, one can obtain 10 more equations on spline coefficients. Combining these 10 equations with the earlier $(5N + 5)$ equations, we get $(5N + 15)$ homogeneous equations in the same number of unknowns. Thus, we have a generalized eigenvalue problem in the form:

$$[M]\{q\} = \lambda^2 [P]\{q\} \quad (25)$$

where $[M]$ and $[P]$ are the square matrices, $\{q\}$ is the column matrix of the spline coefficients and λ is the eigenfrequency parameter. Since the matrices are large order, the eigenvalue problem is solved by applying numerical technique (power method) using FORTRAN programming language to obtain the eigenvalues and eigenvectors as we required.

RESULTS AND DISCUSSION

In this work, the frequency parameters and fundamental

Table 1. Comparison of the fundamental frequency parameter for three layered symmetric angle-ply cylindrical shells of constant thickness with circumferential node number n (H = 0.2, L=20).

n	30°/0°/30°		60°/0°/°60	
	(Lian and Qian, 2000)	Present value	(Lian and Qian, 2000)	Present value
1	0.0539611	0.052035	0.0609682	0.059852
2	0.2348880	0.228728	0.2932790	0.238570
3	0.5297780	0.493655	0.6602740	0.615274
4	0.9063210	0.891402	1.1264500	1.021411
5	1.3414200	1.285620	1.6624800	1.523521

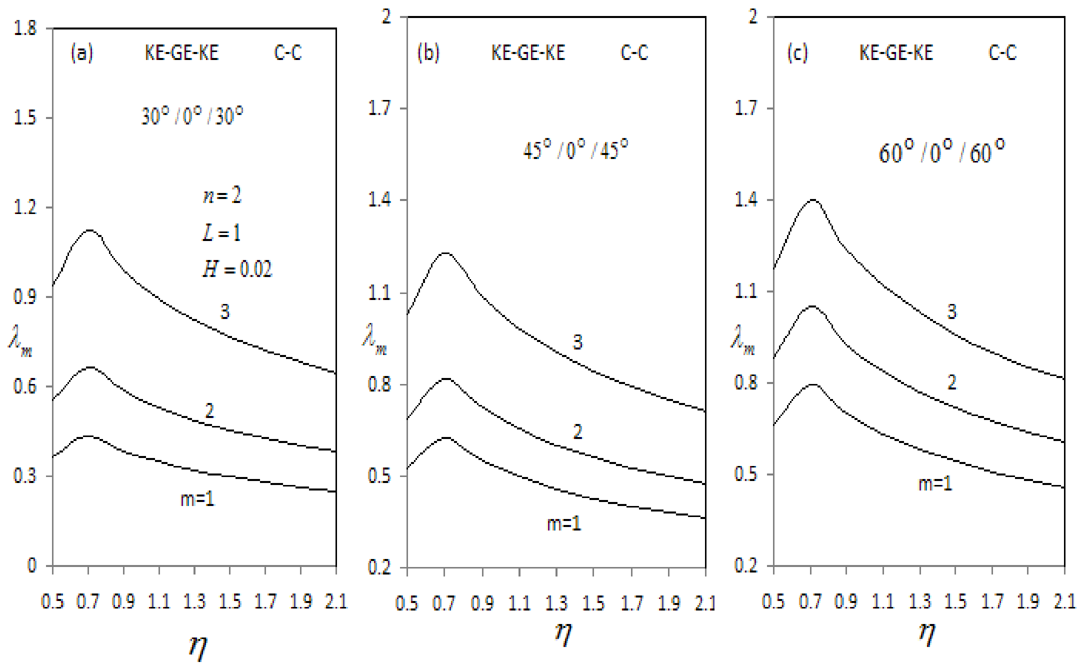

Figure 2. Variation of frequency parameter of linear variation in thickness with taper ratio for three layered symmetric angle-ply shells under C-C boundary conditions.

frequencies are analyzed for three and five layered symmetric angle-ply cylindrical shells analyzed under C-C and C-F boundary conditions using Kevler-49 Epoxy and Graphite Epoxy (AS4/3501-6) materials arranging them in different orders. Convergence study have been made for the frequency parameters of three and four layered shells under various boundary conditions with fixing the length ratio, thickness coefficients, thickness ratio and circumferential node number. The program is performed for N (number of knots) = 2 onwards and finally, it is seen that N = 16 would be enough to achieve the change in percentage of the next value of N as 0.27%. Comparisons are made with the available literature. The present results compared with the results obtained by Lam and Qian (2000) for three layered symmetric angle-ply cylindrical shells (Table 1). The value of shear correction coefficient $K = 5/6$ (Bert and Chen, 1978; Reddy, 1978; Whitney, 1973) is used for comparison and for obtaining new results. The agreement is quite good, which shows that the present method and analysis are accurate.

New results are shown for three and five layered composite cylindrical shells with symmetric ply-angles. Combinations of Kevler-49 epoxy (KE) and AS4 /1350-6 Graphite epoxy (GE) are considered (Bhimaraddi, 1993). Figure 2 corresponds to linear variation in thickness with taper ratio η ranging from 0.5 to 2.1 on frequency parameters $\lambda_m, (m = 1,2,3)$ for three layered symmetric shells of the materials KE and GE arranged in the order of KE-GE-KE, of ply angles $30^0/0^0/30^0$, $45^0/0^0/45^0$ and $60^0/0^0/60^0$ by fixing $H = 0.02$ and $L = 1$ and circumferential node number $n = 2$ under the C-C boundary conditions. For η < 1, the thickness of one

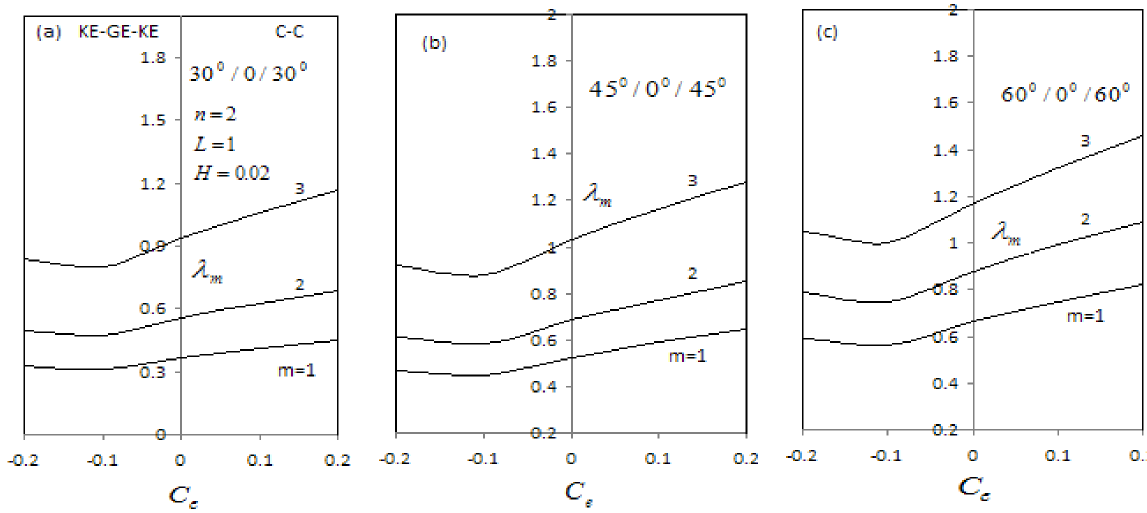

Figure 3. Variation of frequency parameter of three layered symmetric angle-ply shells of exponential variation in thickness under C-C boundary conditions.

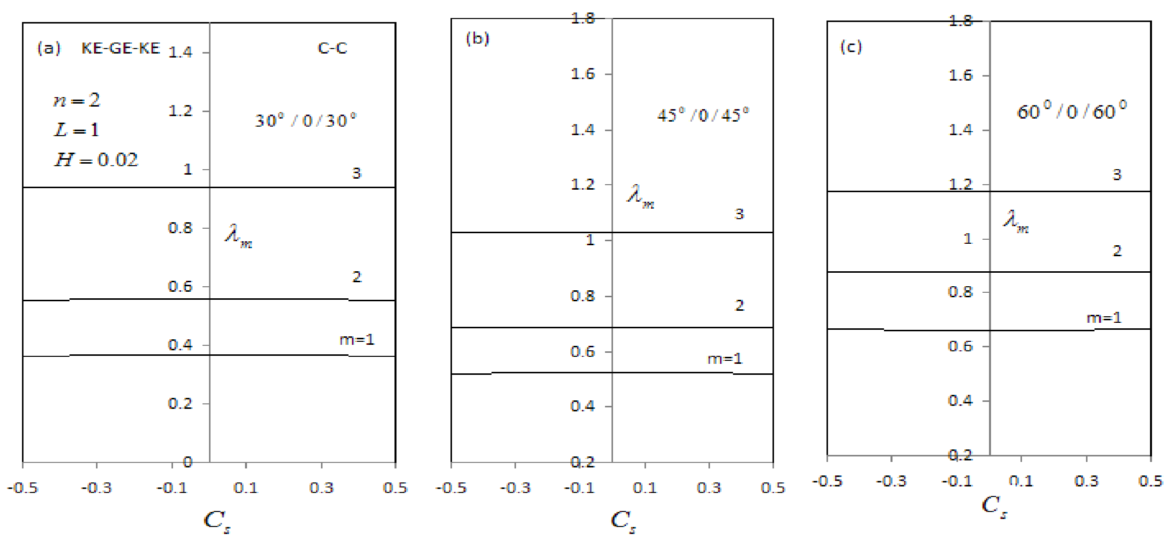

Figure 4. Variation of frequency parameter of three layered symmetric angle-ply shells of sinusoidal variation in thickness under C-C boundary conditions.

end of the cylinder ($x=0$) is smaller than the other end ($x=\ell$). For η > 1, it is the other way, and for η = 1, the considered, λ_m increases and then decreases rapidly with increasing η for small range and then, the decrease of λ_m is almost constant. This trend is same for all the modes and all the angles, but the values of λ_m is higher for higher angles and higher modes.

In Figures 3 and 4, the influence of the coefficient of exponential variation of thickness C_e and the coefficient of sinusoidal variation C_s on λ_m are depicted, along with the C-C boundary conditions. In Figure 3, λ_m decrease slowly for small values of C_e and then increase as C_e increase, that is, the stiffness decrease or increase according as λ_m decreases or increases. It is seen from the figure that the variation of λ_m is almost same for all the values of m. In Figure 4, the sinusoidal thickness variation is studied with λ_m. The stiffness is almost constant for all values of C_s and the nature of variation of λ_m is same for all m. The values are higher for higher

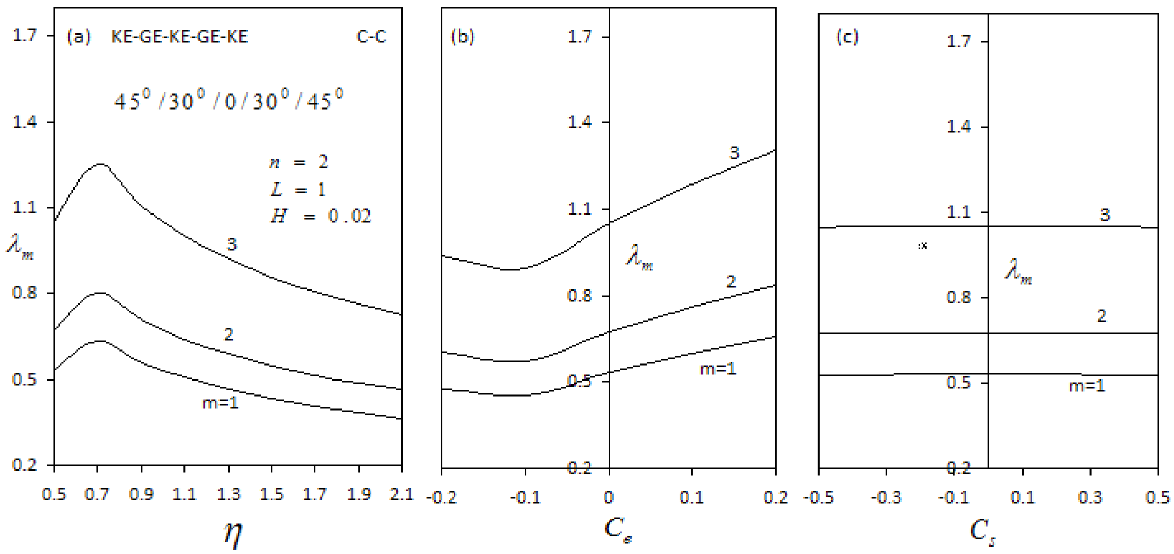

Figure 5. Variation of frequency parameter of five layered symmetric angle-ply shells of linear, exponential and sinusoidal variation in thickness under C-C boundary conditions.

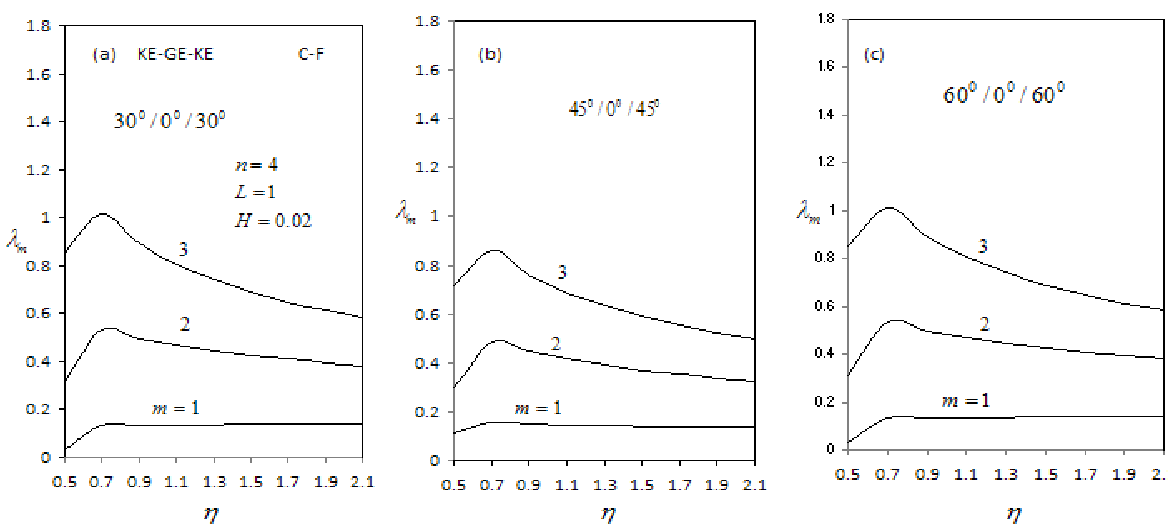

Figure 6. Variation of frequency parameter of linear variation in thickness with taper ratio for three layered symmetric angle-ply shells under C-F boundary conditions.

angles. Figure 5 shows the variation of frequency sinusoidal variation in thickness for five layered angle-ply shells under C-C boundary conditions. The other parameters are fixed and shown in the figure. The nature of variation for five layered shells is also same as three layered shells in all the three variations and modes.

Figures 6 and 7 describe the manner of variation of λ_m with respect to the linear and exponential variations, respectively by fixing $H=0.02$ and $L=1$ and $n=4$ for three layered shells with C-F boundary conditions. The order of the materials is arranged in the form of KG-GE-KG layers. Figure 6a, b and c depicts the variation of λ_m for ply-angles $30^0/0^0/30^0$, $45^0/0^0/45^0$ and $60^0/0^0/60^0$, respectively on taper ratio η. In this case, the values of frequency parameters are lesser than the values obtained for C-C conditions. Similarly, Figure 7a, b and c shows the variation of λ_m for ply-angles $30^0/0^0/30^0$, $45^0/0^0/45^0$ and $60^0/0^0/60^0$, respectively with exponential thickness variation.

Tables 2 and 3 depict how the values of the length parameter L affects ω (in 10^3 Hz) for three and five

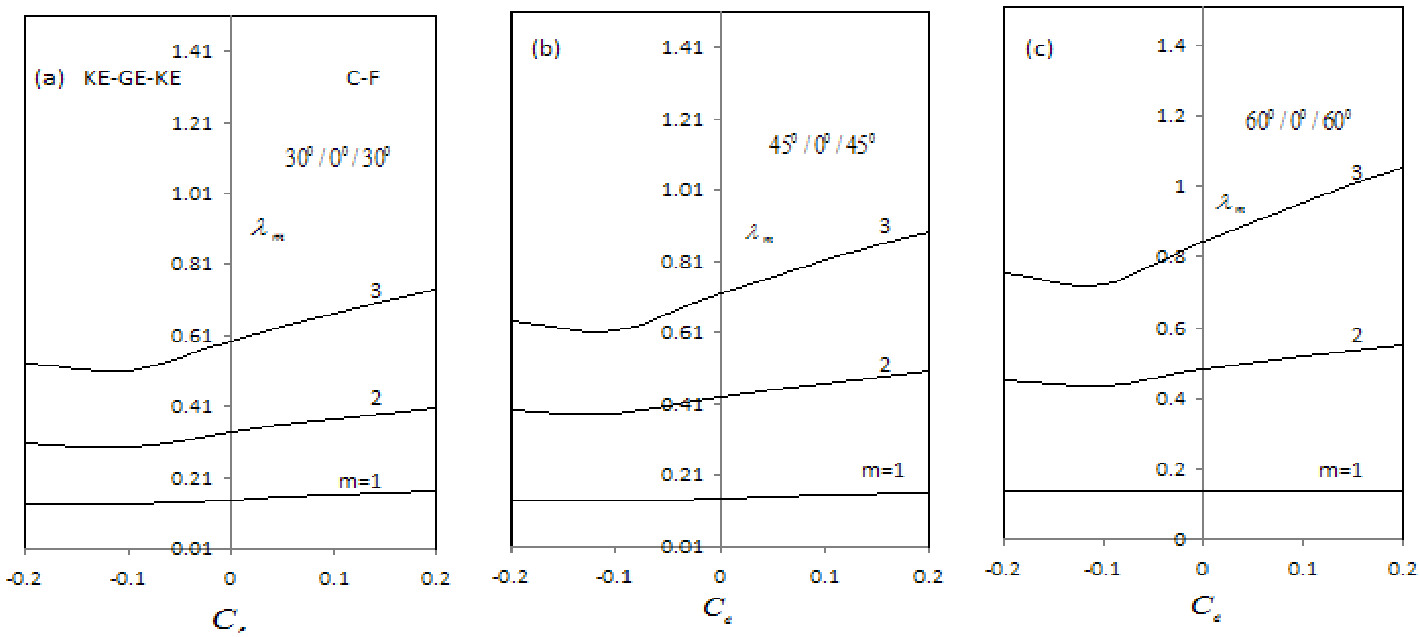

Figure 7. Variation of frequency parameter of three layered symmetric angle-ply shells of exponential variation in thickness under C-F boundary conditions.

Table 2. Effect of the length parameter L on the fundamental frequency $\omega(\times 10^3 Hz)$ of clamped-clamped boundary conditions for three- and five layered symmetric angle-ply using KE and GE materials.

L	30°/0°/30°			45°/30°/0°/30°/45°		
	η = 0.7	C_e = 0.2	C_s = 0.25	η = 0.7	C_e = 0.2	C_s = 0.25
0.5	1.37302	1.50407	1.37302	1.42475	1.56128	1.42342
0.75	1.16578	1.21177	0.97467	1.37273	1.42676	1.14617
1	1.01593	1.056	0.84931	1.26155	1.32226	1.05283
1.25	0.93776	0.97475	0.78398	1.19217	1.23902	0.99443
1.5	0.87979	0.9145	0.73554	1.13074	1.17514	0.94285
1.75	0.8283	0.86098	0.69254	1.0677	1.10962	0.89023
2	0.77919	0.80994	0.65154	1.00036	1.03966	0.83425

$H = 0.02$, $n = 2$. Order of the layered materials: KE-GE-KE (three-layers); KE-GE-KE-GE-KE (five-layers).

Table 3. Effect of the length parameter L on the fundamental frequency $\omega(\times 10^3 Hz)$ of a clamped-free boundary conditions for three and five layered symmetric angle-ply using KE and GE materials.

L	30°/0°/30°			45°/30°/0°/30°/45°		
	η = 0.7	C_e = 0.2	C_s = 0.25	η = 0.7	C_e = 0.2	C_s = 0.25
0.5	1.37302	1.50407	1.37302	1.42475	1.56128	1.42342
0.75	1.16578	1.21177	0.97467	1.37273	1.42676	1.14617
1	1.01593	1.056	0.84931	1.26155	1.32226	1.05283
1.25	0.93776	0.97475	0.78398	1.19217	1.23902	0.99443
1.5	0.87979	0.9145	0.73554	1.13074	1.17514	0.94285
1.75	0.8283	0.86098	0.69254	1.0677	1.10962	0.89023
2	0.77919	0.80994	0.65154	1.00036	1.03966	0.83425

$H = 0.02$, $n = 2$. Order of the layered materials: KE-GE-KE (three-layers); KE-GE-KE-GE-KE (five-layers).

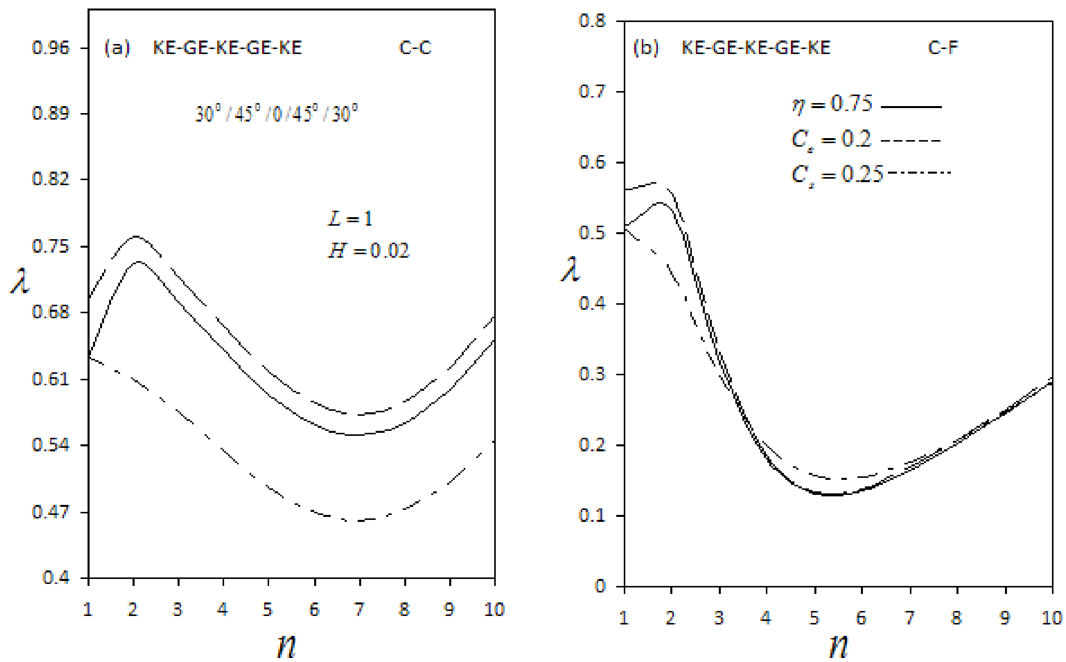

Figure 8. Effect of circumferential node number on fundamental frequency parameter for linear, exponential and sinusoidal variation in thickness with C-C and C-F boundary conditions.

layered shells with ply-angles $30^0/0^0/30^0$ and $45^0/30^0/0^0/30^0/45^0$ under clamped-clamped and clamped-free boundary conditions, respectively. Linear, exponential and sinusoidal thickness variations are analyzed. It is seen from the table that $\omega_m(m=1,2,3)$ decreases as L increases. The increase is fast for very short shells and the rate of decrease is higher for higher modes. Figure 8 presents the variation of frequency parameter with reference to the circumferential node number n. The range of n is considered between 1 and 10. A shell of KE-GE-KE-GE-KE lamination under C-C and C-F boundary conditions is considered with $H=0.02, L=1$. All the three types of variation in thickness of layers are considered, as indicated in the diagrams. Figure 8a shows the effect of n on fundamental frequency parameter λ for C-C conditions. In the case of linear and exponential thickness variations, it is seen that the fundamental frequency parameter values increase up to $n=2$ and then decrease up to $n=7$. Again, there is an increase of λ for n = 7 onwards, but in the case of sinusoidal variation, λ decrease up to $n=7$ and then, increase. Figure 8b shows the effect of n on fundamental frequency parameter λ for C-F conditions. The absolute and relative differences between the maximum and minimum values of λ, caused in the range of values of n considered, is more in the case of C-F boundary conditions than with that of C-C boundary conditions. It seems that the thickness variation in layers does greatly affect the nature of the variation of λ with n.

Conclusion

The variation of frequencies of the three and five layered symmetric angle-ply cylindrical shells of variable thickness with inclusion of shear deformation theory is analyzed. Two types of layered materials, length ratio, coefficients of variable thickness and ply-angles affect the frequency with C-C and C-F boundary conditions. We can choose the desired frequency of vibration from the results by a proper choice of the coefficient of thickness variations and arrangement of ply-angles. The clamped-clamped boundary conditions gave rise to higher frequencies in comparison with the clamped-free boundary conditions. The nature of variation in thickness of layers considerably affects the natural frequencies. The effect of increasing the length of the cylinder is to decrease in frequencies, for all kinds of variation in thickness of layers.

ACKNOWLEDGEMENT

The authors thankfully acknowledge the financial support from Research Management Centre (RMC)-Universiti Teknologi Malaysia under University Short Term Research Grant-Vote No. 4D023 for the completion of

this research work.

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APPENDIX

The quantities $\overline{Q}_{ij}^{(k)}\ (i, j=1,2,4,5,6)$ appearing in Equation 4 are defined by:

$$\overline{Q}_{11}^{(k)} = Q_{11}^{(k)}\cos^4\alpha + Q_{22}^{(k)}\sin^4\alpha + 2(Q_{12}^{(k)} + 2Q_{66}^{(k)})\sin^2\alpha\cos^2\alpha \tag{1}$$

$$\overline{Q}_{22}^{(k)} = Q_{11}^{(k)}\sin^4\alpha + Q_{22}^{(k)}\cos^4\alpha + 2(Q_{12}^{(k)} + 2Q_{66}^{(k)})\sin^2\alpha\cos^2\alpha \tag{2}$$

$$\overline{Q}_{12}^{(k)} = (Q_{11}^{(k)} + Q_{22}^{(k)} - 4Q_{66}^{(k)})\sin^2\alpha\cos^2\alpha + Q_{12}^{(k)}(\cos^4\alpha + \sin^4\alpha) \tag{3}$$

$$\overline{Q}_{16}^{(k)} = (Q_{11}^{(k)} - Q_{12}^{(k)} - 2Q_{66}^{(k)})\cos^3\alpha\sin\alpha - (Q_{22}^{(k)} - Q_{12}^{(k)} - 2Q_{66}^{(k)})\sin^3\alpha\cos\alpha \tag{4}$$

$$\overline{Q}_{26}^{(k)} = (Q_{11}^{(k)} - Q_{12}^{(k)} - 2Q_{66}^{(k)})\cos\alpha\sin^3\alpha - (Q_{22}^{(k)} - Q_{12}^{(k)} - 2Q_{66}^{(k)})\sin\alpha\cos^3\alpha \tag{5}$$

$$\overline{Q}_{66}^{(k)} = (Q_{11}^{(k)} + Q_{22}^{(k)} - 2Q_{12}^{(k)} - 2Q_{66}^{(k)})\cos^2\alpha\sin^2\alpha + Q_{66}^{(k)}(\sin^4\alpha + \cos^4\alpha) \tag{6}$$

$$\overline{Q}_{44}^{(k)} = Q_{55}^{(k)}\sin^2\alpha + Q_{44}^{(k)}\cos^2\alpha \tag{7}$$

$$\overline{Q}_{55}^{(k)} = Q_{55}^{(k)}\cos^2\alpha + Q_{44}^{(k)}\sin^2\alpha \tag{8}$$

$$\overline{Q}_{45}^{(k)} = (Q_{55}^{(k)} - Q_{44}^{(k)})\cos\alpha\sin\alpha \tag{9}$$

Where

$$Q_{11}^{(k)} = \frac{E_x^{(k)}}{1 - \upsilon_{x\theta}^{(k)}\upsilon_{\theta x}^{(k)}}, \quad Q_{12}^{(k)} = \frac{\upsilon_{x\theta}^{(k)}E_\theta^{(k)}}{1 - \upsilon_{x\theta}^{(k)}\upsilon_{\theta x}^{(k)}} = \frac{\upsilon_{\theta x}^{(k)}E_x^{(k)}}{1 - \upsilon_{x\theta}^{(k)}\upsilon_{\theta x}^{(k)}}$$

$$Q_{22}^{(k)} = \frac{E_\theta^{(k)}}{1 - \upsilon_{x\theta}^{(k)}\upsilon_{\theta x}^{(k)}}, \quad Q_{66}^{(k)} = G_{x\theta}^{(k)}, \quad Q_{44}^{(k)} = G_{\theta z}^{(k)}, \quad Q_{55}^{(k)} = G_{xz}^{(k)} \tag{10}$$