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The generalized projective Riccati equations method and its applications for solving two nonlinear PDEs describing microtubules

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Microtubules (MTs) are major cytoskeletal proteins. They are hollow cylinders formed by protofilaments (PFs) representing series of proteins known as tubulin dimers. Each dimer is an electric dipole. These diamers are in a straight position within PFs or in radially displaced positions pointing out of cylindrical surface. In this paper, the authors demonstrate how the generalized projective Riccati equations method can be used in the study of the nonlinear dynamics of MTs. To this end, the authors apply this method to construct the exact solutions with parameters for two nonlinear PDEs describing MTs. The first equation describes the model of microtubules as nanobioelectronics transmission lines. The second equation describes the dynamics of radial dislocations in microtubules. As a result, hyperbolic, trigonometric and rational function solutions are obtained. When these parameters are taken as special values, solitary wave solutions are derived from the exact solutions. Comparison between our recent results and the well-known results is given.

Key words: Generalized projective Riccati equations method, models of microtubules (MTs), exact solutions, solitary solutions, trigonometric solutions rational solutions.

INTRODUCTION

In the recent years, investigations of exact solutions to nonlinear partial differential equations (NPDEs) play an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appear in various scientific and engineering field, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. To obtain traveling wave solutions, many powerful methods have been presented, such as the $\exp(-\varphi(\xi))$ expansion method (Hafez et al., 2014), the tanh-sech method (Malfiieiet, 1992; Malfiieiet and Hereman, 1996; Wazwaz, 2004a), extended tanh-method (EL-Wakil and Abdou, 2007; Fan, 2000; Wazwaz, 2007), sine-cosine method (Wazwaz, 2004b, 2005; Yan, 1996), homogeneous balance method (Fan and Zhang, 1998;

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Wang, 1996), Jacobi elliptic function method (Dai and Zhang, 2006; Fan and Zhang, 2002; Liu et al., 2001; Zhao et al., 2006), F-expansion method (Abdou, 2007; Ren and Zhang, 2006; Zhang et al., 2006), exp-function method (He and Wu, 2006; Aminikhad et al., 2009), trigonometric function series method (Zhang, 2008), $(\frac{G'}{G})$-expansion method (Zhang et al., 2008; Zayed and Gepreel, 2009; Younis and Zafar, 2014; Younis, 2014a, b; Zayed, 2009; Hayek, 2010), the (G'/G,1/G)-expansion method (Zayed and Hoda Ibrahim, 2013a; Zayed and Alurrfi, 2014a, b, c), the modified simple equation method (Jawad et al., 2010; Zayed, 2011; Zayed and Hoda Ibrahim, 2012, 2013b, 2014 ; Zayed and Arnous, 2012), the first integral method (Moosaei et al., 2011; Bekir and Unsal, 2012; Lu et al., 2010; Feng, 2002), the multiple exp-function algorithm method (Ma et al., 2010; Ma and Zhu, 2012), the transformed rational function method (Ma and Lee, 2009), the Frobenius decomposition technique (Ma et al., 2007), the local fractional variation iteration method (Yang et al., 2013), the local fractional series expansion method (Yang et al., 2013), the generalized projective Riccati equations method (Conte and Musette, 1992; Zayed and Alurrfi, 2014d; Zhang et al., 2001; Yan, 2003; Yomba, 2005), the generalized $(\frac{G'}{G})$-expansion method (Alam and Akbar, 2013; 2014a, b, 2015; Alam et al., 2014a, b, c, d) and so on. Conte and Musette (1992) presented an indirect method to seek more solitary wave solutions of some NPDEs that can be expressed as polynomials in two elementary functions which satisfy a projective Riccati equation (Bountis et al. 1986). Using this method, many solitary wave solutions of many NPDEs are found (Zhang et al., 2001; Bountis et al. 1986). Recently, Yan (2003) developed further Conte and Musette's method by introducing more generalized projective Riccati equations.

The objective of this paper is to apply the generalized projective Riccati equations method to construct the exact solutions for the following two nonlinear PDEs of microtubules (MTs):

(i) The nonlinear PDE describing the nonlinear dynamics of MTs as nanobioelectronics transmission lines:

$$m\frac{\partial^2 z(x,t)}{\partial t^2}-kl^2\frac{\partial^2 z(x,t)}{\partial x^2}-qE-Az(x,t)+Bz^3(x,t)+\gamma\frac{\partial z(x,t)}{\partial t}=0, \quad (1)$$

where $z(x,t)$ is the traveling wave, m is the mass of the dimer, k is a harmonic constant describing the nearest-neighbor interaction between the dimers belonging to the same protofilaments (PFs), l is the MT length, E is the magnitude of intrinsic electric field, $q>0$ is the excess charge within the dipole, γ is the viscosity coefficient and A,B are positive parameters. The physical details of the derivation of Equation (1) has been discussed in Zekovic et al. (2014) which are omitted here for simplicity. The authors (Zekovic et al., 2014) have used the Jacobi elliptic function method to find the exact solutions of Equation (1).

(ii) The nonlinear PDE describing the nonlinear dynamics of radial dislocations in MTs:

$$I\frac{\partial^2\phi(x,t)}{\partial t^2}-ch^2\frac{\partial^2\phi(x,t)}{\partial x^2}+pH\phi(x,t)-\frac{pH}{6}\phi^3(x,t)+\Gamma\frac{\partial\phi(x,t)}{\partial t}=0, \quad (2)$$

where $\phi(x,t)$ is the corresponding angular displacement when the whole dimer rotates with the angular displacement $\phi(x,t)$, I is the moment of inertia of the single dimer, c stands for inter-dimer bonding interaction within the same protofilaments (PFs), h is the MT length, p is the electric dipole moment, H is the magnitude of intrinsic electric field and Γ is the viscosity coefficient. The physical details of the derivation of Equation (2) has been discussed in Zdravkovic et al. (2014) which are omitted here for simplicity. The authors (Zdravkovic et al., 2014) have used the simplest equation method to find the exact solutions of Equation (2).

Description of the generalized projective Riccati equations method

Considering the following NPDE:

$$F(u,u_t,u_x,u_{tt},u_{xt},u_{xx},...)=0, \quad (3)$$

where F is a polynomial in $u(x,t)$ and its partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following, the authors give the main steps (Conte and Musette, 1992; Zayed and Alurrfi, 2014d; Zhang et al., 2001; Yan, 2003; Yomba, 2005) of this method.

Step 1. The authors use the wave transformation

$$u(x,t)=u(\xi), \quad \xi=k_1x+\omega t, \quad (4)$$

where k_1, and ω are constants, to reduce Equation (3) to the following ODE:

$$Q(u,u',u'',...)=0, \quad (5)$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, such that $'=\frac{d}{d\xi}$.

Step 2. The authors assume that Equation (5) has the formal solution:

$$u(\xi)=A_0+\sum_{i=1}^{N}\sigma^{i-1}(\xi)\left[A_i\sigma(\xi)+B_i\tau(\xi)\right], \tag{6}$$

where A_0, A_i and B_i are constants to be determined later. The functions $\sigma(\xi)$ and $\tau(\xi)$ satisfy the ODEs:

$$\sigma'(\xi)=\varepsilon\sigma(\xi)\tau(\xi) \tag{7}$$

$$\tau'(\xi)=R+\varepsilon\tau^2(\xi)-\mu\sigma(\xi), \quad \varepsilon=\pm1, \tag{8}$$

Where

$$\tau^2(\xi)=-\varepsilon\left(R-2\mu\sigma(\xi)+\frac{\mu^2+r}{R}\sigma^2(\xi)\right), \tag{9}$$

where $r=\pm1$ and R, μ are nonzero constants.

If $R,=\mu=0,$ Equation (5) has the formal solution:

$$u(\xi)=\sum_{i=0}^{N}A_i\tau^i(\xi), \tag{10}$$

where $\tau(\xi)$ satisfies the ODE:

$$\tau'(\xi)=\tau^2(\xi). \tag{11}$$

Step 3. The authors determine the positive integer N in (6) by using the homogeneous balance between the highest-order derivatives and the nonlinear terms in Equation (5).

Step 4. Substitute (6) along with Equations (7) - (9) into Equation (5) or ((10) along with Equation (11) into Equation (5)). Collecting all terms of the same order of $\sigma^j(\xi)\tau^i(\xi)$ $(j=0,1,...;i=0,1)$ (or $\tau^i(\xi)$, $j=0,1,...$). Setting each coefficient to zero, yields a set of algebraic equations which can be solved to find the values of $A_0, A_i, B_i, k_1, \omega, \mu$ and R .

Step 4. It is well known (Yomba, 2005) that Equations (7) and (8) admit the following solutions:

Case 1. When $\varepsilon=-1, r=-1, R>0,$

$$\sigma_1(\xi)=\frac{R\,\mathrm{sech}\left(\sqrt{R}\,\xi\right)}{\mu\,\mathrm{sech}\left(\sqrt{R}\,\xi\right)+1}, \quad \tau_1(\xi)=\frac{\sqrt{R}\,\tanh\left(\sqrt{R}\,\xi\right)}{\mu\,\mathrm{sech}\left(\sqrt{R}\,\xi\right)+1}, \tag{12}$$

Case 2. When $\varepsilon=-1, r=1, R>0,$

$$\sigma_2(\xi)=\frac{R\,\mathrm{csch}\left(\sqrt{R}\,\xi\right)}{\mu\,\mathrm{csch}\left(\sqrt{R}\,\xi\right)+1}, \quad \tau_2(\xi)=\frac{\sqrt{R}\,\coth\left(\sqrt{R}\,\xi\right)}{\mu\,\mathrm{csch}\left(\sqrt{R}\,\xi\right)+1}, \tag{13}$$

Case 3. When $\varepsilon=1, r=-1, R>0,$

$$\sigma_3(\xi)=\frac{R\sec\left(\sqrt{R}\,\xi\right)}{\mu\sec\left(\sqrt{R}\,\xi\right)+1}, \quad \tau_3(\xi)=\frac{\sqrt{R}\,\tan\left(\sqrt{R}\,\xi\right)}{\mu\sec\left(\sqrt{R}\,\xi\right)+1}, \tag{14}$$

$$\sigma_4(\xi)=\frac{R\csc\left(\sqrt{R}\,\xi\right)}{\mu\csc\left(\sqrt{R}\,\xi\right)+1}, \quad \tau_4(\xi)=-\frac{\sqrt{R}\,\cot\left(\sqrt{R}\,\xi\right)}{\mu\csc\left(\sqrt{R}\,\xi\right)+1}, \tag{15}$$

Case 4. $R,=\mu=0,$

$$\sigma_5(\xi)=\frac{C}{\xi}, \quad \tau_5(\xi)=\frac{1}{\varepsilon\xi}, \tag{16}$$

where C is nonzero constant.

Step 6. Substituting the values of $A_0, A_i, B_i, k_1, \omega, \mu$ and R. as well as the solutions (12) - (16) into (6) the authors obtain the exact solutions of Equation (3).

APPLICATIONS

In this part, the authors will apply the proposed method described in description of the generalized projective Riccati equations method, to find the exact solutions of the two nonlinear PDEs (1) and (2).

Example 1. Exact solutions of the nonlinear PDE (1) describing the nonlinear dynamics of MTS as nanobioelectronics transmission lines

The authors find the exact wave solutions of Equation (1). To this end, the authors use the transformation (4) to reduce Equation (1) into the following ODE:

$$\alpha\psi''(\xi)-\rho\psi'(\xi)-\psi(\xi)+\psi^3(\xi)-\delta=0, \tag{17}$$

where

$$\alpha=\frac{m\omega^2-kl^2k_1^2}{A}, \rho=\frac{\gamma\omega}{A}, \delta=\frac{qE}{A\sqrt{A/B}}, \tag{18}$$

and

$$z(\xi)=\sqrt{\frac{A}{B}}\psi(\xi). \tag{19}$$

Balancing $\psi''(\xi)$ with $\psi^3(\xi)$ in Equation (17), the authors get $N=1$. Consequently, the authors have the formal solution of Equation (17) as follows:

$$\psi(\xi)=A_0+A_1\sigma(\xi)+B_1\tau(\xi). \tag{20}$$

where A_0, A_1 and B_1 are constants to be determined later.
Substituting (20) into (17) and using (7) - (9), the left-hand side of Equation (17) becomes a polynomial in $\sigma(\xi)$ and $\tau(\xi)$. Setting the coefficients of this polynomial to be zero, yields the following system of algebraic equations:

$$\sigma^3(\xi):\quad RA_1^3+(\mu^2+r)(2\alpha A_1\varepsilon^2+3A_1B_1^2)=0,$$

$$\sigma^2(\xi):\quad (\mu^2+r)(3A_0B_1^2-\rho B_1\varepsilon)-2R(2\alpha A_1\varepsilon^2+3A_1B_1^2)\mu-\varepsilon\alpha\mu RA_1+3RA_0A_1^2=0,$$

$$\sigma^2(\xi)\tau(\xi):\quad 3RA_1^2B_1+(\mu^2+r)(2\alpha B_1\varepsilon^2+B_1^3)=0,$$

$$\sigma(\xi):\quad -2\mu(3A_0B_1^2-\rho B_1\varepsilon)+R(2\alpha A_1\varepsilon^2+3A_1B_1^2)-A_1+\varepsilon\alpha A_1R+3A_0^2A_1+\rho\mu B_1=0,$$

$$\sigma(\xi)\tau(\xi):\quad -\varepsilon\rho A_1+6A_0A_1B_1-3\varepsilon\alpha\mu B_1-2(2\alpha B_1\varepsilon^2+B_1^3)\mu=0,$$

$$\tau(\xi):\quad 2\alpha B_1\varepsilon R-B_1+3A_0^2B_1+(2\alpha B_1\varepsilon^2+B_1^3)R=0,$$

$$\sigma^0(\xi):\quad R(3A_0B_1^2-\rho B_1\varepsilon)-A_0+A_0^3-\rho RB_1-\delta=0. \tag{21}$$

Case 1. If authors substitute $\varepsilon=-1$ into the algebraic equations (21) and solve them by Maple 14, the following results were realized:

Result 1. The authors have

$$A_0=\pm\frac{\rho}{6}\sqrt{\frac{-2}{\alpha}},\ A_1=0,\ B_1=\pm\sqrt{-2\alpha},\ R=-\frac{\rho^2+6\alpha}{12\alpha^2},\ \mu=0, \tag{22}$$
$$\delta=\pm\frac{\rho(2\rho^2+9\alpha)\sqrt{-2\alpha}}{27\alpha^2}, r=r$$

where $\alpha<0,\ \rho^2+6\alpha<0$.

From (12), (13), (19), (20) and (22), the authors deduce that if $r=-1$, then the exact wave solution was realized:

$$z(\xi)=\pm\sqrt{\frac{-2A}{\alpha B}}\left[\frac{\rho}{6}+\sqrt{-\frac{\rho^2+6\alpha}{12}}\tanh\left(\sqrt{-\frac{\rho^2+6\alpha}{12\alpha^2}}\xi\right)\right], \tag{23}$$

while if $r=1$, then the authors have the exact wave solution

$$z(\xi)=\pm\sqrt{\frac{-2A}{\alpha B}}\left[\frac{\rho}{6}+\sqrt{-\frac{\rho^2+6\alpha}{12}}\coth\left(\sqrt{-\frac{\rho^2+6\alpha}{12\alpha^2}}\xi\right)\right]. \tag{24}$$

Result 2. The authors have

$$A_0=\pm\frac{\rho}{3}\sqrt{\frac{-2}{\alpha}},\ A_1=0,\ B_1=\pm\sqrt{\frac{-2}{\alpha}},\ R=-\frac{\rho^2+6\alpha}{3\alpha^2},\ \mu=\pm\sqrt{-r}, \tag{25}$$
$$\delta=\pm\frac{\rho(2\rho^2+9\alpha)\sqrt{-2\alpha}}{27\alpha^2},$$

where $\alpha<0, r<0,\ \rho^2+6\alpha<0$.

In this case, the authors deduce that if $r=-1$, then the exact wave solution was realized:

$$z(\xi)=\pm\sqrt{\frac{-A}{2\alpha B}}\left[\frac{\rho}{3}+\frac{\sqrt{-\frac{\rho^2+6\alpha}{3}}\tanh\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{1\pm\operatorname{sech}\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}\right]. \tag{26}$$

Result 3. The authors have

$$A_0=\pm\frac{\rho}{3}\sqrt{\frac{-2}{\alpha}},\ A_1=\pm\sqrt{\frac{3\alpha^3(\mu^2+r)}{2(\rho^2+6\alpha)}},\ B_1=\pm\sqrt{\frac{-2}{\alpha}},\ R=-\frac{\rho^2+6\alpha}{3\alpha^2},\ \mu=\mu, \tag{27}$$
$$\delta=\pm\frac{\rho(2\rho^2+9\alpha)\sqrt{-2\alpha}}{27\alpha^2},$$

where $\alpha<0,\ \rho^2+6\alpha<0,\ \mu^2+r>0$.
In this case, the authors deduce that if $r=-1$, then the exact wave solution was realized:

$$z(\xi)=\pm\sqrt{\frac{-A}{2\alpha B}}\left[\frac{\rho}{3}+\sqrt{-\frac{\rho^2+6\alpha}{3}}\times\left(\frac{\left(\sqrt{\mu^2-1}\right)\operatorname{sech}\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)+\tanh\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{\mu\operatorname{sech}\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)+1}\right)\right], \tag{28}$$

while if $r=1$, then the authors have the exact wave solution

$$z(\xi)=\pm\sqrt{\frac{-A}{2\alpha B}}\left[\frac{\rho}{3}+\sqrt{-\frac{\rho^2+6\alpha}{3}}\times\left(\frac{\left(\sqrt{\mu^2+1}\right)\operatorname{csch}\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)+\coth\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{\mu\operatorname{csch}\left(\sqrt{-\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)+1}\right)\right]. \tag{29}$$

Case 2. If the authors substitute $\varepsilon=1$ and $r=-1$ into the algebraic Equations (21) and solve them by Maple 14, the authors have the following results:

Result 1. The authors have

$$A_0=\mp\frac{\rho}{6}\sqrt{\frac{-2}{\alpha}},\ A_1=0,\ B_1=\pm\sqrt{-2\alpha},\ R=\frac{\rho^2+6\alpha}{12\alpha^2},\ \mu=0, \tag{30}$$
$$\delta=\mp\frac{\rho(2\rho^2+9\alpha)\sqrt{-2\alpha}}{27\alpha^2},$$

where $\alpha < 0$, $\rho^2 + 6\alpha > 0$.

From (14), (15), (19), (20) and (30), the authors deduce the following exact wave solutions

$$z(\xi) = \pm\sqrt{\frac{-2A}{\alpha B}}\left[-\frac{\rho}{6} + \sqrt{\frac{\rho^2+6\alpha}{12}}\tan\left(\sqrt{\frac{\rho^2+6\alpha}{12\alpha^2}}\xi\right)\right], \quad (31)$$

or

$$z(\xi) = \mp\sqrt{\frac{-2A}{\alpha B}}\left[\frac{\rho}{6} + \sqrt{\frac{\rho^2+6\alpha}{12}}\cot\left(\sqrt{\frac{\rho^2+6\alpha}{12\alpha^2}}\xi\right)\right]. \quad (32)$$

Result 2. The authors have

$$A_0 = \mp\frac{\rho}{3}\sqrt{\frac{-2}{\alpha}},\ A_1 = 0,\ B_1 = \pm\sqrt{\frac{-2}{\alpha}},\ R = \frac{\rho^2+6\alpha}{3\alpha^2},\ \mu = \pm 1, \quad (33)$$
$$\delta = \mp\frac{\rho(2\rho^2+9\alpha)\sqrt{-2\alpha}}{27\alpha^2},$$

where $\alpha < 0,\ \rho^2 + 6\alpha > 0$.

In this case, the authors deduce the exact wave solutions

$$z(\xi) = \pm\sqrt{\frac{-A}{2\alpha B}}\left[-\frac{\rho}{3} + \frac{\sqrt{\frac{\rho^2+6\alpha}{3}}\tan\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{1 \pm \sec\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}\right], \quad (34)$$

or

$$z(\xi) = \mp\sqrt{\frac{-A}{2\alpha B}}\left[\frac{\rho}{3} + \frac{\sqrt{\frac{\rho^2+6\alpha}{3}}\cot\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{1 \pm \csc\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}\right]. \quad (35)$$

Result 3. The authors have

$$A_0 = \mp\frac{\rho}{3}\sqrt{\frac{-2}{\alpha}},\ A_1 = \pm\sqrt{-\frac{3\alpha^3(1-\mu^2)}{2(\rho^2+6\alpha)}},\ B_1 = \pm\sqrt{\frac{-2}{\alpha}},\ R = \frac{\rho^2+6\alpha}{3\alpha^2},\ \mu = \mu, \quad (36)$$
$$\delta = \mp\frac{\rho(2\rho^2+9\alpha)\sqrt{-2\alpha}}{27\alpha^2},$$

where $\alpha < 0$, $\rho^2 + 6\alpha > 0$ and $\alpha^3(1-\mu^2) < 0$.

In this case, the authors deduce the exact wave solutions

$$z(\xi) = \pm\sqrt{\frac{-A}{2\alpha B}}\left[-\frac{\rho}{3} + \sqrt{\frac{\rho^2+6\alpha}{3}} \times \left(\frac{\left(\sqrt{1-\mu^2}\right)\sec\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right) + \tan\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{\mu\sec\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)+1}\right)\right], \quad (37)$$

or

$$z(\xi) = \mp\sqrt{\frac{-A}{2\alpha B}}\left[\frac{\rho}{3} + \sqrt{\frac{\rho^2+6\alpha}{3}} \times \left(\frac{\left(\sqrt{1-\mu^2}\right)\csc\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right) + \cot\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)}{\mu\csc\left(\sqrt{\frac{\rho^2+6\alpha}{3\alpha^2}}\xi\right)+1}\right)\right]. \quad (38)$$

Case 3. ($R = 0, \mu = 0$)

Substituting $\psi(\xi) = A_0 + A_1\tau(\xi)$ into (17) and using (11), the left-hand side of Equation (17) becomes a polynomial in $\tau(\xi)$. Setting the coefficients of this polynomial to be zero, yields the following system of algebraic equations:

$$\tau^3(\xi):\quad A_1^3 + 2\alpha A_1 = 0,$$

$$\tau^2(\xi):\quad -\rho A_1 + 3A_0A_1^2 = 0,$$

$$\tau(\xi):\quad -A_1 + 3A_0^2A_1 = 0,$$

$$\tau^0(\xi):\quad A_0^3 - A_0 - \delta = 0.$$

On solving the above, the algebraic equations using the Maple 14, the authors have the following result:

$$A_0 = \pm\sqrt{\frac{1}{3}},\ A_1 = \pm\rho\sqrt{\frac{1}{3}},\ \alpha = -\frac{1}{6}\rho^2,\ \delta = \mp\frac{2}{3}\sqrt{\frac{1}{3}}. \quad (39)$$

From (10), (16), (19) and (39), the authors deduce the following rational solution

$$z(\xi) = \pm\sqrt{\frac{A}{3B}}\left[1 - \frac{\rho}{\xi}\right]. \quad (40)$$

Example 2. Exact solutions of the nonlinear PDE (2) describing the nonlinear dynamics of radial dislocations in MTs

In this subsection, the authors find the exact solutions of Equation (2). To this end, the authors use the transformation (4) to reduce Equation (2) into the following ODE:

$$\alpha\psi''(\xi) - \rho\psi'(\xi) + \psi(\xi) - \psi^3(\xi) = 0, \quad (41)$$

where

$$\alpha = \frac{I\,\omega^2 - ch^2 k_1^{\,2}}{pH}, \rho = \frac{\omega\Gamma}{pH}, \tag{42}$$

and

$$\phi(\xi) = \sqrt{6}\psi(\xi). \tag{43}$$

Balancing $\psi''(\xi)$ with $\psi^3(\xi)$ in Equation (41), the authors get $N = 1$. Consequently, the authors have the formal solution of Equation (41) as follows:

$$\psi(\xi) = A_0 + A_1\sigma(\xi) + B_1\tau(\xi). \tag{44}$$

where A_0, A_1 and B_1 are constants to be determined later.

Substituting (44) into (41) and using (7) - (9), the left-hand side of Equation (41) becomes a polynomial in $\sigma(\xi)$ and $\tau(\xi)$. Setting the coefficients of this polynomial to be zero, yields the following system of algebraic equations:

$$\sigma^3(\xi): \quad -RA_1^3 - \varepsilon(\mu^2 + r)(2\alpha A_1\varepsilon^2 - 3A_1B_1^2) = 0,$$

$$\sigma^2(\xi): \quad -(\mu^2 + r)\varepsilon(-3A_0B_1^2 - \rho B_1\varepsilon) + 2\varepsilon R(2\alpha A_1\varepsilon^2 - 3A_1B_1^2)\mu - \varepsilon\alpha\mu RA_1 - 3RA_0A_1^2 = 0,$$

$$\sigma^2(\xi)\tau(\xi): \quad -3RA_1^2B_1 - \varepsilon(\mu^2 + r)(2\alpha B_1\varepsilon^2 - B_1^3) = 0,$$

$$\sigma(\xi): \quad -2\mu\varepsilon(3A_0B_1^2 + \rho B_1\varepsilon) - \varepsilon R(2\alpha A_1\varepsilon^2 - 3A_1B_1^2) + A_1 + \varepsilon\alpha A_1 R - 3A_0^2A_1 + \rho\mu B_1 = 0,$$

$$\sigma(\xi)\tau(\xi): \quad -\varepsilon\rho A_1 - 6A_0A_1B_1 - 3\varepsilon\alpha\mu B_1 + 2\varepsilon\mu(2\alpha B_1\varepsilon^2 - B_1^3) = 0,$$

$$\tau(\xi): \quad 2\alpha B_1\varepsilon R + B_1 - 3A_0^2B_1 - \varepsilon R(2\alpha B_1\varepsilon^2 - B_1^3) = 0,$$

$$\sigma^0(\xi): \quad R\varepsilon(3A_0B_1^2 + \rho B_1\varepsilon) + A_0 - A_0^3 - \rho RB_1 = 0. \tag{45}$$

If the authors substitute $\varepsilon = -1$ into the algebraic Equations (45) and solve them by Maple 14, the authors have the following results:

Result 1. The authors have

$$A_0 = \pm\frac{1}{2},\ A_1 = 0,\ B_1 = \pm\frac{2}{3},\ R = \frac{9}{16\rho^2},\ \mu = 0, \alpha = \frac{2}{9}\rho^2, r = r. \tag{46}$$

From (12), (13), (43), (44) and (46), the authors deduce that if $r = -1$, then the authors have the exact wave solution

$$\phi(\xi) = \pm\frac{\sqrt{6}}{2}\left[1 + \tanh\left(\frac{3}{4\rho}\xi\right)\right], \tag{47}$$

while if $r = 1$, then the authors have the exact wave solution

$$\phi(\xi) = \pm\frac{\sqrt{6}}{2}\left[1 + \coth\left(\frac{3}{4\rho}\xi\right)\right]. \tag{48}$$

Note that our solution (47) is in agreement with the solution (43) obtained in Zdravkovic et al. (2014).

Result 2. The authors have

$$A_0 = \pm\frac{1}{2},\ A_1 = 0,\ B_1 = \pm\frac{1}{3}\rho,\ R = \frac{9}{4\rho^2},\ \mu = \pm\sqrt{-r}, \alpha = \frac{2}{9}\rho^2, \tag{49}$$

where $r < 0$.

In this case, the authors deduce that if $r = -1$, then the authors have the exact wave solution

$$\phi(\xi) = \pm\frac{\sqrt{6}}{2}\left[1 + \left(\frac{\tanh\left(\frac{3}{2\rho}\xi\right)}{1 \pm \operatorname{sech}\left(\frac{3}{2\rho}\xi\right)}\right)\right], \tag{50}$$

Result 3. The authors have

$$A_0 = \pm\frac{1}{2},\ A_1 = \pm\frac{2\rho^2\sqrt{r}}{9},\ B_1 = \pm\frac{1}{3}\rho,\ R = \frac{9}{4\rho^2},\ \mu = 0, \alpha = \frac{2}{9}\rho^2, \tag{51}$$

where $r > 0$.

In this case, the authors deduce that if $r = 1$, then the authors have the exact wave solution

$$\phi(\xi) = \pm\frac{\sqrt{6}}{2}\left[1 + \operatorname{csch}\left(\frac{3}{2\rho}\xi\right) + \coth\left(\frac{3}{2\rho}\xi\right)\right]. \tag{52}$$

Result 4. The authors have

$$A_0 = \pm\frac{1}{2},\ A_1 = \pm\frac{2\rho^2\sqrt{\mu^2 + r}}{9},\ B_1 = \pm\frac{1}{3}\rho,\ R = \frac{9}{4\rho^2},\ \mu = \mu, \alpha = \frac{2}{9}\rho^2, \tag{53}$$

where $\mu^2 + r > 0$.

In this case, the authors deduce that if $r = -1$, then the authors have the exact wave solution

$$\phi(\xi) = \pm\frac{\sqrt{6}}{2}\left[1 + \frac{\left(\sqrt{\mu^2 - 1}\right)\operatorname{sech}\left(\frac{3}{2\rho}\xi\right) + \tanh\left(\frac{3}{2\rho}\xi\right)}{\mu\operatorname{sech}\left(\frac{3}{2\rho}\xi\right) + 1}\right], \tag{54}$$

while if $r = 1$, then the authors have the exact wave

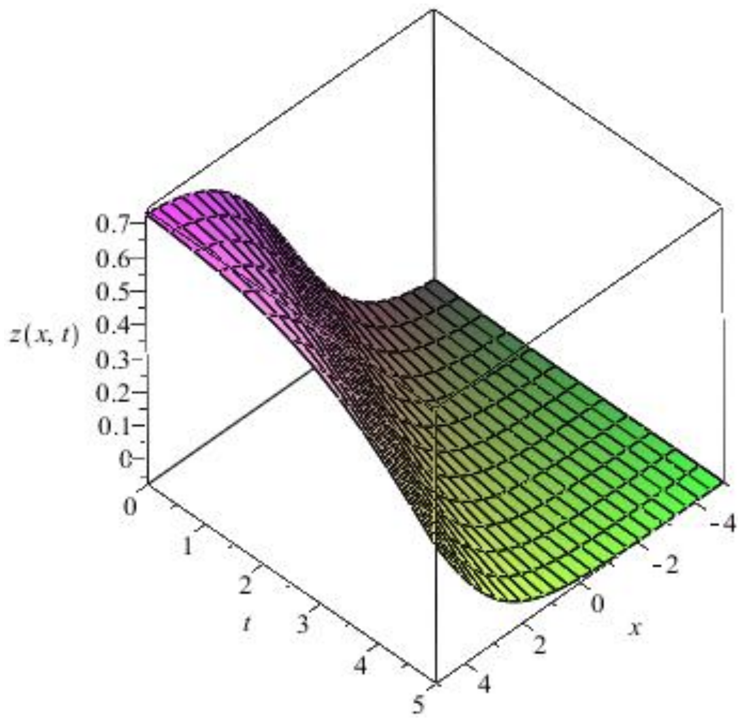

Figure 1. The plot of (23) when $k_1 = 1, \omega = 1, \alpha = -1, \rho = 2, A = 1, B = 2.$

$$\phi(\xi) = \pm\frac{\sqrt{6}}{2}\left[1 + \frac{\left(\sqrt{\mu^2+1}\right)\operatorname{csch}\left(\frac{3}{2\rho}\xi\right) + \coth\left(\frac{3}{2\rho}\xi\right)}{\mu\operatorname{csch}\left(\frac{3}{2\rho}\xi\right) + 1}\right]. \quad (55)$$

solution. Finally, note that the case $\varepsilon = 1, r = -1, R > 0,$ is rejected for example 2, because the authors have complex solutions for Equation (2).

PHYSICAL EXPLANATIONS OF SOME OBTAINED SOLUTIONS

Solitary waves can be obtained from each traveling wave solution by setting particular values to its unknown parameters. In this section, the authors have presented some graphs of solitary waves constructed by taking suitable values of involved unknown parameters to visualize the underlying mechanism of the original equation. Using mathematical software Maple 14, three dimensional plots of some obtained exact traveling wave solutions have been shown in Figures 1 to 6.

The nonlinear PDE (1) describing the nonlinear dynamics of MTs as nanobioelectronics transmission lines

The obtained solutions for the nonlinear PDE (1) incorporate three types of explicit solutions namely, hyperbolic, trigonometric and rational. From these explicit results, it is easy to say that the solution (23) is a kink shaped soliton solution; the solution (24) is a singular kink shaped soliton solution; the solutions (26), (28) are bell-kink shaped soliton solution; the solution (29) is a singular bell-kink shaped soliton solution, the solutions (31), (32), (34), (35), (37), (38) are periodic solutions and the solution (40) is rational solution. The graphical representation of the solutions (23), (26), (34) and (38) can be plotted as shown in Figures 1 to 4.

The nonlinear PDE (2) describing the nonlinear dynamics of radial dislocations in MTs

The obtained solutions for the nonlinear PDE (2) are hyperbolic. From the obtained solutions for this equation, the authors observe that the solution (47) is a kink shaped soliton solution, the solution (48) is a singular kink shaped soliton solution, the solution (50), (54) are bell-kink shaped soliton solutions and the solutions (52), (55) are singular bell-kink shaped soliton solutions. The graphical representation of the solutions (52) and (54) can be plotted as shown in Figure 5 and 6.

Remark: The authors have checked all our solutions with Maple 14 by putting them back into the original Equations (1) and (2).

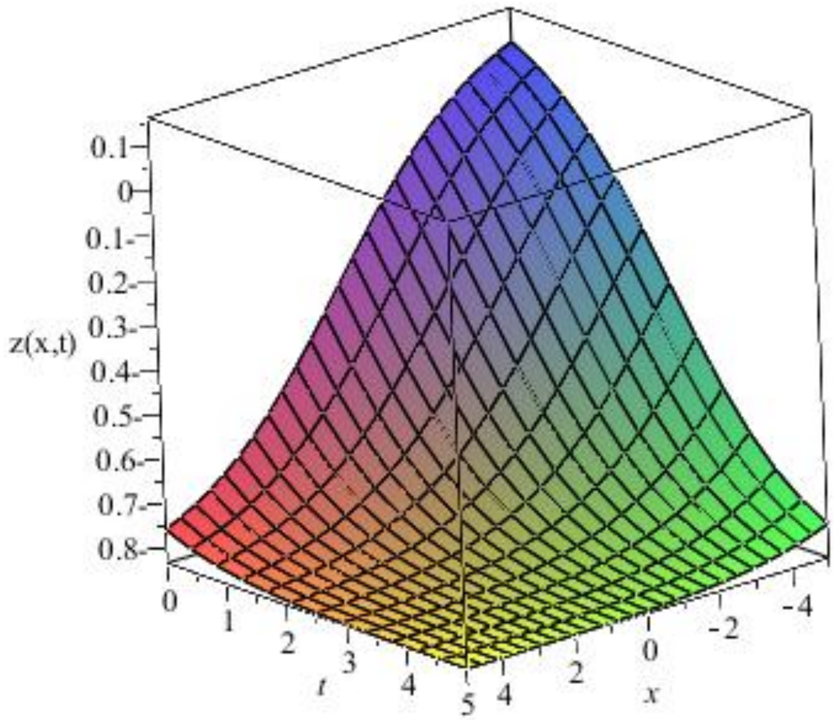

Figure 2. The plot of (26) when $k_1 = 1, \omega = -2, \alpha = -2, \rho = 2, A = 1, B = 1.$

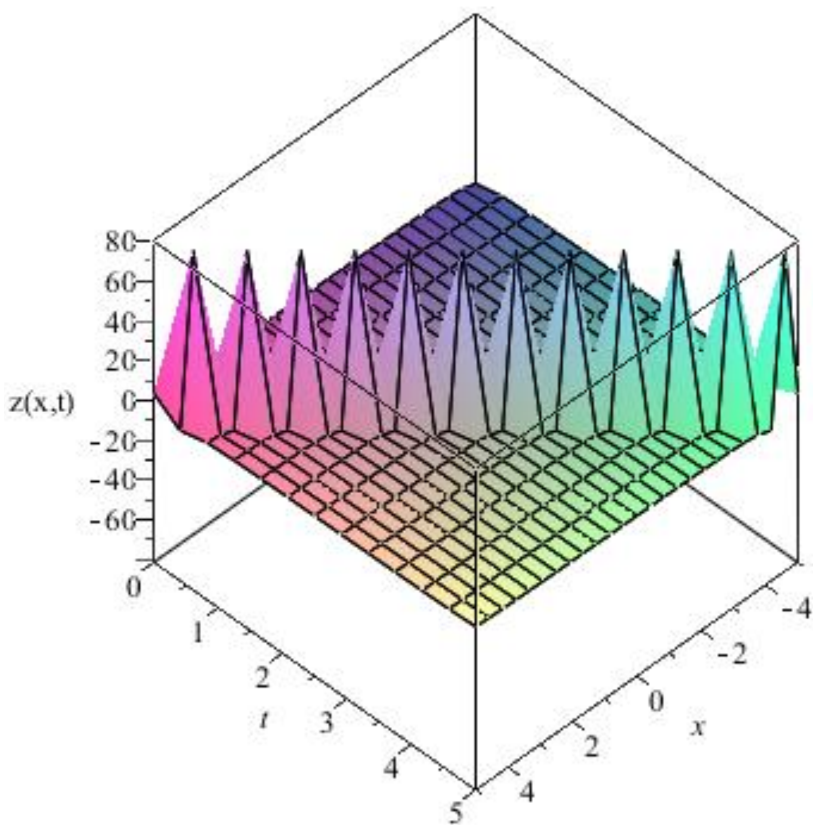

Figure 3. The plot of (34) when $k_1 = 1, \omega = -1, \alpha = -2, \rho = 4, A = 1, B = 1.$

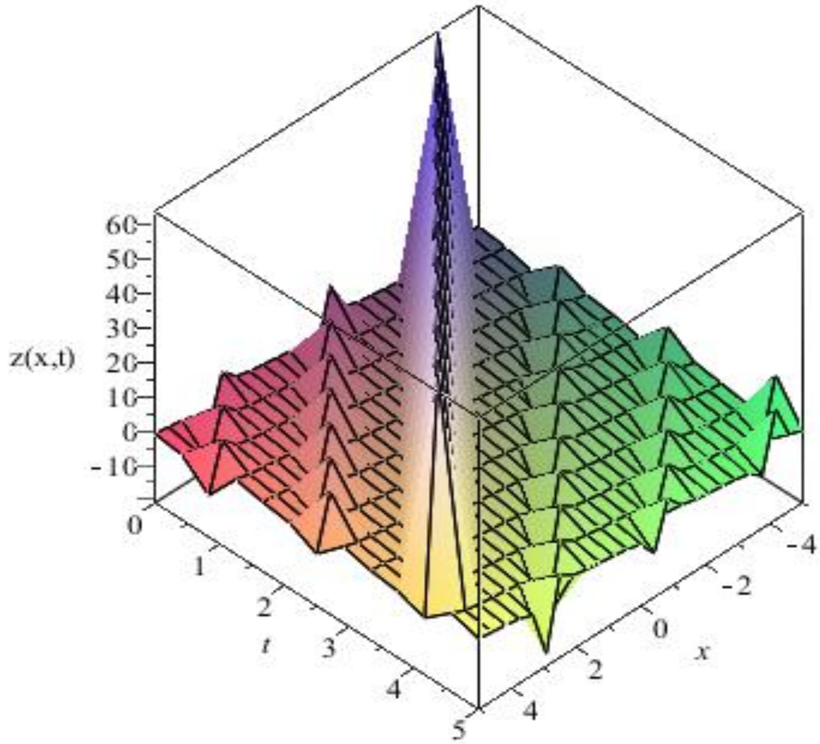

Figure 4. The plot of (38) when $k_1 = 1, \omega = 2, \alpha = -1, \rho = 4, A = 1, B = 1, \mu = \frac{1}{2}$.

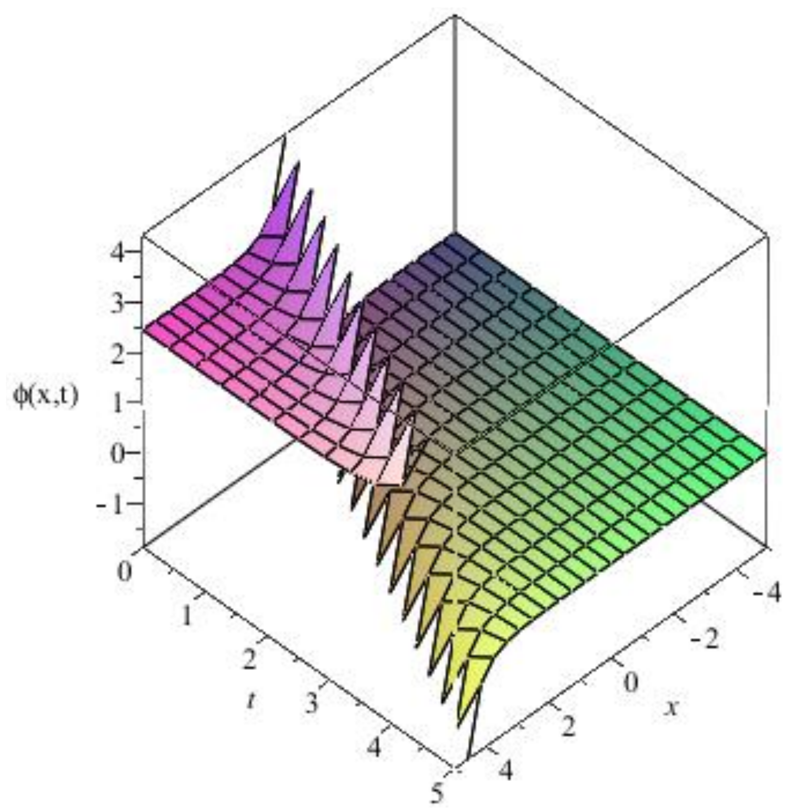

Figure 5. The plot of (52) when $k_1 = 2, \omega = 2, \rho = \frac{3}{2}$.

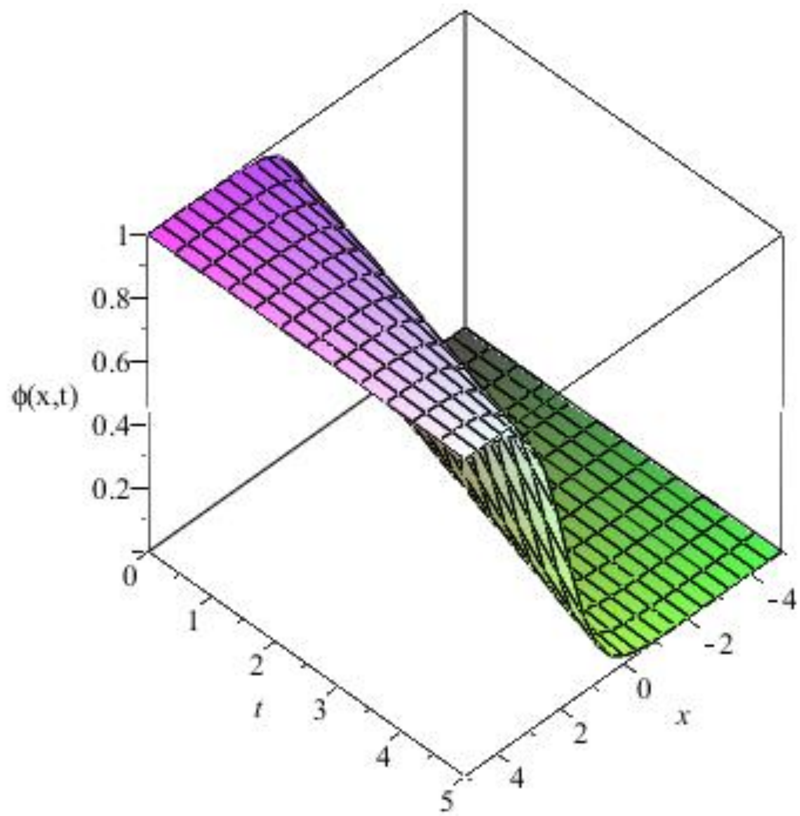

Figure 6. The plot of (54) when $k_1 = 2, \omega = 1, \rho = \frac{3}{2}, \mu = 2.$

Conclusions

The generalized projective Riccati equations method was used in this paper to obtain some new exact solutions of the two nonlinear evolution Equations (1) and (2) which describe the model of MTs as nano-bioelectronics transmission lines and the dynamics of radial dislocations in MTs, respectively. On comparing our results in this paper with the well-known results obtained in Zekovic et al. (2014) and Zdravkovic et al. (2014), the authors deduce that their results are new and not published elsewhere except the result (47) which is in agreement with the result of (43) obtained in Zdravkovic et al. (2014). It is to be noted here that the obtained solutions are of type kink, soliton with singularities and periodic. Solitons are the solutions in the form $\sec h$ and $\sec h^2$, the graph of soliton is a wave that goes up only. It is not like periodic solutions sine, cosine, etc, as in trigonometric function, that goes above and below the horizontal. Kink is also called a soliton; it is in the form $\tanh$ not $\tanh^2$. In kink the limit as $x \to \infty$, the answer is a constant, not like solitons where the limit goes to 0 (Alquran and Al-Khaled, 2011a, b, 2012; Alquran, 2012; Shukri and Al-khaled, 2010; Alquran et al., 2012; Alquran and Qawasmeh, 2014).

Conflict of Interest

The authors declare no conflict of interests.

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