Weibull distribution is often invoked to interpret and predict wind characteristics needed for effective design of wind power systems for different locations. In this paper, daily average wind data for Enugu (6.4°N; 7.5°E), Onitsha (6.8°N; 6.1°E) and Owerri (5.5°N; 7.0°E) over a 25-year period is modeled in terms of the Weibull distribution in order to accurately predict wind potentials for the locations. The monthly and annual wind speed probability density distributions at 10 m meteorological height were analyzed and the Weibull shape and scale factors were empirically determined for the locations. The predicted and measured wind speed probability density distributions of the locations are compared and the accuracy of the model determined for each location using Pearson product moment correlation coefficient ($r$) and root-mean-square error ($\xi$). We find $r$ and $\xi$ to be 0.64, 1.40, 0.67, 1.17 and 0.93, 1.55, respectively, for Enugu, Onitsha and Owerri. The results suggest that the model can be used, with acceptable accuracy, for predicting wind energy output needed for preliminary design assessment of wind machines for the locations.

Key words: Renewable energy-general, wind, Weibull distribution.
The extent to which wind can be exploited as a source of energy depends on the probability density of occurrence of different speeds at the site, which is essentially, site-specific. However, the development of new wind projects continues to be hampered by the lack of reliable and accurate wind resource data in many parts of the developing world. To optimize the design of a wind energy conversion device, data on speed range over which the device must operate to maximize energy extraction is required, which requires the knowledge of the frequency distribution of the wind speed. Among the probability density functions that have been proposed for wind speed frequency distributions of most locations, the Weibull distribution has been the most acceptable and forms the basis for commercial wind energy applications and software (Seyit and Ali, 2009). Some of the wind energy software based on the Weibull distribution includes the Wind Atlas Analysis and Application Program (WAsP) and the recently developed Nigerian Wind Energy Information System (WIS).

In previous papers (Enibe, 1987; Ugwuoke et al., 2008; Odo et al., 2010), the theoretical potentials of wind at various heights above the ground, based on annual average values of wind speed, have been assessed for many Nigerian locations. These analyses were carried out using measured data over various periods ranging from 1 to 10 years. In these analyses, little or no attention was given to the frequency distribution patterns of wind speed over the studied periods for the locations. In this paper, the frequency distribution of daily averages of wind speed for three locations in south-eastern Nigeria, namely: Enugu, Onitsha and Owerri, over a longer period of up to 25 years are examined. The observed data for these three locations are modeled in terms of the Weibull distribution, to enable an accurate prediction of the wind potentials of the locations and the results are compared. The results of this analysis are expected to be very useful to designers of wind turbines, for various wind energy applications, for the locations.

MATERIALS AND METHODS

In this study, we use 25 years (1978 to 2003) daily averages of wind speed data at 10 m meteorological height, for Enugu (6.4°N; 7.5°E), Onitsha (6.8°N; 6.1°E) and Owerri (5.5°N; 7.0°E) obtained from the data bank of Nigerian Meteorological Agency (NIMET). The data gives information on the daily average wind speed distributions of the locations over the study period, from which the monthly and yearly average data were calculated for the current analysis.

Weibull probability density function

The Weibull probability density distribution is a two-parameter function characterized by a dimensionless shape (k) parameter and scale (c) parameter (in unit of speed). It is a mathematical idealization of the distribution of wind speed over time for most locations. The function gives the probability of wind speed being in a range of 1 m/s about a particular speed (v), taking into account all variations for the period covered by the statistics. The Weibull distribution is a statistical function given (Walker and Jenkins, 1997; Gipe, 2004) by:

\[
f(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp\left(-\frac{v}{c}\right),
\]

(1)

Where \( f(v) \) is the probability density defined as the frequency of occurrence of wind speed (v), c is the scale parameter (in unit of m/s), which is closely related to the wind speed for the location, and k is the dimensionless shape parameter, which describes the width of the distribution and measures the probability of extraction of wind energy at a given characteristic wind speed. The Weibull distribution is therefore characterized for any location by the two parameters c and k. The cumulative form of the Weibull distribution \( F(v) \) which gives the probability of the wind speed exceeding the value v is expressed (Justus et al., 1978; Walker and Jenkins, 1997) as:

\[
F(v) = \exp\left(-\frac{v}{c}\right)^k
\]

(2)

On the other hand, the power derivable from the wind is a cubic function of the wind speed such that, in the Weibull distribution, the power density \( P_A \) of the wind at any speed is given (Seyit and Ali, 2009) by:

\[
P_A = \frac{1}{2} \rho c^3 \int_{0}^{v} v^2 f(v) dv.
\]

(3)

Where \( \rho \) is the density of air. However, the power derivable from the wind scales with the height \( h \) above the ground according to the Hellman’s exponential law given (Walker and Jenkins, 1997; Gipe, 2004) by:

\[
\frac{v}{v_0} = \left(\frac{h}{h_0}\right)^\alpha
\]

(4)

Where \( h_0 \) is any reference height and \( v_0 \) is the wind speed at \( h_0 \), while \( \alpha \) is the Hellman’s constant which varies from one location to another. Equation 4 suggests that the derivable power increases with increasing height only if the change in density of air is negligible. It has been shown (Walker and Jenkins, 1997) that within the troposphere (\( h \leq 10 \) km) the density of air varies very little for any location.

Analysis of Equation 3 using Equation 1 shows that the power density could be expressed as a gamma function \( (\Gamma) \) defined in general x-variable (Dass, 1998) as:

\[
\Gamma x = \int_{0}^{x} x^{a-1} e^{-x} dx.
\]

(5)

Using Equations 5 and 1 in Equation 3, the average wind power
Equation 7 therefore suggests that the probability of capturing the wind by a turbine at a mean wind speed is small if the shape factor is high for that location, since $k$ could be used as a measure of dispersion (Pallabazzer, 2003) in a distribution. This shows that knowledge of the exact value of $k$ provides preliminary information on the wind speed regime for which wind turbines should be designed for optimum performance in any given location. Similarly, wind speed is a real valued random variable and most locations show wide dispersion (Justus et al., 1978) so that the use of mean values as the characteristic speed for designs may not be very reliable (Jaramillo and Borja, 2004).

In this paper, we use analytic method in which $F_v$ is plotted against $v$ on double logarithm scales and apply a one-dimensional regression on the plots to obtain values for $k$ and $c$ for the locations.

### RESULTS AND DISCUSSION

We calculated the monthly and annual average wind speed distributions at 10 m meteorological height for the three locations over the studied period. The time series distributions of the monthly average values for the locations are shown in Figure 1, while the annual average values are shown in Table 1. The distributions give annual mean values of $5.5 \pm 0.6$ m/s, $3.6 \pm 0.4$ m/s and $3.3 \pm 0.3$ m/s, respectively, for Enugu, Onitsha and Owerri. It could easily be observed from Figure 1 that the distributions of the monthly average wind speed for the three locations are fairly similar, peaking in the month of March and having minimum values in November. However, wind speed is highest in Enugu and lowest in Owerri. Perhaps, the result is as expected since difference in wind speed distributions may be related to the difference in altitude between the locations. To model the data in terms of the Weibull distribution, we took twice logarithm of Equation 2 to obtain

$$\ln(-\ln F_v) = k \ln v - k \ln c.$$  

(8)

The plots of $\ln (-\ln F_v)$ as a function of $\ln v$ for the three locations, on the same scale, are shown in Figure 2. Linear regression of the plots gives

$$\ln(-\ln F_{v_E}) = 2.0 \ln v - 3.7,$$

$$\ln(-\ln F_{v_O}) = 1.5 \ln v - 2.6$$

and

$$\ln(-\ln F_{v_W}) = 1.9 \ln v - 3.1$$

respectively, for Enugu, Onitsha and Owerri. By comparing each of the equations with Equation 8, the values of $k$ and $c$ were deduced for each location. The summary of the results is shown in Table 1.

### Table 1. Wind speed distributions parameters for Enugu, Onitsha and Owerri.

<table>
<thead>
<tr>
<th>Location</th>
<th>Latitude</th>
<th>Longitude</th>
<th>$V_{mean}$ (m/s)</th>
<th>$k$</th>
<th>$c$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enugu</td>
<td>6.4°N</td>
<td>7.5°E</td>
<td>5.5 ± 0.6</td>
<td>2.0</td>
<td>6.4</td>
</tr>
<tr>
<td>Onitsha</td>
<td>6.8°N</td>
<td>6.1°E</td>
<td>3.6 ± 0.4</td>
<td>1.5</td>
<td>5.6</td>
</tr>
<tr>
<td>Owerri</td>
<td>5.5°N</td>
<td>7.0°E</td>
<td>3.3 ± 0.3</td>
<td>1.9</td>
<td>5.1</td>
</tr>
</tbody>
</table>

METHODS

Density ($P_v$) based on Weibull distribution can be expressed (Ucar and Balo, 2009) in the form:

$$P_{(av)} = \frac{1}{2} \rho \omega^3 \frac{\Gamma \left( 1 + \frac{3}{k} \right)}{\left[ \Gamma \left( 1 + \frac{1}{k} \right) \right]^3}$$  

(6)

Where $\omega$ is the characteristic wind speed of the location. However, meteorologists have characterized the distributions of wind speeds for many of the world’s wind regimes in terms of the speed distribution patterns. For example, in temperate climate (mid latitudes), a typical shape parameter $k = 2$ offers a good approximation (Gipe, 2004). For $k = 2$, Equation 1 or 2 reduces to Rayleigh wind speed distribution. Thus, the Rayleigh distribution is a special case of the Weibull distribution developed for estimation of wind potential in temperate climate locations. Wind characteristics are essentially location specific and performance of real wind conversion devices which are designed based on the Rayleigh distribution may greatly differ if actual wind conditions at the location differ from those standard speed distributions.

A method has been suggested (Iheonu et al., 2002; Gipe, 2004) for estimating the shape ($k$) factor of a set of wind speed data using the mean wind speed ($v$) and standard deviation ($\sigma$) in a simple relation of the form:

$$k = \left( \frac{\sigma}{v} \right)^{-1.086}$$  

(7)
Furthermore, using the results for $k$ and $c$ obtained from the regression analysis for each location in Equation 1, the probability densities ($\%$) of occurrence of different wind speeds were determined. The different speeds used for this analysis were chosen from the range $0 \leq v \leq 10$ m/s as covered by observational data in eight (8) comparable speed bins, for effective overlap. The results of the prediction were compared with those calculated from observed data. The measured and predicted probability density distributions for different wind speed bins for the three locations are shown in Figure 3, while the summary of the comparison is displayed in Table 2. Similarly, the suitability of the model in predicting wind potential for each location was determined using two non-parametric statistics, namely, the Pearson product moment correlation coefficient ($r$) and root-mean-square error ($\xi$). The Pearson correlation coefficient is defined (Aalen, 1978) as:

$$r = \sqrt{1 - \frac{\sum_{i=1}^{N} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2}},$$

(9)

Where $N$ is the number of observations in each data set, $y$ and $x$, respectively are the measured and predicted probability density, while $\bar{y}$ is the mean of the measured values. This statistic varies from 0 (for a null association) to ±1 (for a perfect association). A correlation is statistically significant at a set level of significance if $r \geq 0.5$, otherwise, it is not significant. The present analyses give correlation coefficients $r \approx 0.6$, 0.7 and 0.9, respectively, for Enugu, Onitsha and Owerri at 5% level of significance. Thus, all the correlations are statistically significant at 5% level.

On the other hand, the root-mean-square error is a statistic that determines the degree of departure of two data sets from a supposed association and is defined (Joanes and Gill, 1998) as:

$$\xi = \left[\frac{1}{N} \sum_{i=1}^{N} (y_i - x_i)^2\right]^{\frac{1}{2}},$$

(10)

The results give $\xi \approx 1.40$, 1.17 and 1.55, respectively, for Enugu, Onitsha and Owerri. Results of all these analyses are summarized in Table 2.

Modeling and prediction of wind characteristics are major design inputs in the development of wind power systems for any location. However, the wind speed distribution for many of the world’s wind regimes have been characterized and wind power systems are

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**Figure 2.** Plot of ln(-ln $F_v$) against ln $v$ for the locations.

**Figure 3.** Observed and predicted probability density for: (a) Enugu, (b) Onitsha, (c) Owerri.
optimized based on these standard distributions. In fact, the Rayleigh distribution \((k = 2)\) is often employed by most wind power system developers (Vosburgh, 1983). The result is that many wind power systems perform poorly in many different locations because the actual wind conditions at the locations differ largely from those standard distributions (Ramachandra et al., 2005; Al-Mohamad and Karmeh, 2003).

It could easily be observed from the time series distributions of the monthly average wind speed that the three locations studied in south-eastern Nigeria show similarity in climatology of wind. Perhaps, this observation could be attributed to their proximity in geographical extent. The three locations lie within a latitudinal stretch of about 1.3° and longitudinal stretch of 1.4°. Similarly, the distributions do not show any latitudinal or longitudinal dependence either. Thus, wind speed in south-eastern locations of Nigeria is essentially location specific, which may be driven by environmental factors, rather than geographical dependence.

We have also shown in the results that the Weibull shape factor is 2.0, 1.5 and 1.9, respectively for Enugu, Onitsha and Owerri, while the corresponding scale factor is 6.4, 5.6 and 5.1 m/s. The values of the shape factor presented in this paper suggest that while the data for Enugu and Owerri are in close agreement with Rayleigh distribution, the data for Onitsha departs significantly from the standard Rayleigh distribution. The results further suggest that the wind speed distributions in the studied locations are widely dispersed, with those of Enugu and Owerri being much wider than that of Onitsha. The implication of these results is that any wind turbine which is optimized based on Rayleigh distribution may be suitable for Enugu and Owerri, but not for Onitsha. It therefore becomes necessary that wind turbines for utility generation in these locations be designed locally rather than relying on importation of already designed systems.

A more comprehensive wind speed evaluation and energy assessment is achieved by the use of real life frequency distribution. The frequency distribution obviously indicates the percentage of the time of occurrence of the various wind bins/spectra and provides information on when a particular rated turbine in the location is expected to yield power (Amonye and Hassan, 2010). Thus, the most frequently occurring wind spectra defines the characteristic wind speed for the location, which is a more reliable value for the design of wind turbines for any location than the average value, which is affected by the skewness of the distribution (Iheonu et al., 2002; Ucar and Balı, 2009). A major outcome of our result is the statistical accuracy of the Weibull distribution model in predicting wind potentials of the locations as determined by the correlation coefficient. In fact, for Owerri, the model gives almost a perfect prediction of the characteristic wind speed. However, for all the locations, the correlation coefficients are statistically significant. These results show that the predicted and measured data are acceptably related by the Weibull distribution. Hence, within the region of overlap, the Weibull distribution can be used, with acceptable accuracy, to predict wind potentials for these locations.

On the other hand, it is obvious in Figure 3 that there is a wide departure of the theoretical predictions from the real life wind speed frequency distribution patterns of the studied locations. In fact, for each of the locations, within the region of overlap, the theoretical model appears to underestimate the probability density of every speed bin. This is further supported statistically by the large values of root-mean-square error calculated for all the locations. Perhaps, this departure is attributable to the coarse approximations arising from daily averages which fail to account for the short time-scale variations of wind characteristics of the locations. Wind speed is a real valued random variable and observations over smaller time-scales, such as hourly averages, may help to improve the results.

**REFERENCES**


