International Journal of Physical Sciences Vol. 5(3), pp. 208-215, March 2010
Available online at http://www.academicjournals.org/IJPS
ISSN 1992 - 1950 © 2010 Academic Journals

Full Length Research Paper

Effect of thermally stratified ambient fluid on MHD convective flow along a moving non-isothermal vertical plate

Gurminder Singh[1], P. R. Sharma[2] and A. J. Chamkha[3]*

[1]Birla Institute of Technology (Ranchi), Ext. Center Jaipur, 27 Malviya Industrial Area, Jaipur-302017, India.
[2]Department of Mathematics, University of Rajasthan, Jaipur - 302055, India.
[3]Manufacturing Engineering Department, the Public Authority for Applied Education and Training, Shuweikh, 70654, Kuwait.

Accepted 21 December, 2009

Aim of the paper is to investigate the effect of linear thermal stratification in stable stationary ambient fluid on steady convective flow of a viscous incompressible electrically conducting fluid along a moving, non-isothermal vertical plate in the presence of transverse magnetic field. The governing equations of continuity, momentum and energy are transformed into ordinary differential equations using local similarity transformation. The resulting coupled non-linear ordinary differential equations are solved using Runge-Kutta fourth order method along with shooting technique. The velocity and temperature distributions are discussed numerically and presented through graphs. The numerical values of skin-friction coefficient and Nusselt number at the plate are derived, discussed numerically for various values of physical parameters and presented through Tables.

Key words: Thermal stratification, MHD, convection, boundary layer flow, non-isothermal plate, skin friction, nusselt number.

INTRODUCTION

Convective heat transfer in thermal stratified ambient fluid occurs in many industrial applications and is an important aspect in the study of heat transfer. If stratification occurs, the fluid temperature is function of distance and convection in such environment exists in lakes, oceans, nuclear reactors where coolant (generally liquid metals) is present in magnetic field etc. Sakiadis (1961) pioneered the study of fluid flow due to continuously moving flat surface. Tsou et al. (1967) analyzed the flow and heat transfer along a continuously moving surface. Cheesewright (1967) examined the natural convection along an isothermal vertical surface in non-isothermal surroundings. Chen and Eichhorn (1976) studied natural convection along an isothermal vertical plate in thermally stratified medium. Fumizawa (1980) experimentally analyzed the effect of magnetic field on natural convection in liquid metal (NaK) used as coolant in nuclear reactor. Moutsoglou and Chen (1980) considered the buoyancy effect on a continuously moving inclined sheet. Venkatachala and Nath (1981) obtained the non-similarity solution for natural convection in thermally stratified fluid. Uotani (1987) experimentally studied the natural convection in thermally stratification for liquid metal (PbBi). Kulkarni et al. (1987) investigated the problem of natural convection from an isothermal flat plate suspended in a linearly stratified fluid medium. Ramachandran et al. (1987) analyzed the correlation for laminar mixed convection flow along an inclined continuously moving surface. Chen (1999) studied flow along a non-isothermal flat plate in the presence of free stream. Takhar et al. (2001) obtained the non-similar solution along a continuously moving vertical surface immersed in thermally stratified medium. Saha and Hossain (2004) examined

*Corresponding author. E-mail: achamkha@yahoo.com.

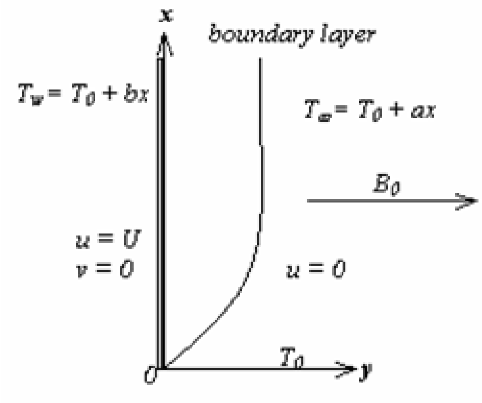

Figure 1. Physical model.

the natural convection with mass transfer in thermally stratified medium.

Similarity solution is an easy and useful way to understand the interaction between flow field and heat transfer. The convective flow along a continuously moving non-isothermal vertical plate whose temperature varies linearly with the distance when immersed in linearly thermally stratified medium permits similarity solution, hence give insight into the effect of thermal stratification on flow and temperature distribution inside the boundary layer. In view of the above, aim of the paper is to investigate effect of linear thermal stratification in stationary ambient fluid on steady convective flow of a viscous incompressible electrically conducting fluid along a moving, non-conducting, non-isothermal vertical plate in the presence of transverse magnetic field.

FORMULATION OF THE PROBLEM

Consider steady laminar convective flow of a viscous incompressible electrically conducting fluid along a non-conducting, non-isothermal vertical plate moving with constant velocity U, kept at temperature T_w, and the ambient fluid far away from plate has temperature T_∞. The x-axis is taken along the plate and y-axis is normal to the plate. The ambient fluid has temperature T_0 at $x = 0$. Magnetic field of uniform intensity B_o is applied in y-direction. The physical model is given in Figure1.

It is assumed that the external field is zero, also electrical field due to polarization of charges and Hall effect are neglected. Incorporating the Boussinesq's approximation within the boundary layer, the governing equations of continuity, momentum and energy [Jeffery (1966), Bansal (1994), Schlichting and Gersten (1999)], respectively are given by;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_o^{\,2}}{\rho}u \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

The boundary conditions [Bejan (1984)] are given by:

$$y = 0: \quad u = U, v = 0, T = T_w = T_0 + bx,$$

$$y \to \infty: u \to 0, T \to T_\infty = T_0 + ax. \tag{4}$$

METHOD OF SOLUTION

Introducing the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}, \tag{5}$$

Where; $\psi(x, y) = \sqrt{\nu x U}\, f(\eta)$ and the similarity variable

$$\eta = y\left(\frac{U}{\nu x}\right)^{1/2}. \tag{6}$$

It is observed that the equation (1) is identically satisfied by equation (6). Substituting equation (6) into the equations (2) and (3), the resulting coupled non-linear ordinary differential equations are

$$f''' + \frac{1}{2}ff'' + G\theta - Mf' = 0 \tag{7}$$

and

$$\theta'' - Pr\left(f'\theta - \frac{1}{2}f\theta' + Sf'\right) = 0 \tag{8}$$

The boundary conditions are reduced to

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0, \theta(0) = 1 - S \text{ and } \theta(\infty) = 0. \tag{9}$$

The governing locally-similar boundary-layer equations (7) and (8) with boundary conditions (9) are solved using Runge-Kutta fourth order technique along with double shooting technique [Conte and Boor (1981)].

SKIN-FRICTION COEFFICIENT

The local skin-friction coefficient at the plate is given by;

Table 1. Numerical values of $f''(0)$ and $-\theta'(0)$ for different values of physical parameters *M*, *G*, *Pr* and *S*.

	***f″*(0)**			**-*θ′*(0)**		
M = 0.5	Pr = 0.01	Pr = 0.05	Pr = 0.1	Pr = 0.01	Pr = 0.05	Pr = 0.1
G = 1.0						
S = 0.0	0.4619	0.2545	0.1480	0.1025	0.2246	0.3156
S = 0.1	0.3290	0.1386	0.0399	0.0948	0.2095	0.2958
S = 0.2	0.1968	0.0240	-0.0664	0.0869	0.1937	0.2752
S = 0.3	0.0653	-0.0892	-0.1714	0.0786	0.1773	0.2537
Pr = 0.05, G = 1.0	M = 0.5	M = 1.0	M = 1.5	M = 0.5	M = 1.0	M = 1.5
S = 0.0	0.2545	-0.2205	-0.5580	0.2246	0.1946	0.1744
S = 0.1	0.1386	-0.3115	-0.6344	0.2095	0.1812	0.1623
S = 0.2	0.0240	-0.4015	-0.7104	0.1937	0.1672	0.1498
S = 0.3	-0.0892	-0.4909	-0.7855	0.1773	0.1527	0.1364
Pr = 0.05, M = 0.5	G = 1.0	G = 2.0	G = 3.0	G = 1.0	G = 2.0	G = 3.0
S = 0.0	0.2545	1.1611	1.9911	0.2246	0.2682	0.2976
S = 0.1	0.1386	0.9478	1.6891	0.2095	0.2495	0.2766
S = 0.2	0.0240	0.7369	1.3906	0.1937	0.2300	0.2546
S = 0.3	-0.0892	0.5283	1.0951	0.1773	0.2097	0.2317

$$C_f = 2(Re)^{-1/2} f''(0).... \quad (10)$$

NUSSELT NUMBER

The local rate of heat transfer in terms of the local Nusselt number at the plate is given by;

$$Nu = -(Re)^{1/2} \theta'(0).... \quad (11)$$

RESULTS AND DISCUSSION

It is observed from Table 1 that the local skin-friction coefficient decreases with the increase in the Prandtl number. Thus, drag exerted by fluid on the plate is reduced with the increase in Prandtl number. Further, the rate of heat transfer at the plate increases with the increase in Prandtl number. This is attributed to the fact that the low Prandtl number fluid has high thermal conductivity and hence absorbs and convects heat readily, raising the fluid temperature. This in turn reduces temperature gradient at surface causing reduction in surface heat flux.

It is also seen from the Table 1 that with the increase in stratification parameter both the local skin-friction coefficient and heat transfer at surface decrease. It is understood, since factor $(T_w - T_\infty)$ reduces with the increase in stratification factor, thus buoyancy effect very close to the plate is marginalized thereby reducing the drag and fluid velocity. In extension, effect of low velocity near plate results in reduced heat transfer.

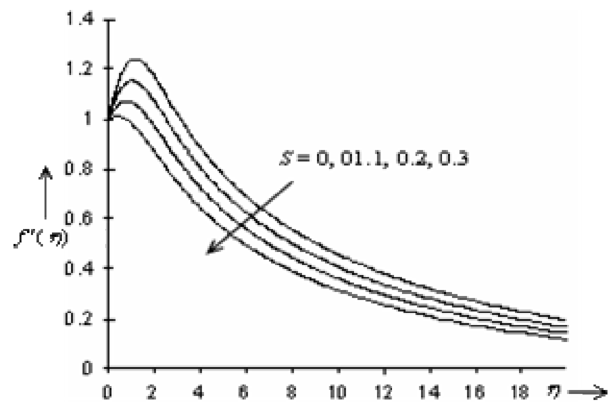

Figure 2. Velocity distribution versus η when M = 0.5, G = 1.0 and Pr = 0.01.

It can be commented looking at the Table 1 that with the increase in magnetic parameter the drag exerted by fluid on surface reduces and in fact at higher values of magnetic parameter plate exerts drag on fluid [$f''(0) < 0$]. Since, fluid velocity near the plate is reduced; it results in reduction of heat transfer at the plate. Further, with the increase in buoyancy parameter the fluid velocity inside boundary layer increases causing increase in local skin-friction coefficient. Also increased fluid velocity near plate thereby increases the heat transfer at the plate.

Figure 2 shows that with the increase in stratification parameter the fluid velocity decreases. This is because

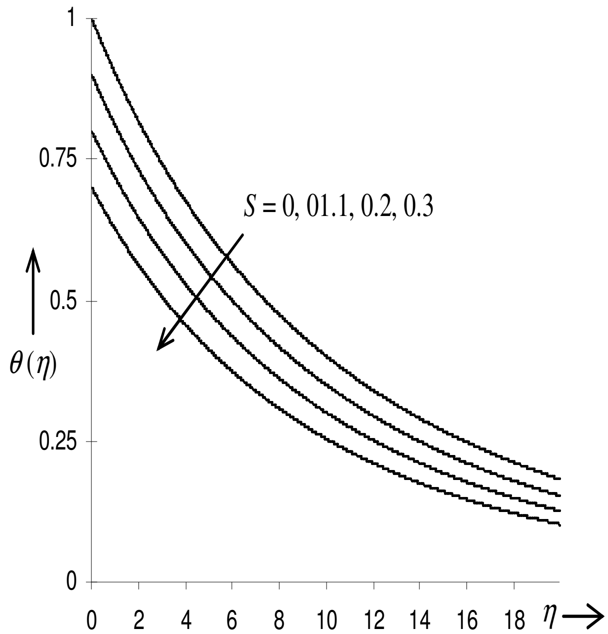

Figure 3. Temperature distribution versus η when M = 0.5, G = 1.0 and Pr = 0.01.

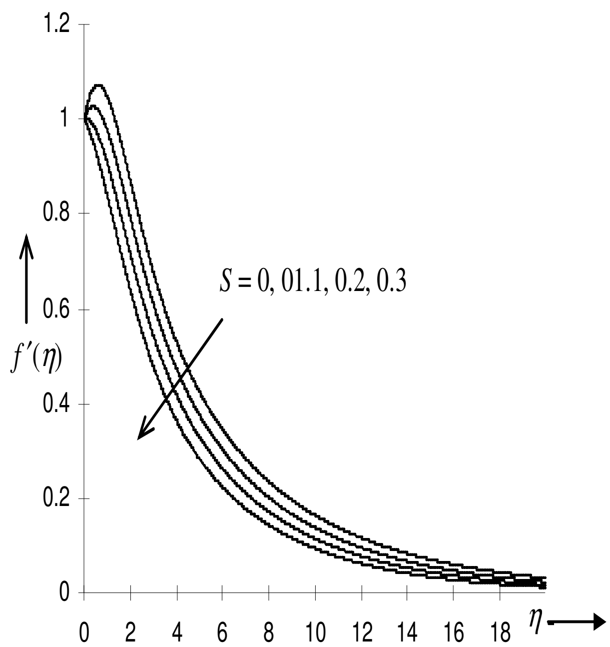

Figure 4. Velocity distribution versus η when M = 0.5, G = 1.0 and Pr = 0.05.

the buoyancy factor $(T_w - T_\infty)$ reduces within the boundary layer, with the increase in stratification parameter. Figure 3 depicts that fluid temperature decreases with the increase in stratification parameter.

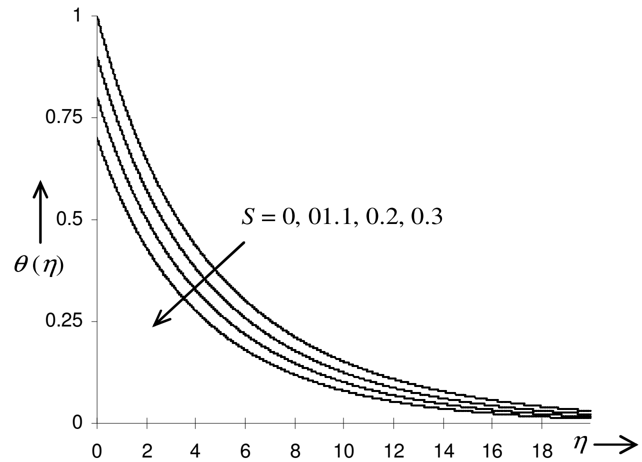

Figure 5. Temperature distribution versus □ when M = 0.5, G = 1.0 and Pr = 0.05.

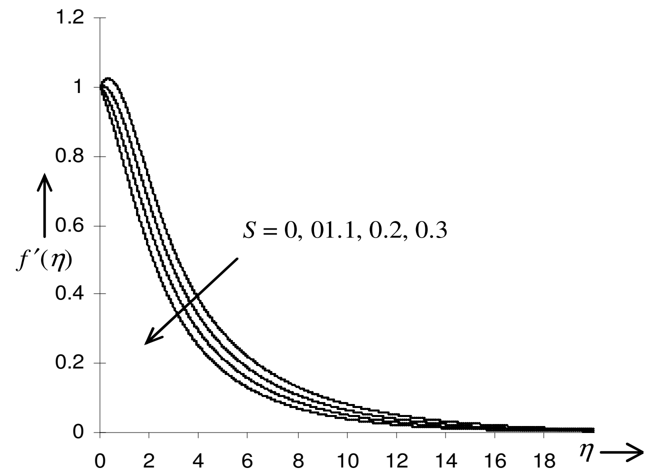

Figure 6. Velocity distribution versus η when M = 0.5, G = 1.0 and Pr = 0.1.

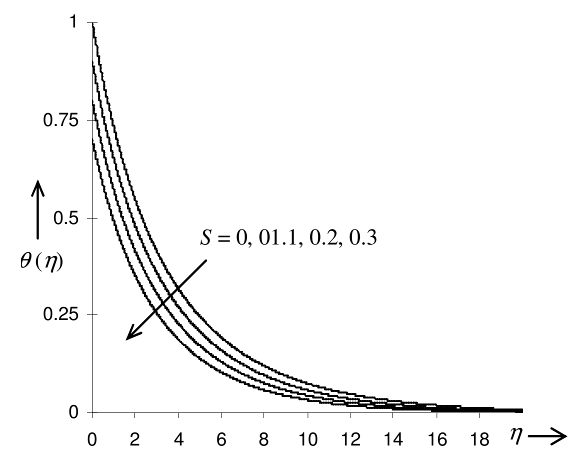

Figure 7. Temperature distribution versus η when M = 0.5, G = 1.0 and Pr = 0.1.

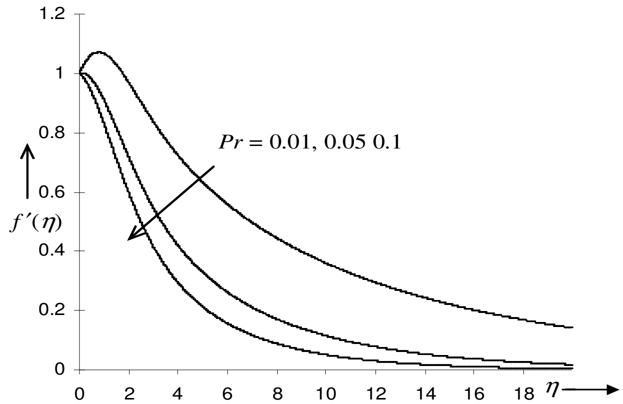

Figure 8. Velocity distribution versus η when M = 0.5, G = 1.0 and S = 0.2.

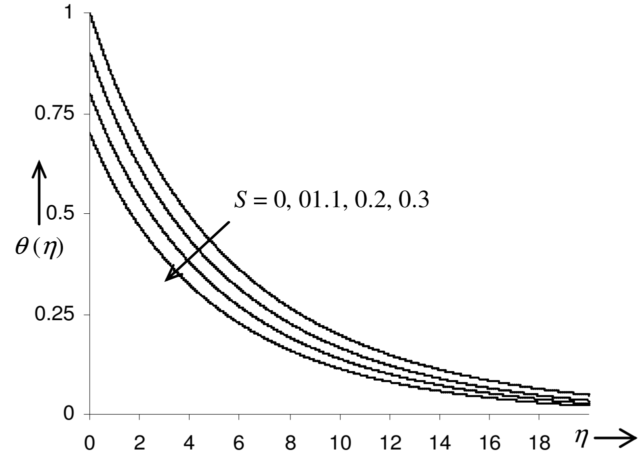

Figure 11. Temperature distribution versus η when M = 1.0, G = 1.0 and Pr = 0.05.

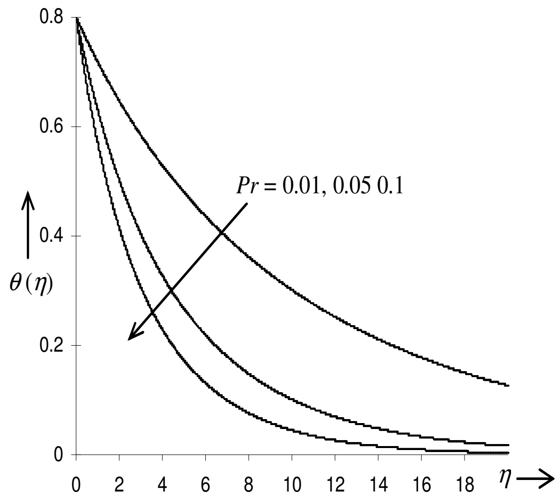

Figure 9. Temperature distribution versus η when M = 0.5, G = 1.0 and S = 0.2.

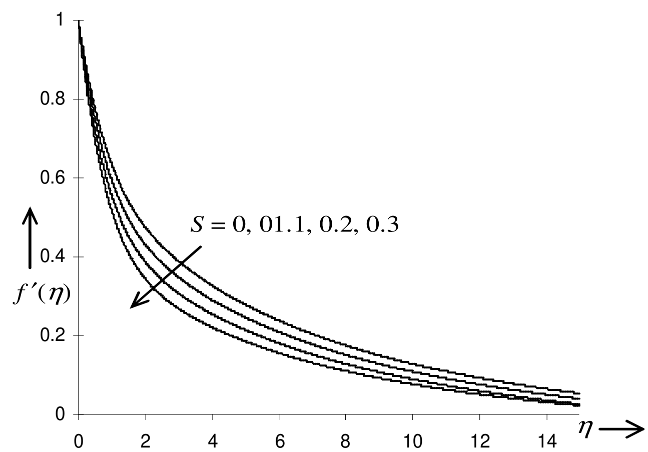

Figure 12. Velocity distribution versus η when M =1.5, G = 1.0 and Pr = 0.05.

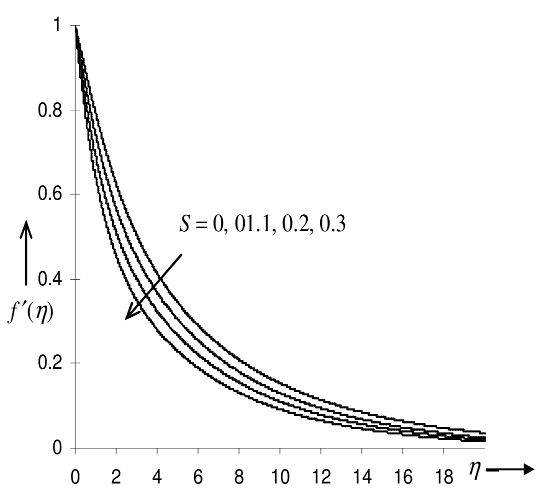

Figure 10. Velocity distribution versus η when M = 1.0, G = 1.0 and Pr = 0.05.

A comparative study of Figures 2 to 7 indicates that the effect of stratification parameter is marginalized with the increase in Prandtl number, as the separateness among the fluid velocity and temperature profiles reduce. Also, for given value of Prandtl number the velocity and thermal boundary layer thicknesses are almost same while with the increase in Prandtl number, the boundary layer thickness reduces. Figures 8 and 9 show that fluid velocity and temperature decrease with the increase in Prandtl number.

A comparative study of Figures 4, 5 and 10 - 13 reveal the effect of magnetic parameter. It is noted looking at the increased separateness among the temperature profiles that the effect of stratification parameter is more pronounced on fluid temperature at high value of magnetic parameter. Figures 14 and 15 indicate that with the increase in magnetic parameter the fluid

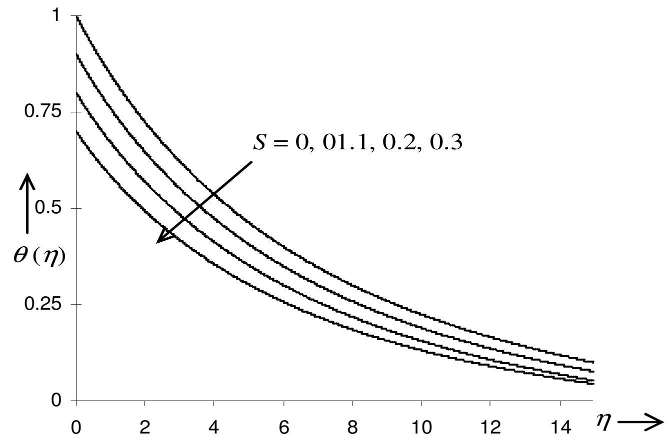

Figure 13. Temperature distribution versus η when M = 1.5, G = 1.0 and Pr = 0.05.

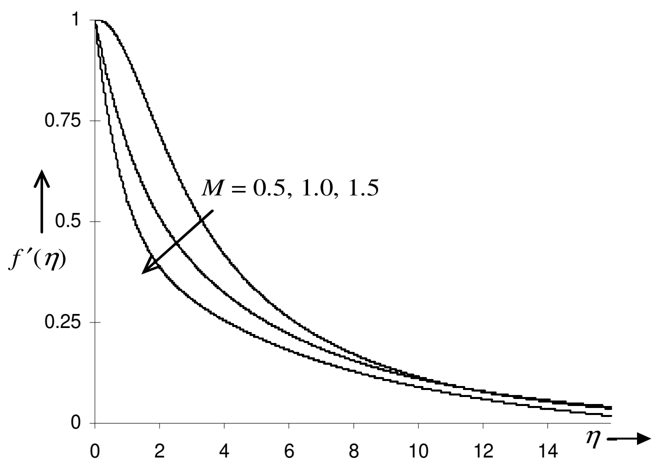

Figure 14. Velocity distribution versus η when S =0.2, G = 1.0 and Pr = 0.05.

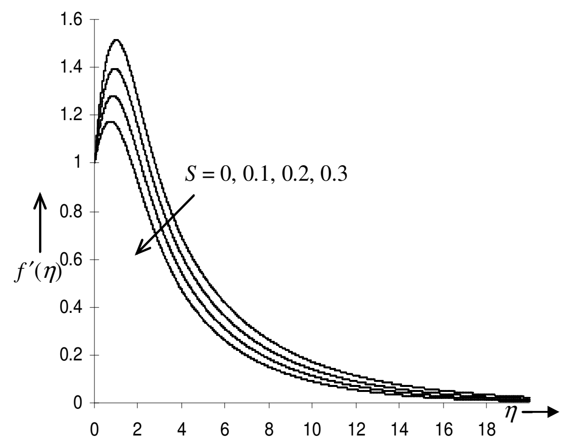

Figure 16. Velocity distribution versus η when M =0.5, G = 2.0 and Pr = 0.05.

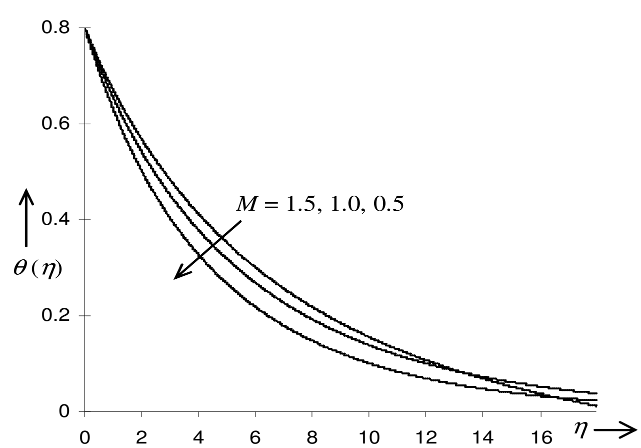

Figure 15. Temperature distribution versus η when S = 0.2, G = 1.0 and Pr = 0.05.

velocity decreases, while the fluid temperature increases. This is due to the fact that in presence of transverse magnetic field, Lorentz force sets in, which retards the fluid velocity. Since the fluid velocity reduced the heat reduced the heat is not convected readily, hence the fluid temperature is higher. This would mean that heat transfer must reduce at the surface, this is vindicated looking at the table. Through comparative study of Figures 4, 5 and 16 - 19, it can be suitably remarked that the increase in buoyancy parameter does not practically vary the effect of stratification factor on fluid and temperature profiles. Figures 20 and 21 shows that with the increase in buoyancy parameter fluid velocity increases, while the fluid temperature decreases. This must happen because buoyancy force assists the flow by increasing fluid velocity and hence the heat is convected readily thereby reducing fluid temperature.

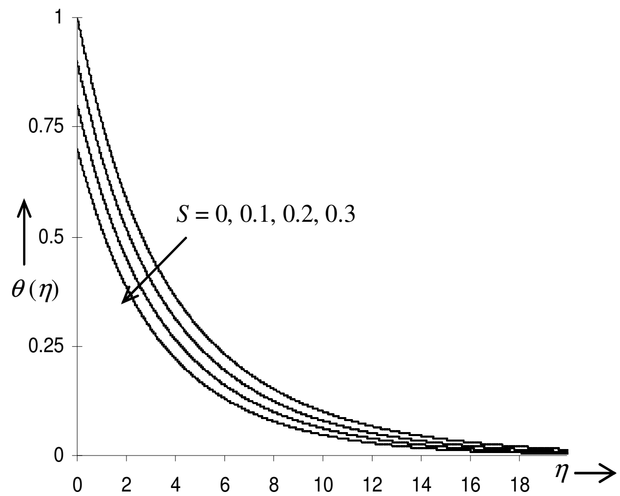

Figure 17. Temperature distribution versus η when M = 0.5, G = 2.0 and Pr = 0.05.

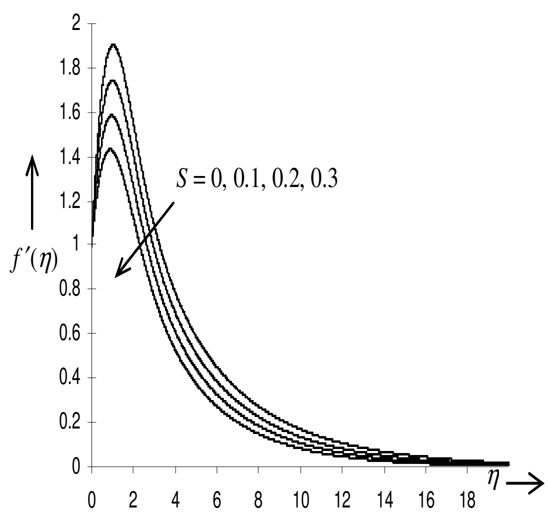

Figure 18. Velocity distribution versus η when M =0.5, G = 3.0 and Pr = 0.05.

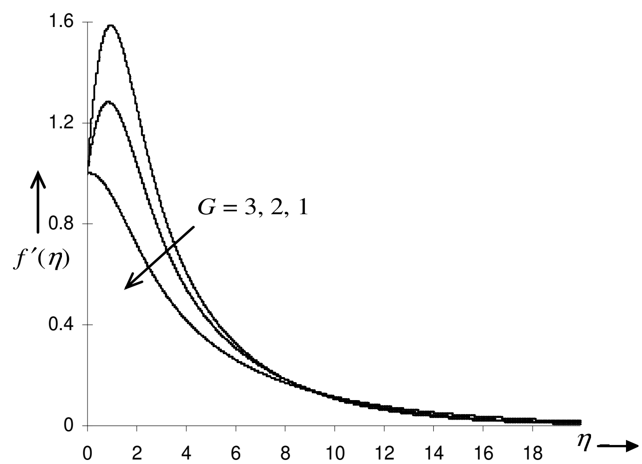

Figure 20. Velocity distribution versus η when M =0.5, S = 0.2 and Pr = 0.05.

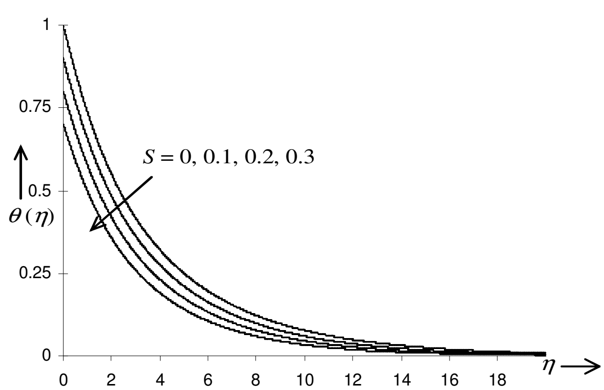

Figure 19. Temperature distribution versus η when M = 0.5, G = 3.0 and Pr = 0.05.

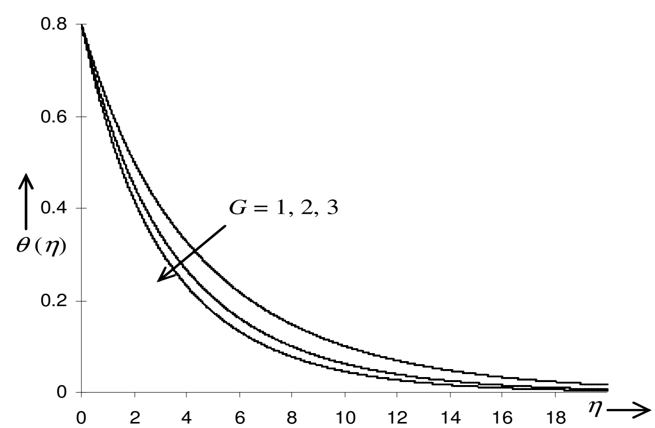

Figure 21. Temperature distribution versus η when M = 0.5, S = 0.2 and Pr = 0.05.

CONCLUSION

1) The fluid velocity and temperature decrease with the increase in stratification parameter.
2.) The local skin-friction coefficient and heat transfer at surface decrease with the increase in stratification parameter.
3.) The effect of stratification parameter is marginalized with the increase in Prandtl number.
4.) The effect of stratification parameter is pronounced on fluid temperature at high value of magnetic parameter.
5.) The effect of stratification factor on fluid and temperature distribution is practically same at different values of buoyancy parameter.

Nomenclature: ***g***; Acceleration due to gravity of the Earth, $\boldsymbol{T_0}$**;** ambient fluid temperature at $x = 0$, ***G;*** buoyancy parameter $\left\{= \frac{Gr}{Re^2}\right\}$, ***x, y;*** Cartesian coordinates, ***f ;*** dimensionless stream function, ***Gr ;*** Grashof number $\boldsymbol{B_o}$*;* magnetic field intensity, ***M;*** magnetic parameter $\left\{= \frac{\sigma B_o{}^2 x^2}{\mu}(Re)^{-1}\right\}$, ***Nu;*** local Nusselt number, ***a, b;*** positive constants, ***Pr;*** Prandtl number $\left\{= \frac{\mu C_p}{\kappa}\right\}$, ***Re;*** Reynolds Number $\left\{= \frac{Ux}{\nu}\right\}$, $\boldsymbol{C_f}$*;* local skin-friction coefficient, $\boldsymbol{C_p}$*;* specific heat at constant pressure, ***S;*** stratification parameter $\left\{= \frac{a}{b} < 1\right\}$, ***T;*** temperature of the fluid, $\boldsymbol{T_\infty}$*;* temperature of ambient fluid far away from plate $\{= T_0 + ax\}$, $\boldsymbol{T_w}$*;* temperature of the plate $\{= T_0 + bx\}$, ***U;*** uniform velocity of plate, ***u, v;*** velocity components along *x*- and *y*-directions, respectively.

Greek letters: $\boldsymbol{\kappa}$***;*** coefficient of thermal conductivity, $\boldsymbol{\mu}$***;*** coefficient of viscosity, $\boldsymbol{\beta}$***;*** coefficient of thermal expansion, $\boldsymbol{\rho}$***;*** density of fluid, $\boldsymbol{\theta}$***;*** dimensionless temperature $\left\{= \frac{T - T_\infty}{T_w - T_0}\right\}$, $\boldsymbol{\sigma}$***;*** electrical conductivity, $\boldsymbol{\nu}$***;*** kinematic viscosity $\left\{= \frac{\mu}{\rho}\right\}$, $\boldsymbol{\psi}$***;*** stream function, $\boldsymbol{\eta}$***;*** similarity variable. **Superscript: ′;** differentiation with respect to η.

REFERENCES

Bansal JL (1994). “Magnetofluiddynamics of Viscous Fluids”, Jaipur Publishing. House, Jaipur, India.

Bejan A (1984). “Convection Heat Transfer”, John Wiley & Sons, New York, USA.

Cheesewright R (1967). “Natural convection from a plane, vertical surface in non-isothermal surroundings”, Int. J. Heat Mass Trans. 10: 1847-1859.

Chen CC, Eichhorn R (1976). “Natural convection from a vertical surface to stratified fluid”, ASME J. Heat Trans. 98: 446-451.

Chen CH (2000). “Heat transfer characteristic of a non-isothermal surface moving parallel to a free stream”, Acta Mechanica. 142: 195-205.

Conte SD, Boor C (1981). Elementary Numerical Analysis, McGraw-Hill Book Co., New York, USA.

Fumizawa M (1980). “Natural convection experiment with NaK under transverse magnetic field”, J. Nucl. Sci. Tech. 17(2): 98-105.

Jeffery A (1966). “Magnetohydrodynamics”, Oliver and Boyed, New York, USA.

Kulkarni AK, Jacob HR, Hwang JJ (1987). “Similarity solution for natural convection flow over an isothermal vertical wall immersed in a thermally stratified medium”. Int. J. of Heat and Mass Trans. 30: 691-698.

Moutsoglou A, Chen TS (1980). “Buoyancy effects in boundary layers on inclined continous moving sheets”, ASME J. of Heat Trans.102: 171-173.

Ramachandran N, Armaly BF, Chen TS (1987). “Correlation for laminar mixed convection in boundary layer adjacent to inclined continuous moving plate”. Int. J. Heat Mass Trans. 102: 171-173.

Saha SC, Hossain MA (2004). “Natural Convection flow with combined buoyancy effects due to thermal and mass diffusions in a thermally stratified media”. Non Linear Analysis: Modell. Control. 9(1): 89-102.

Sakiadis BC (1961). “Boundary layer behaviour on continuous solid surface- II: Boundary layer on a continuous flat surface”, AIChE J. 7(1): 221-225.

Schlichting H, Gersten K (1999). “Boundary Layer Theory”, McGraw-Hill Book Co., New York, USA.

Takhar HS, Chamkha AJ, Nath G (2001). “Natural convection flow from a continuously moving vertical surface immersed in thermally stratified medium”. Heat Mass Transf. 38: 17-24.

Tsou FK, Sparrow EM, Goldstein RJ (1967). “Flow and heat transfer in the boundary layer on a continuously moving surface”, Int. J. Heat Mass Trans. 10: 219-235.

Uotani M (1987). “Natural convection heat transfer in thermally stratified liquid metal”. J. Nucl. Sci. Tech. 24(6): 442-451.

Venkatachala BJ, Nath G (1981). “Non-similar laminar natural convection in a thermally stratified fluid”, Int. J. Heat Mass Trans. 24: 1848-1859.