Full Length Research Paper

New extraction method of the scattering parameters of a physical system starting from its causal bond graph model: application to a microwave filter

Hichem Taghouti^{1*} and Abdelkader Mami^{1,2}

¹Laboratory of Analysis and Command Systems (L.A.C.S), ENIT, Tunis, C. U. Belvedere 1060, Tunis, Tunisia. ²Department of Physics, Faculty of Sciences of Tunis, Electronic Laboratory, 2092 El Manar, Tunis, Tunisia.

Accepted 23 May, 2011

The general idea consists of using jointly the bond graph technique and the scattering formalism. For that, we propose to extract the scattering parameters of a microwave filter with localized or distributed elements from its causal bond graph model while starting with the determination of the integrodifferentials operators which is based, in their determination, on the causal ways and causal loops present in the associated bond graph model, and which gives rise to the wave matrix which gathers the incident and reflected waves propagation of the studied filter. The scattering parameters, founded from the wave matrix, were checked by comparison of the simulation results. Thereafter, we used a procedure to model these scattering parameters on a particular type of bond graph model often named Scattering Bond Graph Model.

Key words: Waves propagation, bond graph modelling, high frequency domain, scattering parameters, integrodifferentials operators, microwave filter, modelling and simulation.

INTRODUCTION

The scattering or the wave-scattering formalism (Paynter and Busch-Vishniac, 1988) was used in vast physique fields such as the characterization of the electric circuits.

Several work, since the invention of the bond graph approach (Di Filippo et al., 2002), showed that the scattering formalism (Newton, 2002) constitutes an alternative approach for the physical systems modelling (Kamel and Dauphin-Tanguy, 1993). They pointed out, on one hand, some properties and in particular, the orthogonality of wave matrix (Magnusson et al., 2001) respectively the scattering matrix (Ferrero and Pirola, 2006) which represent intrinsically the causal relations and includes explicitly the conservation laws (Pedersen, 2003). They showed in addition that the scattering representation exists for systems having neither impedance nor admittance such as the junctions of Kirchhoff, the gyrateurs and the transformers (Patrick and Adrien, 2008).

In addition, the bond graph language (Di Filippo et al.,

2002) is based on a graphic representation of the physical systems. These representations are based on the identification and the idealization of the intrinsic characteristics of the physical environments and on the structuring of a complex physical system in the networks form (Belevich, 1968).

Moreover, in physics, the analogies theory allows bond graph technique (Di Filippo et al., 2002) to generalize the representation networks with all the traditional physics fields of the systems with localized and/or distributed parameters (Christopher et al., 1999).

The purpose of this paper is to present and apply a new extraction method of the scattering parameters of any physical systems while basing on its causal and reduced bond graph model (Taghouti and Mami, 2009; Taghouti and Mami, 2010a, b, c).

At first, and after having to present the method, wepropose to use the causal bond graph model of a high frequency low-pass filter (Trabelsi et al., 2003) to find, on one hand, the integro-differentials operators (Khachatryan and Khachatryan, 2008) which is based on the causal ways and loops present in the bond graph model and, on the other hand, to extract the wave matrix

^{*}Corresponding author. E-mail: hichem.taghouti@live.fr.



Figure 1. Complex system representation with the wave scattering variables.

(Magnusson et al., 2001) from these operators.

Then, we extract directly the scattering parameters (Newton, 2002) from the found wave matrix (Magnusson et al., 2001) and, with the aim to validate the found results; we make a comparison by the simulation of these scattering parameters with a simple program and the classic techniques of conception and simulation of the microwave circuits (Vendelin et al., 2005).

Finally, we propose to build a particular type of bond graph model often named "scattering bond graph model" (Kamel and Dauphin-Tanguy, 1996) which is able to highlight these transmission and reflection coefficients (Scattering parameters) (Duclos and Clement, 2003).

EXTRACTION METHOD OF THE SCATTERING PARAMETERS

The new extraction method of the scattering parameters (Newton, 2002) is an analytical method which makes it possible to establish, for a linear complex system, the scattering relations between a fixed entry and exit (Taghouti and Mami, 2010a). However, this method implies the succession of the following stages:

- Decomposition of the system (causal bond graph model) in subsystems (causal bond graph sub-model) put in cascades which are characterized by their respective wave matrix (Magnusson et al., 2001).
- Calculating the total wave matrix (Magnusson et al., 2001) of the whole system by carrying out the product of the elementary wave matrix.
- Finally the application to this matrix of a linear transformation to extract the scattering parameters (Newton, 2002) characterizing the complex system.

The step that we propose was thought in this objective and consists of establishing a systematic method which binds the bond graph technique (Di Filippo et al., 2002) to the wave-scattering formalism (Paynter and Busch-Vishniac, 1988). This method is based on an algebragraphic procedure (Maher and Scavarda, 1991) which uses the causal ways notions and the Mason's rule (Bolton, 1998) applied to a causal bond graph transformed and reduced (Taghouti and Mami, 2009; Taghouti and Mami, 2010b, c).

Wave-scattering representation and decomposition of complex system

Generally, we can represent any complex system functioning in low or high frequency by the following model of the Figure 1 where the process is represented by the quadripole Q with different wave scattering.

These three subsystems (source, process and load) are inter-connected and communicate between them by the means of a power transfer which is done in a continuous way from the source to the load as Figure 1 indicates it (Taghouti and Mami, 2010a). It is considered that the process, in its quadripole form, is in complex form and can be decomposed to subsystems which are connected by the intermediary of the wave-scattering variables (Paynter and Busch- Vishniac, 1988) as Figure 2 indicates it.

Let us consider the two processes A and B with share where the signal entering B is directed in the same direction as the outgoing signal of process A. In a similar way, the outgoing signal of B is in the same direction as the signal entering A as Figure 3 indicates it.

If these two processes are coupled, the assumption of the power continuity (Paynter and Busch-Vishniac, 1988) will imply:

$$\mathbf{a}_{\mathrm{A}} = \mathbf{b}_{\mathrm{B}} \tag{1}$$

$$\mathbf{b}_{\mathrm{A}} = \mathbf{a}_{\mathrm{B}} \tag{2}$$

The a_A and a_B quantities entering the A and B processes are called incident waves in the same way, the quantities b_A and b_B associated with the signals leaving the A and B processes are called reflected waves (Duclos and



Figure 2. Wave scattering representation of the quadripole Q.



Figure 3. Representation of the wave scattering variables.

Clement, 2003). Classically, we express the power circulating in a bond connecting two systems in the shape of a product of the two variables effort (noted : ε) and flow (noted : ϕ) in reduced form (Maher and Scavarda, 1991).

$$P = \left(\frac{a_i}{\sqrt{2}}\right)^2 - \left(\frac{b_i}{\sqrt{2}}\right) = \varepsilon_i, \phi_i$$
(3)

$$\left(\frac{\mathbf{a}_{i} + \mathbf{b}_{i}}{\sqrt{2}}\right) \left(\frac{\mathbf{a}_{i} + \mathbf{b}_{i}}{\sqrt{2}}\right) = \boldsymbol{\varepsilon}_{i}, \boldsymbol{\varphi}_{i}$$
(4)

So we can introduce the following linear transformation:

$$\begin{bmatrix} \boldsymbol{\varepsilon}_{i} \\ \boldsymbol{\varphi}_{i} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_{i} \\ b_{i} \end{bmatrix} = \mathbf{H} \begin{bmatrix} a_{i} \\ b_{i} \end{bmatrix}$$
(5)

The linear opposite transformation of the H transformation allows the passage of the intrinsic variables effort and flows (ϵ, ϕ) with the wave variables (a_i, b_i) as the following relation indicates it:

$$\begin{bmatrix} a_i \\ b_i \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \varepsilon_i \\ \varphi_i \end{bmatrix} = H \begin{bmatrix} \varepsilon_i \\ \varphi_i \end{bmatrix}$$
(6)

The processes A and B constitute two processes with 2-

ports of entry and exit whose wave matrices are:

$$\begin{bmatrix} b_1^{A} \\ a_1^{A} \end{bmatrix} = \begin{bmatrix} w_{11}^{A} & w_{12}^{A} \\ w_{12}^{A} & w_{22}^{A} \end{bmatrix} \begin{bmatrix} a_2^{A} \\ b_2^{A} \end{bmatrix}$$
(7)

$$\begin{bmatrix} \mathbf{b}_{1}^{\mathrm{B}} \\ \mathbf{a}_{1}^{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_{11}^{\mathrm{B}} & \mathbf{w}_{12}^{\mathrm{B}} \\ \mathbf{w}_{12}^{\mathrm{B}} & \mathbf{w}_{22}^{\mathrm{B}} \end{bmatrix} \begin{bmatrix} \mathbf{a}_{2}^{\mathrm{B}} \\ \mathbf{b}_{2}^{\mathrm{B}} \end{bmatrix}$$
(8)

The chain of n processes with 2-ports of entry and exit constitutes a process with 2-ports of entry and exit where the global wave matrix W is:

$$W = W^{(A)} * W^{(B)} * \dots * W^{(N)} = \prod_{i=1}^{N} W^{(i)}$$
(9)

So:

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{a}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{w}_{11} & \mathbf{w}_{12} \\ \mathbf{w}_{12} & \mathbf{w}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a}_2 \\ \mathbf{b}_{21} \end{bmatrix} = \begin{bmatrix} \mathbf{W} \end{bmatrix} \begin{bmatrix} \mathbf{a}_2 \\ \mathbf{b}_2 \end{bmatrix}$$
(10)

The scattering parameters are given by the following scattering matrix:

$$\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{a}_2 \\ \mathbf{a}_2 \end{bmatrix}$$
(11)

The relations between these matrixes are given by the following equations:

$$\begin{cases} w_{11} = -S_{21}^{-1} * S_{22} \\ W_{12} = S_{21}^{-1} \\ W_{21} = S_{12} - S_{11} * S_{22} * S_{21}^{-1} \\ W_{22} = S_{11} * S_{21}^{-1} \end{cases}$$
(12)

And:

$$\begin{cases} S_{11} = -W_{22}^{-1} * W_{12} \\ S_{21} = W_{22}^{-1} \\ S_{12} = W_{11} - W_{21} * W_{12} * W_{22}^{-1} \\ S_{22} = W_{21} * W_{22}^{-1} \end{cases}$$
(13)

Wave scattering parameters and causal bond graph model

It is considered that the process, in its quadripole form and when inserted between two particular ports P_1 and P_2 which represent respectively the entry (source) and the exit (load) of the complex system can be represented by the following bond graph model transformed and reduced such as:

 ϵ_1 and ϵ_2 are respectively the reduced variable (effort) at the entry and the exit of the system.

 ϕ_1 and ϕ_2 are respectively the reduced variable (flow) at the entry and the exit of the system.

$$\varepsilon_{i} = \frac{\text{effert}}{\sqrt{R_{0}}}$$
(14)

$$\varphi_i = \text{flow} * \sqrt{R_0}$$
 (15)

These are the reduced effort (e) and flow (f) with respect to R_0 (scaling resistance).

And to establish the entry-exit analytical relations, the bond graph model of the studied system must be transformed, reduced and especially be causal since these relations rest on the concepts of causal way and causal loops which can comprise the reduced bond graph model (Taghouti and Mami, 2010a, b, c).

The causality assignment to the reduced bond graph model of Figure 4 enables us to notice that they are four different cases of causality assignment in input-output of the process (Maher and Scavarda, 1991).

For each type of reduced and causal bond graph model given below, we will have one matrix which connects the reduced variables to the integral-differentials operators H_{ij} . Figure 5 to 8. From each matrix, we can deduce the

corresponding wave matrix by referring to the Equations 5, 6 and 10.

These wave matrices can give us the corresponding scattering parameters by referring to the Equation 13 and the following equations.

$$\begin{bmatrix} \varepsilon_1 \\ -\phi_1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$$
(16)
$$\begin{bmatrix} \varepsilon_2 \\ -\phi_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$$
(17)

Case 1

$$\begin{pmatrix} \boldsymbol{\varepsilon}_{1} \\ \boldsymbol{\varphi}_{2} \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varphi}_{1} \\ \boldsymbol{\varepsilon}_{2} \end{pmatrix}$$
(18)

$$W = \begin{bmatrix} \frac{1 - H_{11} + H_{22} - \Delta H}{2H_{21}} & \frac{-1 - H_{11} + H_{22} - \Delta H}{2H_{21}} \\ \frac{-1 - H_{11} + H_{22} - \Delta H}{2H_{21}} & \frac{1 - H_{11} + H_{22} - \Delta H}{2H_{21}} \end{bmatrix}$$
(19)

$$\Delta H = H_{11} + H_{22} - H_{12}H_{21}$$
(20)

Case 2

$$\begin{pmatrix} \boldsymbol{\varphi}_1 \\ \boldsymbol{\varepsilon}_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varphi}_2 \end{pmatrix}$$
(21)

$$W = \begin{bmatrix} \frac{1 - H_{11} + H_{22} - \Delta H}{2H_{21}} & \frac{1 - H_{11} - H_{22} + \Delta H}{2H_{21}} \\ \frac{-1 + H_{11} + H_{22} - \Delta H}{2H_{21}} & \frac{1 + H_{11} - H_{22} - \Delta H}{2H_{21}} \end{bmatrix}$$
(22)

Case 3

$$\begin{pmatrix} \boldsymbol{\epsilon}_{1} \\ \boldsymbol{\epsilon}_{2} \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varphi}_{1} \\ \boldsymbol{\varphi}_{2} \end{pmatrix}$$
(23)
$$W = \begin{bmatrix} \frac{-1 + H_{11} - H_{22} + \Delta H}{2H_{21}} & \frac{-1 + H_{11} + H_{22} - \Delta H}{2H_{21}} \\ \frac{1 + H_{11} + H_{22} + \Delta H}{2H_{21}} & \frac{1 + H_{11} - H_{22} - \Delta H}{2H_{21}} \end{bmatrix}$$
(24)

Case 4

$$\begin{pmatrix} \boldsymbol{\varphi}_{1} \\ \boldsymbol{\varphi}_{2} \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{pmatrix} \begin{pmatrix} \boldsymbol{\varepsilon}_{1} \\ \boldsymbol{\varepsilon}_{2} \end{pmatrix}$$
(25)



Figure 4. The reduced bond graph representation.



Figure 5. Reduced bond graph model with flow- effort causality.



Figure 6. Reduced bond graph model with effort-flow causality.



Figure 7. Reduced bond graph model with flow-flow causality.



Figure 8. Reduced bond graph model with effort-effort causality.



Figure 9. The low-pass filter with localized elements and its tow ends P1 and P2.

$$W = \begin{bmatrix} \frac{-1 + H_{11} - H_{22} + \Delta H}{2H_{21}} & \frac{1 - H_{11} - H_{22} + \Delta H}{2H_{21}} \\ \frac{-1 - H_{11} - H_{22} - \Delta H}{2H_{21}} & \frac{1 + H_{11} - H_{22} - \Delta H}{2H_{21}} \end{bmatrix}$$
(26)

Now, if we like to find the scattering matrix we need to use the Equation 13.

We note that H_{ij} are the integral-differentials operators which are based, in their determination, on the causal ways and causal loops present in the associated bond graph model (Maher and Scavarda, 1991; Taghouti and Mami, 2010a).

$$H_{ij} = \sum_{k=1}^{N} \frac{G_k \Delta_k}{\Delta}$$
(27)

$$\Delta = 1 - \sum L_i + \sum L_i L_j - \sum L_i L_j L_k + ... + (-1)^m \sum ... + ...$$
(28)
Where:

 Δ = the determinant of the causal bond graph.

 $P_i = input port.$

- P_i= output port.
- H_{ii} = complete gain between P_i and P_i .

N = total number of forward paths between P_i and P_i.

 G_k = gain of the kth forward path between P_i and P_i.

 L_i = loop gain of each causal algebraic loop in the bond graph model.

 L_iL_j = product of the loop gains of any two non-touching loops (no common causal bond).

 $L_i L_j L_k$ = product of the loop gains of any three pairwise nontouching loops.

 Δ_k = the cofactor value of Δ for the kth forward path, with the loops touching the kth forward path removed; that is remove those parts of the causal bond graph which form the loop, while retaining the parts needed for the forward path.

APPLICATION TO A LOW-PASS FILTER WITH LOCALIZED ELEMENTS

In this paragraph, we will try to apply and check the procedure described previously to a low-pass filter with localized elements as Figure 9 indicates it (Taghouti and Mami, 2010b, c).

This filter is a Chebyshev filter with order 4 and having the following characteristics:

100 MHz of cut-off frequency (f_c). Sensibility: k = 0.5. Attenuation: $A_{max} = 0.1$ dB, $A_{min} = 20$ dB.

Extraction of the scattering parameters from the bond graph model

The bond graph model of this studied filter is given by Figure 10. So that, to extract the scattering parameters from the bond graph model and by using the new extraction method which is described previously; we must transform the bond graph model into a causal bond graph model often named reduced bond graph model (Maher and Scavarda, 1991; Taghouti and Mami, 2009) only containing the reduced variables with respect to a scaling resistance R_0 (internal source resistance) such as:

$$\mathbf{t}_{ci} = \mathbf{R}_0 * \mathbf{C}_i \tag{29}$$

$$\tau_{\rm Li} = \frac{\rm L_i}{\rm R_0} \tag{30}$$

 $z_{i:}$ the reduced equivalent impedance of the i element put in series.

y_i: the reduced equivalent admittance of the i element put in parallel.









20-sim 4.1 Viewer (c) CLP 2009

Figure 11. The transformed and reduced causal bond graph model.

So we have:

 $\mathbf{z}_{1} = \boldsymbol{\tau}_{L1} * \mathbf{P} \tag{31}$

 $\mathbf{z}_2 = \boldsymbol{\tau}_{L2} * \mathbf{P} \tag{32}$

 $\mathbf{y}_1 = \boldsymbol{\tau}_{c1} * \mathbf{P} \tag{33}$

 $\mathbf{y}_2 = \mathbf{\tau}_{c2} * \mathbf{P} \tag{34}$

p: Laplace operator.

The bond graph model given above in Figure 11 can be broken up into two cells (sub-model) put in cascade form while respecting the assumption of the power continuity (Paynter and Busch-Vishniac, 1988; Taghouti and Mami, 2010c) between all sub-models.

Each cell is made up with an impedance z in parallel with an admittance y often noted [z---y], if the studied filter is with T form, or [y---z] if the studied filter is with Π form (type). So we have the first sub-model such as:

This model is in conformity with case 1 described

previously. So we have the integro-differentials operators by taking account to the previously equations.

$$L_1 = \frac{-1}{z_1 y_1}$$
 : Loop gain of the algebraic loop.

 $\Delta{=}1{+}\frac{1}{z_1y_1}$: Determinant of the associated causal bond graph.

$$\begin{cases} H_{11} = \frac{z_1}{z_1 y_1 + 1} \\ H_{12} = \frac{1}{z_1 y_1 + 1} \\ H_{21} = \frac{1}{z_1 y_1 + 1} \\ H_{22} = \frac{-y_1}{z_1 y_1 + 1} \end{cases}$$
 : All the integro-differentials operators
$$H_{22} = \frac{-y_1}{z_1 y_1 + 1} \\ \Delta H = \frac{-1}{z_1 y_1 + 1} \end{cases}$$

From these operators, we can deduce directly the wave matrix by taking into account the equations of case 1:

$$W^{(1)} = \frac{1}{2} \begin{bmatrix} z_1 y_1 - z_1 - y_1 + 2 & -z_1 y_1 + z_1 + y_1 \\ -z_1 y_1 - z_1 + y_1 & z_1 y_1 + z_1 + y_1 \end{bmatrix} \quad : \quad \text{Wave}$$

matrix of the first sub-model.

And now we have the second sub-model (Figure 13). And in the same manner we can extract the second wave matrix such as:

$$L_2 = \frac{-1}{z_2 y_2} : \text{Loop gain of the algebraic loop.}$$

$$\Delta = 1 + \frac{1}{z_2 y_2} : \text{Determinant of the associated causal}$$

bond graph.

$$\begin{cases} H_{11} = \frac{Z_2}{Z_2 y_2 + 1} \\ H_{12} = \frac{1}{Z_2 y_2 + 1} \\ H_{21} = \frac{1}{Z_2 y_2 + 1} \\ H_{22} = \frac{-y_2}{Z_2 y_2 + 1} \\ \Delta H = \frac{-1}{Z_2 y_2 + 1} \end{cases}$$
 : All the integro-differentials operators.

And the second wave matrix is:

$$\mathbf{W}^{(2)} = \frac{1}{2} \begin{bmatrix} z_2 y_2 - z_2 - y_2 + 2 & -z_2 y_2 + z_2 + y_2 \\ -z_2 y_2 - z_2 + y_2 & z_2 y_2 + z_2 + y_2 \end{bmatrix}$$

The total wave matrix is given by the product of the first and the second wave matrix such as:

$$\mathbf{W}^{(\mathrm{T})} = \mathbf{W}^{(1)} * \mathbf{W}^{(2)} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} \\ \mathbf{W}_{21} & \mathbf{W}_{22} \end{bmatrix}$$
(35)

So the corresponding scattering matrix is given in Equation 36:

$$\mathbf{S}^{(\mathrm{T})} = \begin{bmatrix} \mathbf{W}_{22}^{-1} * \mathbf{W}_{12} & \mathbf{W}_{11} - \mathbf{W}_{21} * \mathbf{W}_{12} * \mathbf{W}_{22}^{-1} \\ \mathbf{W}_{22}^{-1} & -\mathbf{W}_{21} * \mathbf{W}_{22}^{-1} \end{bmatrix}$$
(36)

From this matrix we can deduce these following scattering parameters:

$$S_{11} \frac{z_{1}z_{2}y_{1}y_{2} + z_{1}y_{2}(y_{1} - z_{2}) + z_{1}(y_{1} - y_{2} - l) + z_{2}(y_{1} + y_{2} - l) + y_{1} + y_{2}}{d(p)}$$
(37)

$$S_{12} = S_{21} = \frac{2}{d(p)}$$
(38)

$$S_{22} \frac{-z_1 z_2 y_1 y_2 + z_1 y_2 (y_1 - z_2) + z_1 (y_2 - y_1 - 1) - z_2 (y_1 + y_2 - 1) + y_1 + y_2}{d(p)}$$
(39)

$$d(p) = z_1 z_2 y_1 y_2 + z_1 y_2 (y_1 + z_2) + (y_2 + y_1)(z_1 + z_2) + (z_1 + z_2) + (y_1 + y_2) + 2$$
(40)

SIMULATION RESULTS AND CHECKING

Thus, the validation is carried out by simulating the scattering parameters of equations 37, 38 and 39.

A simple programming of the previously scattering parameters equations is given in Figures 14, 17, 15 and 16 below which represent respectively the reflection and transmission coefficients of the studied filter (Taghouti and Mami, 2010c).

We notice that the reflection coefficients S_{11} and S_{22} are equal in module. This result is also checked by the figures as follows.

$$|S_{11}| = |S_{22}|$$
(41)

To validate and checked the found results, by simulation, it is enough to simulate the low-pass filter of the Figure 18 to find the representative curves of the reflection and transmission coefficients respectively Sii and S_{ij} by the



Figure 12. The first causal bond graph sub-model.



Figure 13. The second causal bond graph sub-model.



Figure 14. Reflection coefficient S_{11} seen at entry.



Figure 15. Transmission coefficient S_{12} seen from exit to entry.



Figure 16. Transmission coefficient S_{21} seen from entry to exit.



Figure 17. Reflection coefficient S₂₂ seen at exit.

ADS software (Advanced Design System) (Dirk Jansen 2005) often used in microwave and regarded as a traditional method in the line's theory (Magnusson et al., 2001). Figure 18 thus represents the system's model studied with adapted entry and exit and the numerical

values of its elements necessary for simulation (Taghouti and Mami, 2010c).

The simulation of the low-pass filter above gives the graphical representation of the reflection and transmission coefficients S_{ii} and S_{ij} (i≠j and i, j=1...2) according



Figure 18. The low-pass filter under the HP-ADS software.

to the frequency (Figure 19).

MODELLING OF THE SCATTERING PARAMETERS WITH BOND GRAPH TECHNIQUE

We noted that we will use the method which is developed by Pr. A. Kamel (Kamel and Dauphin-Tanguy, 1993; 1996).

Procedure used to model the scattering matrix by the bond graph technique

The scattering matrix of our studied process is a 2-2 matrix having a particular form, no matter the expressions complexity of the series impedance or parallel admittance. It is orthogonal since the process is considered without loss, and it admits the following general form.

$$S = \frac{\begin{bmatrix} b_n^{11}p^n + \dots + b_0^{11} & b_n^{12}p^n + \dots + b_0^{12} \\ b_n^{21}p^n + \dots + b_0^{21} & b_n^{22}p^n + \dots + b_0^{21} \end{bmatrix}}{a_np^n + a_{n-1}p^{n-1} + \dots + ao}$$
(42)

We note that:

$$d(p) = a_n p^n + a_{n-1} p^{n-1} + \dots + a_0$$
(43)

Indeed, if we consider the scattering matrix form found earlier for the process alone, we can say that it is not a

true transfer matrix (Belevich, 1968). Moreover, it is not in the adequate form since its various S_{ii} parameters and sometimes S_{ij} have the numerator's degree equal to that of denominator and that poses a major problem to determinate the scattering bond graph model of any physical system studied in a general way.

The solution with this problem is to regard the scattering matrix of a process as a transfer matrix from an input-output point of view, connecting the incident and the reflected waves in a symbolic system form.

We start by carrying out an Euclidean division of each term of the numerator matrix (scattering parameters) by the common denominator d(s) that leads to the new shape of the scattering matrix (Belevich, 1968) such as:

$$S = S' + D \tag{44}$$

S: the new scattering matrix with degrees in the numerator at most one less than that of d(s).

D: direct transmission matrix.

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_1 & \mathbf{d}_3 \\ \mathbf{d}_4 & \mathbf{d}_2 \end{bmatrix}$$
(45)

Thereafter we seek for the new matrix $\langle S \rangle$ its development in continuous fraction in alpha-beta starting from the Routh method (Shamash, 1980) and build the corresponding bond graph model, since it is about a multivariable system (Molisch et al., 2002) while being based on the systematic procedure according to:

• Calculate the α-Routh table from the common





Figure 19. Simulation results of the low-pass filter.

denominator d(s) and the $\beta\mbox{-Routh}$ table from the new numerator of the S-matrix.

- Construct the direct chain by using the adequate number of elements I-C (which α_i coefficients are their modules) in integral causality, equal to the degree of d(s).
- Duplicate this chain and construct the two entries of the quadruple.
- Construct the tow outputs by using information bonds and a sufficient number of TF and GY elements whose modules are precisely the β^{ij}_n coefficients.
- Add the direct part (transmission matrix D) by using information bonds.

To obtain the scattering bond graph model of the physical system, it is enough to add the reflection coefficient

 $\rho_g = \frac{z_0 - 1}{z_0 + 1}$ of the source and the reflection coefficient

 $\rho_c = \frac{z_L - 1}{z_L + 1}$ of the load to the scattering bond graph model

of the process to the adequate sites.

Scattering bond graph model of the low-pass filter

To obtain the scattering bond graph model of the studied circuit, it is enough to add the reflexion coefficient

$$\rho_g = \frac{z_0 - 1}{z_0 + 1}$$
 of the source and the reflexion coefficient

 $\rho_{c} = \frac{z_{L} - l}{z_{L} + 1}$ of the load to the scattering bond graph model

of the process to the adequate sites (Figure 20).

It is interesting to notice that the structure of the scattering bond graph of the process remains the same whatever the degree of the common denominator of scattering matrix. The only thing that changes is the corresponding number of I and C linked to the α -Routh expansion (Kamel and Dauphin-Tanguy, 1993; 1996; Taghouti and Mami, 2010c).

Conclusion

In this paper, we tried to present a method which appears new for the determination of the scattering parameters of any physical system functioning in high frequency. Then, we applied this technique to a low-pass filter based on



20-sim4.1 Viewer (c) OLP 2009

Figure 20. Scattering bond graph model of the low-pass filter connecting to its source and load.

localized elements.

Lastly, we validated the results found by a simple comparison between two methods of simulation: simulation by the traditional methods used in microwave under the ADS software and simulation by the our own method of the reduced bond graph which is based on the causal and simplified bond graph model of the studied system like on the minimum of the causal ways and loops present in this model often decomposed to sub-models as we showed previously.

Generally, this new analysis method leads us to use this new method which combines at the same time the bond graph technical and the scattering formalism for modelling and simulation of the scattering matrices of any electrical circuits often functioning in high frequency and based on localized or distributed elements giving rise to the famous model often named: Scattering Bond Graph. This new type of modelling will enable us to capture the power transfers in a simple and direct manner at the same time and it proposes us a temporal approach of the phenomena usually modeled with the frequencies tools.

ACKNOWLEDGMENTS

We thank all those who contributed to this work. We thank all the members of laboratory of analysis and commands systems under the direction of Pr. MEKKI KSOURI, like Pr. MAMI ABDELKADER and without forgetting the members of the laboratory of electronics and high frequency circuits.

REFERENCES

Belevich V (1968). Classical Network Theory. Holden-Day, San Francisco.

- Bolton, Newnes W (1998). Control Engineering Pocketbook CONTROL ENGINEERING POCKETBOOK. Oxford: Newness.
- Christopher I, Byrnes, David S, Gilliam, Victor I, Shubov (1999). Boundary control, stabilization and zero-pole dynamics for a nonlinear distributed parameter syst., Int. J. Robust. Nonlinear Cont., 9(11): 737 – 768.
- Di Filippo JM, Delgado M, Brie C, Paynter HM (2002). A survey of bond graphs: Theory, applications and programs. J. Franklin Inst., 328 (5-6): 565-606.
- Dirk J (2005). Advanced Design System: The Electronic Design Automation Handbook. Kluwer Academic Publishers, 2003. ISBN: 1-4020-7502-2, German ISBN 3-446-21288-4 (2005), p. 684.
- Duclos G, Clément AH (2003). A new method for the calculation of transmission and reflection coefficients for water waves in a small basin. Comptes Rendus Mecanique, 331(3): 225-230.
- Ferrero A, Pirola M (2006). Generalized Mixed-Mode S-Parameters. IEEE Transactions on Microwave Theory and Techniques. 54(1): 458-463.
- Kamel A, Dauphin-Tanguy G (1993). Bond Graph Modeling of Power Waves in the Scattering Formalism. J. title: SIMULATION SERIES. 25(2): 41. Publisher: SCS Society for Computer Simulation, USA, And ISSN: 0735-9276.
- Kamel A, Dauphin-Tanguy G (1996). Power transfer in physical systems using the scattering bond graph and a parametric identification approach. Syst. Anal. Modelling Simulation, 27(1): 1-13.
- Khachatryan A (2008). Factorization of a convolution-type integrodifferential equation on the positive half line. 60(11): 1555–1567.
- Magnusson PC, Alexander GC, Tripathi VK, Weisshaar A (2001). Transmission lines and wave propagation. 4th ed.CRC Press.
- Maher A, Scavarda S (1991). A procedure to match bond graph and scattering formalisms. J. Franklin Inst., 328(5-6): 887-89.
- Molisch AF, Steinbauer M, Toeltsch M, Bonek E, Thomä RS (2002). Capacity of MIMO Systems Based on Measured Wireless Channels. IEEE J. on Selected Areas in Communications, 20(3): 561-569.
- Newton RG (2002). Scattering theory of waves and particles. New York: Springer-Verlag; Dover Edition.
- Patrick J, Adrien S (2008). Construction and analysis of improved Kirchhoff conditions for acoustic wave propagation in a junction of thin slots. ESAIM Proceeding, 25:44-67.
- Paynter HM, Busch-Vishniac I (1988). Wave scattering Approaches to conservation and causality. J. Franklin inst., 325(3): 295-313.
- Pedersen G (2003). Energy conservation and physical optics for discrete long wave equations. J. Wave Motion, 37(1): 81-100.

- Shamash Y (1980). Stable biased reduced order models using the Routh method of reduction. Int. J. Sys. Sci., 1464-5319. 11(5): 641-654.
- Taghouti 0H, Mami A (2010c). Modelling Method of a Low-pass Filter based on Microstrip T-Lines with Cut-Off Frequency 10 GHz by the Extraction of its Wave-Scattering Parameters from its Causal Bond Graph Model. Am. J. Eng. Appl. Sci., 3(4): 631-642.
- Taghouti H, Mami A (2009). Application of the reduced bond graph approaches to determinate the scattering parameters of a high frequency filter. Proceedings of 10th Int. conference on Sci., and Techniques of Automatic Control & Computer Engineering. Hammamet, Tunisia, December 20-22, 2009, STA'2009-SSI-548: 379-391.
- Taghouti H, Mami A (2010a). How to Find Wave-Scattering Parameters from the Causal Bond Graph Model of a High Frequency Filter. Am. J. Appl. Sci., 7(5): 702-710.
- Taghouti H, Mami A (2010b). Extraction, Modelling and Simulation of the Scattering Matrix of a Chebychev Low-Pass Filter with cut-off frequency 100 MHz from its Causal and Decomposed Bond Graph Model. ICGST Int. J. Auto. Cont. Sys. Eng., 10(1): 29-37.
- Trabelsi H, Gharsallah A, Baudrand H. (2003). Analysis of microwave circuits including lumped elements based on the iterative method. Int. J. RF Microwave CAE. 13(4): 269-275.
- Vendelin GD, Pavio AM, Rohde UL (2005). Microwave Circuit Design Using Linear and Non Linear Techniques. 2nd Edition, ISBN: 978-0-471-41479-7, Hardcover, 1080 pages.