

Full Length Research Paper

Magneto-Stark effect on exciton in parabolic band $GaAs/Ga_{0.7}Al_{0.3}As$ quantum well heterostructure

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The effect of applied crossed electric and magnetic fields on the heterostructure semiconductor is used in the scientific investigation on electronic and optical exciton properties. The aim of this work is to study the magneto-Stark effect for confined excitons in single $GaAs - Ga_{0.7}Al_{0.3}As$ QWs. The magnetic field B is taken as perpendicular to the z -growth direction of the heterostructure, whereas the applied electric field E is along the z -growth direction. The data we used includes intrinsic parameters of the systems and manipulated external magnetic and electric fields. In the model equation, we utilized variational non-degenerate parabolic band approximations using 1 s hydrogen like ion ground state to calculate the position at which spatial distance b/n electron and hole ($\Delta = 0$), that is, overlap $e-h$ occurred where $B \rightarrow \infty$ and $E \rightarrow 0$; we also used Matlab version R2017a to simulate our result as depicted in graphs. As electric field (E) increases along growth z -direction, the spatial distance (Δ) increases due to a reduction of Coulomb interaction b/n $e-h$, whereas increasing the magnetic field (B) perpendicular to the growth z -direction has the reverse effect and shrinks the wave function in the QW plane. This shrinkage enhances the $e-h$ interaction, which in turn, more likely localizes the electron within the same QW as the whole and thus keeps the ground state in a direct exciton which is efficient in photonics.

Key words: Spatial distance, exciton, growth direction, non-correlation, magneto-Stark.

INTRODUCTION

Quantum confinement effect (QCE) of carriers in various nanostructures plays an essential role in modern science and nanotechnology. QCE changes significantly the electronic structure, the density of state, and consequently influences the transport and optical properties of systems (Xiao-Jing and Chan, 2013). External perturbations such as uniform electric field and magnetic fields are effective tools to study the impurity related properties in semiconductor nanostructures. As well known the

presence of magnetic field introduces another electronic confinement depending on direction of magnetic field (parallel or perpendicular the growth direction) some interesting physical phenomena can take place (Brozak et al., 1990) and it requires extensive studies for the novel fundamental physical properties of semiconductor nanostructure arises due to dependence of external parameters.

Furthermore applied electric field causes an asymmetric

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asymmetric distribution of carriers' density which strongly modifies the electronic and optical properties of semiconductor nanostructures (Zeng et al., 2012). External electric and magnetic fields reduces the symmetry of the system, therefore leading to level structures possessing numerous complex splitting of excitonic absorption lines. The analysis of excitonic absorption spectra in both electric and magnetic field strengths has been a long-standing subject from the theoretical as well as experimental point of view (Duque CA et al., 2006). Due to specific material parameters, excitonic properties such as Bohr radius and electric/magnetic field strength units provide the possibility to access exotic regimes more easily compared to standard atomic systems. Excitons are often considered to be a solid-state quasi-particle analogue to the hydrogen atom.

A detailed knowledge of the optical properties of semiconductor heterostructures is of paramount importance for possible device applications. In that respect, the study of exciton properties in those systems is of great interest as such coupled electron-hole (e - h) excitations, which arise from the e-h. Coulomb interaction, may considerably modify the interband optoelectronic properties of semiconductor heterostructures (Duque et al., 2006). The development of semiconductor nanostructures, such as quantum boxes and dots, by the use of different techniques, has attracted much theoretical and experimental attention (Petroff et al., 1987). Exciton effects on the optical spectrum of such systems usually consist of discrete states each having a dispersion that reflects the movement of the coupled e-h pair as a whole. As the photon momentum is quite small. Optical experimental studies usually would not give much information about the finite center-of-mass momentum of exciton states. Studies in exciton dispersion and related properties have been recently performed both experimentally, through photoluminescence PL and magneto absorption experiments, (Whittaker et al., 1991; Fritze et al., 1996; Butov et al., 2000; Parlangeli et al., 2000) measurement of the Fano line shape for resonant states (Oberli et al., 1994) the exciton-mass dependence of the recombination time experimental data on polariton effects, (Houdré et al., 1994; Tredicucci et al., 1995) and recent PL measurements (Ashkinadze et al., 2005) of modulation-doped GaAs/AlGaAs QWs and heterojunctions, as well as theoretically (Gorkov and Dzyaloshinskii, 1967; Paquet et al., 1985; Duque CA et al., 2006).

In undoped quantum dots the optical properties are dominated by excitonic effects. In this paper, we consider quantum dots exhibiting a nearly parabolic confinement for both electrons and holes. It examines theoretical values based on intrinsic properties of the studied systems ($GaAs - Ga_{0.7}Al_{0.3}As$) QW heterostructure of excitons in the presence of external applied electric field (\mathbf{E}) parallel to the \mathbf{z} -axis and applied external magnetic

(\mathbf{B}) fields in plane to the growth direction \mathbf{z} -axis.

METHOD OF CALCULATION

The total Hamiltonian for the exciton in a parabolic band QW heterostructure in the presence of electric (\mathbf{E}) and a magnetic (\mathbf{B}) field applied in the same direction (\mathbf{z} axis) (here we assume that we have an Harmonic oscillator) can be expressed as:

$$H = H_{0(z_e)} + H_{0(z_h)} + H_{mag}(\rho) + V_{coul}(\rho, |z_e, z_h|) \quad (1)$$

$H_{0(z_e)}$ is the 1-D Hamiltonian for electrons, $H_{0(z_e)} = \frac{p_{z_e}^2}{2m_e^*} + V_{z_e}(z_e)$, $H_{0(z_h)}$, is the 1-D Hamiltonian for holes

$$H_{0(z_h)} = \frac{p_{z_h}^2}{2m_h^*} + V_{z_h}(z_h)$$

and $V_e(z_e)$ and $V_h(z_h)$ is the potential that defines the double quantum well for electrons (holes) in the five regions of z , $H_{mag}(\rho)$ is the magnetic Hamiltonian in the symmetric gauge, which depends on the relative coordinates of electrons and holes in the $x - y$ plane.

$$H_0 = \frac{p_x^2}{2M} + \frac{p_x^2}{2\mu} + k_y l_B (eEl_B) - \frac{(eEl_B)^2}{2\beta_0} + \sum_{i=e,h} \left[\frac{p_{iz}^2}{2m_i^*} + V_i(z_i) + \frac{1}{2} m_i^* \omega_i^2 (z_i - z_i^0) \right] \quad (2)$$

And the eigenvalue

$$E_0 = \frac{p_x^2}{2M} + \frac{p_x^2}{2\mu} - k_y l_B (eEl_B) - \frac{(eEl_B)^2}{2\beta_0} + E_\eta^e(z_e^0) + E_\alpha^h(z_h^0) \quad (3)$$

Where $\eta, \alpha = 0, 1, 2, 3 \dots$ are the Landau magnetic sub band indices $\beta_0 = \frac{\hbar^2}{Ml_B^2}$, $l_B = \sqrt{\frac{\hbar c}{eB}}$ is the Landau magnetic length (or cyclotron radius), μ is the e-h reduced mass, M total mass, $z_{e,h}^0$ are the non correlated e and h orbit-center positions along the growth direction, and $\omega_{e,h}$ are the corresponding cyclotron frequencies. Note that:

$$\Delta = z_e^0 - z_h^0 = l_B \left(k_y l_B + \frac{eEl_B}{\beta_0} \right) \quad (4)$$

This equation represents the spatial distance between the centers of the non-correlated electron and hole magnetic parabolas. The above result for the non-correlated e - h pair allows one to obtain the ground-state exciton energy via a variation procedure with a hydrogenic-like 1 s-like type of trial envelope exciton wave function. We will first present the theoretical results for bulk GaAs. The heavy-hole exciton binding energy E_b , the $e - h$ overlap integral I overlap, and the average $e - h$ distance $\langle z_e - z_h \rangle$ along the growth direction. The exciton binding energy is a decreasing function of both Δ and \mathbf{E} , for each value of \mathbf{B} . These results may be understood as follows: As in the bulk the non-correlated electron and hole oscillate around the points z_e^0 and z_h^0 respectively, and the binding energy is determined by the e-h attractive Coulomb interaction, which tends to zero with the $e - h$ distance, it is obvious that E_b must diminish with increasing Δ and applied electric field $\mathbf{E} \sim \mathbf{B}^2 \Delta$ Equation 4 with ($k_y = 0$) for a fixed value of the magnetic field. As expected (Dignam and Sipe, 1992) the present calculations for $\Delta =$

0 and $\mathbf{E} = 0$ show an increase in the exciton binding energy with increasing values of the magnetic field, that is, with increasing confinement associated to the magnetic field. That binding energy (E_b) decreases with increasing values of the magnetic field for relatively high (fixed) value of Δ . This behavior is due to the increasing confinement of the electron and hole around their corresponding equilibrium positions as the magnetic field increases, for a given value of Δ . The e- h overlap integral as a function of Δ and of the growth-direction applied electric field.

The electric and magnetic field dependence of the in-plane Bohr radius, $a_B = \sqrt{\langle \rho \rangle^2}$ increases as electric field E increases due to a reduction in the e-h Coulomb interaction which is caused by increased spatial separation of e-h in the z direction. Increasing the magnetic field has the reverse effect and shrinks the wave function in the QW plane. This shrinkage of spatial separation enhances the binding energy of e-h interaction which, in turn, leads to a more likely localization of the electron within the same QW as the hole and thus the ground state becomes a direct exciton. The trial envelope variational wave function is chosen and simplified form of Equation 1 as:

$$\Phi = \Phi_0(\boldsymbol{\rho}, z_e, z_h) f(\mathbf{r}) \quad (5)$$

$\Phi_0(\boldsymbol{\rho}, z_e, z_h)$ is the eigenfunction of $\hat{H} = \hat{H}_0 - \frac{e^2}{\epsilon|\vec{r}_e - \vec{r}_h|}$ and $\boldsymbol{\rho}$ is the in plane internal exciton coordinate

$$f(\mathbf{r}) \propto e^{-\lambda r} \quad (6)$$

With $\vec{r} = \vec{r}_e - \vec{r}_h$ is the relative electron-hole position.

Equation 6 is a hydrogenic 1s-like wave function and λ is a variational parameter the exciton binding energy is given as $E_b = E_{gs}^0 - E_{ex}$ where E_{gs}^0 is the ground-state energy associated to the non correlated e-h pair which depends on the confining heterostructure potentials, applied electric and magnetic fields and $k_y = \frac{p_y}{\hbar}$ (de Dios-Leyva et al., 2007). The E_{gs}^0 obtained from Equation 3 with the parameter $\alpha, \eta = 0$ through a minimization procedure with respect to z_e^0 or z_h^0 for a given value of $\Delta = z_e^0 - z_h^0$. By using corresponding $f_{gs} = (z_e - z_h)$ ground-state eigenfunction of H_0 . We now define

$$h(z_e, z_h) = f_{gs}^2(z_e, z_h) \quad (7)$$

$$g(z) = \int_{-\infty}^{\infty} h(z_e, z_h) dR \quad (8)$$

$z = z_e - z_h$ is the corresponding e-h distance in the growth direction, and straightforwardly obtains the binding energy through a maximization variational procedure as follows:

$$E_b(\lambda) = -\frac{\hbar^2 \lambda^2}{2\mu} + \frac{e^2 \int \frac{g f^2}{r} d^3 r}{\int g f^2 d^3 r} \quad (9)$$

Here we note that, for bulk GaAs, there are no confining potentials, one obtains

$$E_0 = \frac{p_x^2}{2M} + \frac{p_x^2}{2\mu} - k_y l_B (eEl_B) - \frac{(eEl_B)^2}{2\beta_0} + \frac{\hbar\omega_{exc}}{2} \quad (10)$$

$$F_{gs} = \frac{1}{\sqrt{\pi l_B}} \exp - \left[\frac{(z_e - z_e^0)^2}{2l_B^2} \right] \times \frac{1}{\sqrt{\pi l_B}} \exp - \left[\frac{(z_h - z_h^0)^2}{2l_B^2} \right] \quad (11)$$

And therefore Equation 11 is reduced into

$$g(z) = \frac{1}{\sqrt{2\pi l_B}} \exp \left[-\frac{(z - \Delta)^2}{2l_B^2} \right] \quad (12)$$

Excitons are often considered to be a solid-state quasi-particle analogue to the hydrogen atom (Hayashi and Katsuki, 1952). Taking as of hydrogen like ion model that we could calculate ion in its ground state $|1s\rangle$ help us to approximate the position at which the overlap integral electron and hole can occurred as follows:

$$a = He^+(1s|1s)H = \int_0^{\infty} \frac{2}{a^2} \exp -\frac{r}{a} \cdot \frac{2}{a^2} \exp -\frac{2r}{a} dr \int Y_{00}^2 d\tau = \frac{12\sqrt{2}}{27} \quad (13)$$

According to the probability of finding He^+ in ground state is $W(1s) = |a|^2 \approx 0.702$

In the case of a semiconductor $GaAs/Ga_{1-x}Al_xAs$ heterostructure grown along the z axis, the determination of the eigenfunctions and eigenvalues of Equation 3 involves a simple characteristic problem for the electron or hole, associated with the Hamiltonian which may be readily solved via an expansion in terms of sine or harmonic-oscillator functions. Finally, in the quantitative study of the optical properties of excitons in semiconductor heterostructures, it is necessary to evaluate the matrix elements for the interband optical transitions, which by neglecting the photon wave vector are proportional to the Equation 13 in the model of hydrogen like ion.

RESULTS AND DISCUSSION

The result obtained from Equation 3 when $k_y = 0$ stand for perpendicular momentum as depicted in Figure 1. $\Delta^* = (0-30)$ arbitrary units effective spatial separation versus applied electric field ($E=0-60$ KV/cm) for different values of applied magnetic field ($B_1 = 2T, B_2 = 4T, B_3 = 6T, B_4 = 8T, B_5 = 12T$ and $B_6 = 20T$). For low value of external applied magnetic field the slope of external applied electric field is slightly proportional to spatial distance (e-h). As the value of applied magnetic field B increased the increment of spatial distance (e-h) affected even external applied electric field kept increasing it indicate that applied magnetic field has decisive role on the importance of another parameters in the study of the exciton states in $GaAs/Ga_{1-x}Al_xAs$ semiconductor heterostructures under growth-direction applied electric E and in-plane magnetic B fields, the relation between Δ, E and B contained in Equation 3, for $k_y = 0$. It follows that, the spatial distance between the centers of the electron and hole magnetic parabolas, for a fixed value of the in plane magnetic field is an increasing linearly $\Delta \sim E$ function of the growth-direction applied electric field. This is the expected result as the applied electric field tends to

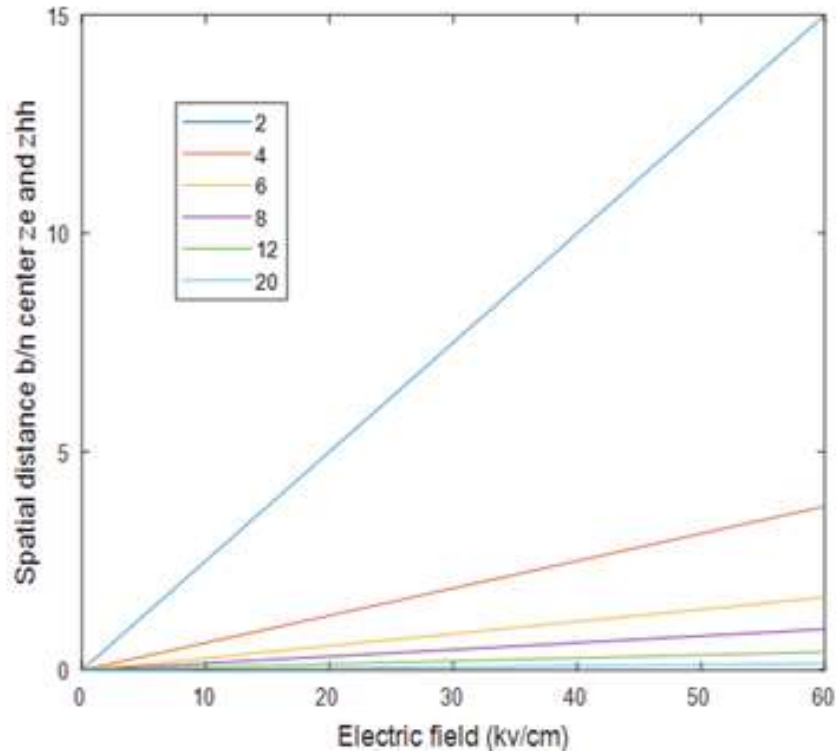


Figure 1. Results obtained from Eq.5 for $k_y = 0$, by using the bulk GaAs intrinsic parameters $\frac{\Delta}{a^*} = \Delta^*$ as a function of the growth-direction applied electric field, for various crossed-direction in-plane applied magnetic fields ($B_1 = 2T, B_2 = 4T, B_3 = 6T, B_4 = 8T, B_5 = 12, B_6 = 20T$)

spatially separate or polarize the e-h pair. As E increases the system changed from direct band gap to indirect band gap material even it goes to state transition such as semiconductor to insulator increasing the band gap between valence band to conduction band.

As it is depicted in Figure 2 for fixed different values of applied electric field (E) when the external applied magnetic field (B) increased the spatial distance e-h pair got reduced or the polarization of the e-h decreased. Thus the excitonic optical properties of the system completely change due to state to state transitions of the system that is, it behaved as free charge carrier in which excitonic properties quenched. For this, a precise knowledge of the exciton requires certain space level between electron-hole in such increasing magnetic field is responsible for increasing e-h wave-function overlaps. The binding energies are more sensitive to the magnetic field than the electric field. It should be noted that $\Delta \rightarrow 0$ when $B \rightarrow \infty$ for each value of $E \neq 0$. This overlap can be tuned by an external electric field, a magnetic field.

Thus this physical phenomenon entails transitions of materials from semiconductors to conductors that electron and hole may act as one free carrier. As shown in Figure 3 the applied electric field and the applied magnetic fields are a parabolic function to each other for

each fixed value of the effective spatial distance ($\Delta^* = \frac{\Delta}{a^*}$) between the electron and heavy-hole exciton Bohr radius (a^*) in case it depends on intrinsic material parameter. For the lower value of effective spatial distance of e-h that we observe relatively more parabolic function because the Landau magnetic length (or cyclotron radius) influence the system that the binding energy is highest in this regime. Additionally, for such field strengths the excitonic states, that is mostly determined by the inner Coulomb potential. While at larger value of effective spatial distance of e-h (Δ^*) the function relatively approaches to linear in which both parameters (applied electric field and magnetic fields) this implies both parameters affect the system equally at low the binding energy (E_b) regime. The externally applied electric field E is related to the spatial Δ separation between the electron and hole externally applied magnetic parabolas by the relation $E = \frac{\beta_0 \Delta^*}{e l_B^2} \sim B^2 \Delta^*$

Figure 4 the critical position at which e-h overlaps approximated as eigenvalue of ground wave function of single exciton 1 s hydrogen like ion variational approximation is about 0.702 arbitrary units. At low applied electric field (E) the magnetic field kept relatively constant up to overlap position and as applied electric

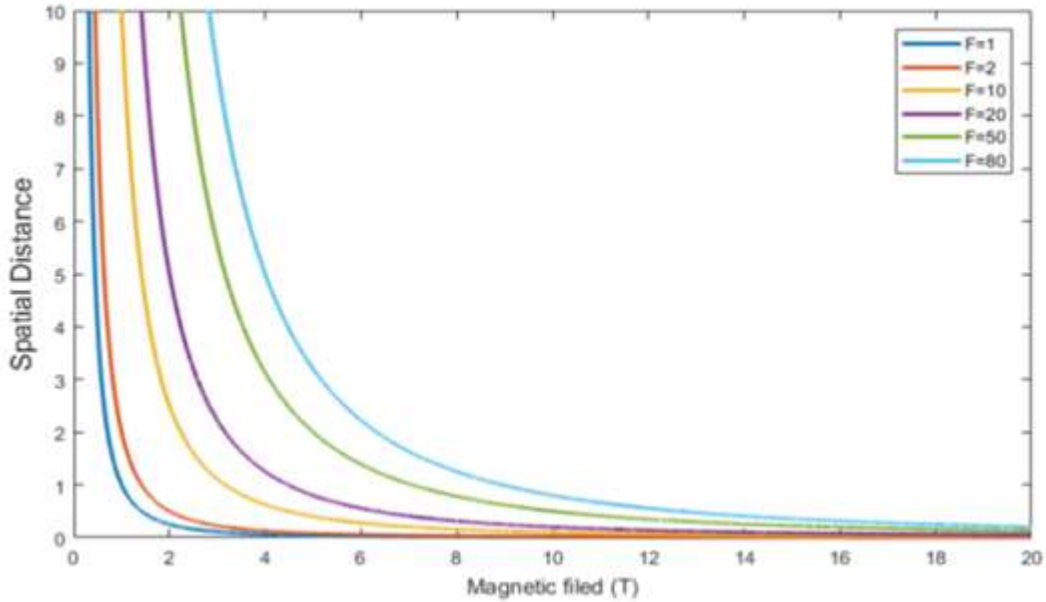


Figure 2. Magnetic-field dependence of the spatial distance between the centers of the electron and heavy-hole magnetic parabolas, for various values of the crossed-direction growth-direction applied electric field. The distance is given in units of the $a^*=118 \text{ \AA}$ heavy-hole exciton Bohr radius $E_1 = 1 \text{ KV/cm}$, $E_2 = 2 \text{ KV/cm}$, $E_3 = 10 \text{ T}$, $E_4 = 20 \text{ KV/cm}$, $E_5 = 50 \text{ KV/cm}$, $E_6 = 80 \text{ KV/cm}$

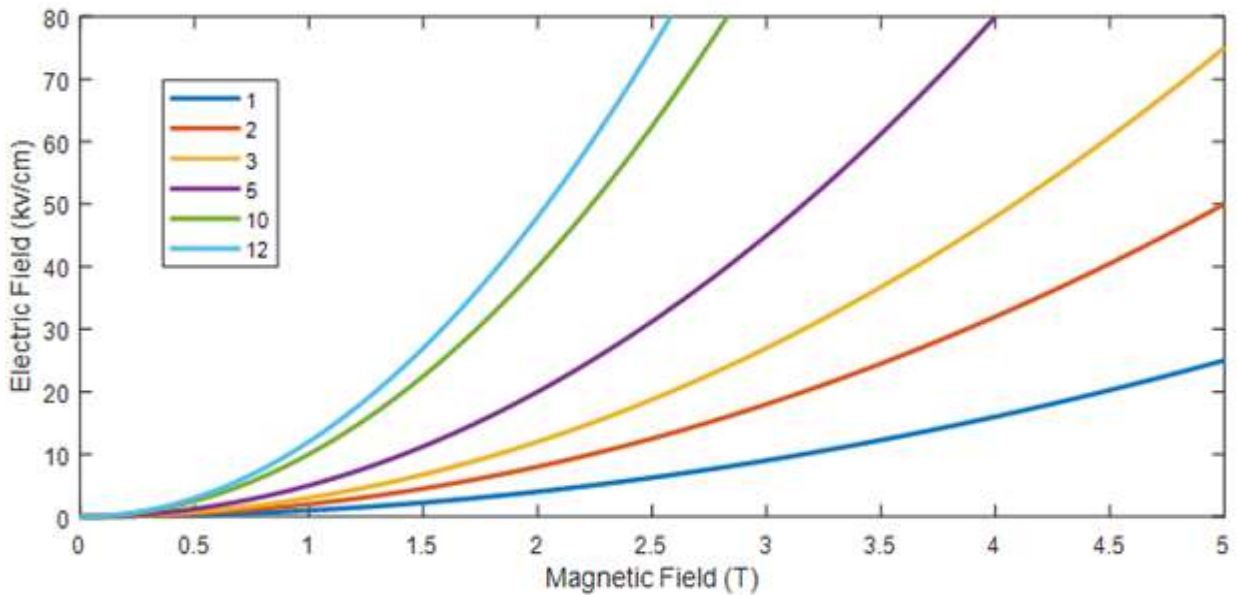


Figure 3. Magnetic-field dependence of the crossed-direction growth-direction applied electric field for fixed values of the parameter ($\Delta_1^* = 1, \Delta_2^* = 2, \Delta_3^* = 3, \Delta_4^* = 5, \Delta_5^* = 10$ and $\Delta_6^* = 12$)

increases the critical position reached slowly and at this critical point the applied electric field got quenched ($E \rightarrow 0$). While applied magnetic field increase goes to infinity ($B \rightarrow \infty$) at the critical overlap position for each values of

applied external electric field E as crossed electric and magnetic fields. This conclusion is, of course, in agreement with de Dios-Leyva et al. (2007) the analytical expression for the e-h overlap integral, which may be

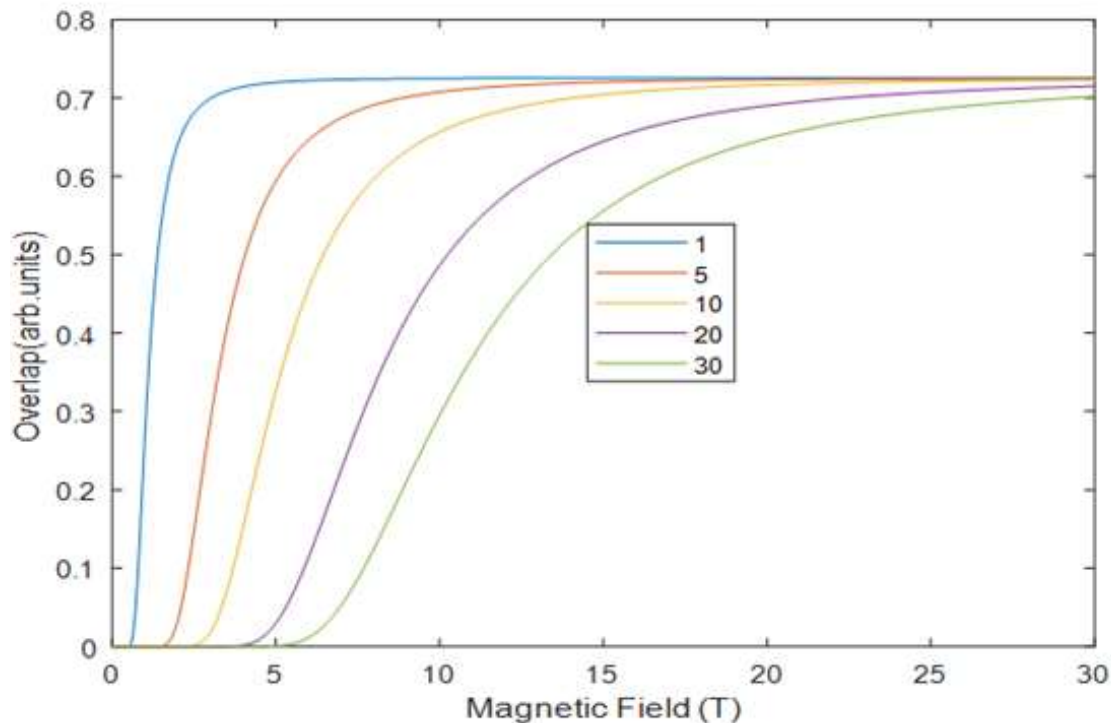


Figure 4. Electron –hole overlap integral $I_{overlap}$ arbitrary .units with respect to magnetic field, for bulk GaAs, under crossed electric and magnetic fields, as functions correspond to $(E_1 = 1 \text{ KV/cm}, E_2 = 5 \text{ KV/cm}, E_3 = 10 \text{ KV/cm}, E_4 = 20 \text{ KV/Cm}, E_5 = 30 \text{ KV/cm})$

obtained from corresponding Equation 8, 9, and 14.

$$I_{overlap} \propto \exp \frac{-\Delta^2}{4l_B^2} \quad (14)$$

Conclusion

Exciton is the only physical feature observed semiconductor physics. It is physical properties of semiconductors and could be affected dependence of some parameters like the external electric and magnetic fields that used to probe interface of heterostructures quantum well and quantum dot of semiconductors nanostructures. In this paper we tried to reviewed the effect of external applied magnetic field in plane to growth direction and applied external electric field along growth direction interacting with electron-hole coupling which has direct influences on the spatial position of electron and hole (z_e, z_h) entail with different physical properties. As such increasing external magnetic field in plane to the growth direction decrease the spatial distance between electron and hole i.e. it enhance with its coulomb correlation energy as well the binding energy got increased this effect has vital role on material systems have attracted much attention for their applications in

optoelectronic devices for that matter it requires more intensive study. Furthermore increasing external electric field increases the spatial distance between hole and electron. As result the system might be changed from direct band to indirect band semiconductors and also it could entail phase transition from semiconductors to insulators or conductors depend on external parameters that result with different the electronic and optical properties of the system. This is an exciting physical phenomenon that has been the interesting subject of investigation with the recent progress in semiconductor nanotechnology and its fundamental importance in semiconductor physics.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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