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Full Length Research Paper

Some iterative algorithms for trifunction equilibrium variational inequalities

Muhammad Aslam Noor[1,2], Khalida Inayat Noor[1] and Eisa Al-Said[2]

[1]Department of Mathematics, COMSATS Institute of Information Technology, Park Road, Islamabad, Pakistan.
[2]Department of Mathematics, College of Science, King Saud University, Riyadh, Saudi Arabia.

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In this paper, we use the auxiliary principle technique to suggest and analyze some iterative methods for solving a new class of equilibrium problems and variational inequalities, which is called the trifunction equilibrium variational inequality. Convergence of these iterative methods is proved under very mild and suitable assumptions. Several special cases are also considered. Results proved in this paper continue to hold for these known and new classes of equilibrium and variational inequalities.

Key words: Mixed variational inequality, equilibrium problems, convergence, auxiliary principle technique.

INTRODUCTION

In recent years, variational inequalities have appeared as an interesting and dynamic filed of pure and applied sciences. Variational techniques are being used to study a wide class of problem with applications in industry, structural engineering, mathematical finance, economics, optimization, transportation and optimization problems. This has motivated us to introduce and study several classes of variational inequalities. Blum and Oettili (1994) introduced the equilibrium problems and shows that the equilibrium problems include the variational inequalities, fixed problem and optimization problems as special cases. Motivated by the work of Blum and Oettili (1994), Noor et al. (1994) considered the equilibrium problems involving the trifunction, called the trifunction equilibrium problems which include the equilibrium problems as special cases. For the formulation, applications and numerical methods of equilibrium problems (Blum and Oettili, 1994; Noor, 1975, 2003, 2004, 2004a, 2004b, 2004c; Noor et al., 1993, 1994, 2004, 2004a, 2008) .

We would like to mention that variational inequalities and trifunction equilibrium problems are quite different problems. It is natural to consider the unification of these problems. Inspired and motivated by the research going on in these different directions, we introduce and consider a new class of variational inequalities and trifunction equilibrium problems, which is called the trifunction equilibrium variational inequality. This new class includes the variational inequalities and trifunction equilibrium problems as special cases.

There are a substantial number of numerical methods for solving the variational inequalities and trifunction equilibrium problems. Due to the nature of the trifunction variational inequality problem, projection methods its variant form, such as Wiener-Hopf equations cannot be used for solving the trifunction equilibrium variational inequality. This fact motivated us to use the auxiliary principle technique of Glowinski et al. (1981) as developed by Noor (2004, 2004a, 2004b) and Noor et al (1993, 2004, 2004a, 2008). This technique is quite flexible and a general one. We again use this technique to suggest some explicit and proximal-point iterative methods for these problems. We also consider the convergence criteria of the proposed methods under suitable mild conditions, which are the main results (Theorem 1, Theorem 2 and Theorem 3) of this paper. Several special cases of our main results are also considered. Results obtained in this paper may be viewed as an improvement and refinement of the previously known results. Recent results in this direction is found in Noor et al. (2011, 2011a, 2011b, 2011c). The ideas and techniques of this paper stimulate further

*Corresponding author. E-mail: mnoor.c@ksu.edu.sa or noormaslam@hotmail.com.

research in this area of pure and applied sciences.

PRELIMINARIES

Let *H be* a real Hilbert space, whose inner product and norm are denoted by $\langle .,. \rangle$ and $\|.\|$, respectively. Let K be a closed and convex set in H.

For a given trifunction $F(.,.,.)$: $\mathrm{H}\times\mathrm{H}\times\mathrm{H}\rightarrow R$ and an operator T: $\mathrm{H}\rightarrow H$, we consider the problem of finding $u\in K$ such that:

$$F(u,Tu,v)+\langle Tu,v-u\rangle\geq 0,\ \forall v\in K\ , \quad (1)$$

which is called the trifunction equilibrium variational inequality. We note that, if $F(.,.,.)\cong F(.,.),$ then the problem of Equation 1 is studied (Takhashi and Takhashi, 2008; Yao et al., 2009, 2011; Noor et al., 2011d, 2011h).
We now discuss some important special cases of the problem of Equation 1.

Special cases

1. If $F(u,Tu,v)=0$, then the problem of Equation 1 is equivalent to finding $u\in K$ such that:

$$\langle Tu,v-u\rangle\geq 0,\ \forall v\in K, \quad (2)$$

which is known as a variational inequality**,** introduced and studied (Stampacchia, 1964). A wide class of problems arising in elasticity, fluid flow through porous media and optimization can be studied in the general framework of the problems of Equation 1. For the applications, formulation of numerical results and other aspects of the variational inequalities and their generalizations (Giannessi et al., 1995, 2001; Noor, 1975, 2003, 2004, 2004a, 2004b, 2004c; Noor et al., 1993, 1994, 2004, 2004a, 2008; Yao et al., 2009, 2011).
2. If $\langle Tu,v-u\rangle=0$, then the problem of Equation 1 turns into the problem of finding $u\in K$ such that:

$$F(u,Tu,v)\geq 0,\ \forall v\in K, \quad (3)$$

which is known as trifunction equilibrium problems considered (Noor, 2004, 2004a, 2004b). For suitable and appropriate choice of the operator and spaces, one can obtain several new and known problems as special cases of the mixed equilibrium variational inequalities problems of Equation 1. The applications, formulations, numerical methods and other aspects of the equilibrium problems and variational inequalities were reported by Giannessi et al. (1995, 2001), Noor (1975, 2003, 2004, a, b, c) Noor et al., 1993, 1994, 2004, 2004a, 2008), and Yao et al. (2009, 2011).

Definition 1

An operator $T:H\rightarrow H$ is said to be:

1. Monotone, if and only if, $\langle Tu-Tv,u-v\rangle\geq 0,\quad \forall u,v\in H.$
2. Partially relaxed strongly monotone, if there exists a constant $\alpha>0$ such that

$$\langle Tu-Tv,z-v\rangle\geq -\alpha\|z-u\|^2,\quad \forall u,v,z\in H\ .$$

For $z=u$, partially relaxed strong monotonicity reduces to monotonicity of the operator *T*.

Definition 2

A trifunction $F(.,.,.):H\times H\times H\rightarrow R$ with respect to an operator T is said to be:

1. Jointly monotone, if and only if, $F(u,Tu,v)+F(v,Tv,u)\leq 0,\quad \forall u,v\in H.$
2. Partially relaxed strongly jointly monotone, if and only if, there exists a constant $\alpha>0$ such that

$$F(u,Tu,v)+F(v,Tv,z)\leq \mu\|z-u\|^2,\qquad \forall u,v,z\in H\ .$$

It is clear that for $z=u,$ partially relaxed strongly jointly monotone trifunction is simply jointly monotone.

MAIN RESULTS

Here, we suggest and analyze an iterative method for solving the mixed equilibrium variational-inequality problem of Equation 1 by using the auxiliary principle technique. This technique is mainly due to Glowinski et al. (1981) as developed by Noor (2004, 2004a, 2004b) and Noor et al. (2011, 2011a, 2011b, 2011c) in their work.

For a given $u\in K$, consider the problem of finding a $w\in K$, such that:

$$\rho F(u,Tu,v)+\langle \rho Tu,v-w\rangle+\langle w-u,v-w\rangle\geq 0,\quad \forall v\in K, \quad (7)$$

where $\rho>0$ is a constant.

If $w = u$, then $w \in H$ is a solution of Equation 1. This observation enables us to suggest and analyze the following iterative method for solving trifunction equilibrium variational inequality of Equation 1.

Algorithm 1

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\rho F(u_n, Tu_n, v) + \langle \rho Tu_n, v - u_{n+1} \rangle + \langle u_{n+1} - u_n, v - u_{n+1} \rangle \geq 0, \quad \forall v \in K, \quad (8)$$

We now discuss some special cases.

If $F(u, Tu, v) = 0$, then Algorithm 1 reduces to the following scheme for variational inequalities of Equation 2.

Algorithm 2

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\langle \rho Tu_n, v - u_{n+1} \rangle + \langle u_{n+1} - u_n, v - u_{n+1} \rangle \geq 0, \quad \forall v \in K.$$

If $\langle Tu, v - u \rangle = 0$, then Algorithm 1 reduces to Algorithm 3.

Algorithm 3

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$F(u_n, Tu_n, v) + \langle u_{n+1} - u_n, v - u_{n+1} \rangle \geq 0, \quad \forall v \in K,$$

which is used for finding the solution of mixed equilibrium problems of Equation 3.

For suitable and appropriate choice of $F(.,.), T, \varphi(.)$ and spaces, one can define iterative algorithms to find the solutions to different classes of equilibrium problems and variational inequalities.

We now study the convergence analysis of Algorithm 1 using the technique of Noor et al (2011), and this is the main motivation of our next result.

Theorem 1

Let $u \in H$ be a solution of Equation 1 and $u_{n+1} \in H$ be an approximate solution obtained from Algorithm 1. If the trifunction $F(.,.,.)$ and the operator $T(.)$ are partially relaxed, strongly monotone operators with constants $\mu > 0$ and $\sigma > 0$, respectively, then:

$$\|u - u_{n+1}\|^2 \leq \|u - u_n\|^2 - (1 - 2\rho(\mu + \sigma))\|u_{n+1} - u_n\|^2 \quad (9)$$

Proof

Let $u \in K$ be a solution of Equation 1. Then, replacing v by u_{n+1} in Equation 1, we have:

$$\rho F(u, Tu, u_{n+1}) + \rho \langle Tu, u_{n+1} - u \rangle \geq 0, \quad \rho > 0. \quad (10)$$

Let $u_{n+1} \in K$ be the approximate solution obtained from Algorithm 1. Taking $v = u$ in equation 8, we have:

$$\rho F(u_n, Tu_n, u) + \langle \rho Tu_n, u - u_{n+1} \rangle + \langle u_{n+1} - u_n, u - u_{n+1} \rangle \geq 0. \quad (11)$$

Adding Equations 10 and 11, we have:

$$\rho[F(u, Tu, u_{n+1}) + F(u_n, Tu_n, u)] + \rho\langle Tu - {}_n, u_{n+1} - u \rangle + \langle u_{n+1} - u_n, u - u_{n+1} \rangle \geq 0,$$

which implies that

$$\langle u_{n+1} - u_n, u - u_{n+1} \rangle \geq -\rho[F(u_n, Ru_n, u) +$$
$$\geq -\rho(\mu + \sigma)\|u_{n+1} - u_n\|$$
$$+ F(u, Tu, u_{n+1})] + \rho\langle Tu_n - Tu, u_{n+1} - u \rangle$$
$$\|^2 \quad (12)$$

where we have used partially relaxed strong monotonicity of the trifunction $F(.,.,.)$ and operator T.

Using the relation $2\langle u, v \rangle = \|u + v\|^2 - \|u\|^2 - \|v\|^2, \quad \forall u, v \in H$, and from Equation 12, one can have:

$$\|u - u_{n+1}\|^2 \leq \|u - u_n\|^2 - (1 - 2\rho(\mu + \sigma))\|u_{n+1} - u_n\|^2$$

,

which is the required result of Equation 9.

Theorem 2

Let H be a finite dimensional space. If u_{n+1} is the approximate solution obtained from Algorithm 1 and $u \in K$ be a solution to the problem of Equation 1. Then $\lim_{n\to\infty} u_n = u$.

Proof

Let $u \in H$ be a solution of Definition 1. For $0 < \rho < \frac{1}{2(\mu+\sigma)}$, we see that the sequence $\{\|u - u_n\|\}$ is non increasing and consequently $\{u_n\}$ is bounded. Also from Equation 1, we have:

$$\sum_{n=0}^{\infty} (1 - 2(\mu+\sigma)) \|u_{n+1} - u_n\|^2 \leq \|u - u_0\|^2 ,$$

which implies that

$$\lim_{n\to\infty} \|u_{n+1} - u_n\| = 0. \quad (12)$$

Let $\hat{u}$ be the cluster point of $\{u_n\}$ and the subsequence $\{u_{n_j}\}$ of this sequence converges to $\hat{u} \in H$. Replacing u_n by u_{n_j} in Equation 8 and taking the limit as $n_j \to \infty$ and using Equation 12, we have:

$$F(\hat{u}, T\hat{u}, v) + \langle T\hat{u}, v - \hat{u} \rangle \geq 0, \qquad \forall v \in K ,$$

which shows $\hat{u}$ solves the mixed equilibrium-variational inequality of Equation 1 and $\|u_{n+1} - \hat{u}\|^2 \leq \|u_n - \hat{u}\|^2$. Thus, it follows from the aforementioned inequality that the sequence $\{u_n\}$ has exactly one cluster point and $\lim_{n\to\infty} u_n = \hat{u}$, the required result.

We again use the auxiliary principle technique to suggest and analyze several proximal point algorithms for solving the mixed equilibrium-variational inequalities of Equation 1 and this is another motivation of this paper.

For a given $u \in K$, consider the problem of finding $w \in K$, such that:

$$\rho F(w, Tw, v) + \langle \rho Tw, v - w \rangle + \langle w - u - \gamma(u - u), v - w \rangle \geq 0, \ \forall v \in K, \quad (14)$$

where $\rho \geq 0$ and $\gamma \geq 0$ are constants.

If $w = u$, then $w \in K$ is a solution of Equation 1. This observation enables us to suggest and analyze the following iterative method for solving trifunction equilibrium variational inequalities of Equation 1.

Algorithm 4

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\rho F(u_{n+1}, Tu_{n+1}, v) + \langle \rho Tu_{n+1}, v - u_{n+1} \rangle + \langle u_{n+1} - u_n - \gamma(u_n - u_{n-1}), v - u_{n+1} \rangle \geq 0, \quad \forall v \in K. \quad (15)$$

Algorithm 4 is called the inertial proximal point method for solving the trifunction equilibrium variational inequality Equation 1.

Note that, if $\gamma = 0$, then Algorithm 1 reduces to the following proximal point method for solving Equation 1.

Algorithm 5

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\rho F(u_{n+1}, Tu_{n+1}, v) + \langle \rho Tu_{n+1}, v - u_{n+1} \rangle + \langle u_{n+1} - u_n, v - u_{n+1} \rangle \geq 0, \quad \forall v \in K.$$

For a given $u \in K$, consider the problem of finding a $w \in K$ such that:

$$\rho F(u, Tu, v) + \langle \rho Tw, v - w \rangle + \langle w - u - \gamma(u - u), v - w \rangle \geq 0, \ \forall v \in K.$$

Note that if $w = u$, then $w \in K$ is a solution of Equation 1. This observation enables us to suggest and analyze the following proximal iterative method for solving trifunction equilibrium variational inequalities of Equation 1.

Algorithm 6

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\rho F(u_n, Tu_n, v) + \langle \rho Tu_{n+1}, v - u_{n+1} \rangle + \langle u_{n+1} - u_n \gamma(u_n - u_{n-1}), v - u_{n+1} \rangle \geq 0, \quad \forall v \in K.$$

For a given $u \in K$, consider the problem of finding a $w \in K$, such that:

$$\rho F(w,Tw,v)+\langle \rho Tu, v-w\rangle + \langle w-u - \gamma(u-u), v-w\rangle \geq 0, \ \forall v \in K.$$

If $w=u$, then $w \in K$ is a solution of Equation 1. This observation enables us to suggest and analyze the following iterative method for solving mixed equilibrium-variational inequalities of Equation 1.

Algorithm 7

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\rho F(u_{n+1},Tu_{n+1},v)+\langle \rho Tu_n, v-u_{n+1}\rangle + \langle u_{n+1}-u_n-\gamma(u_n-u_{n-1}), v-u_{n+1}\rangle \geq 0, \ \forall v \in K.$$

Some special cases of these algorithms are as follow:

If $F(u,Tu,v)=0$, then Algorithm 4 reduces to the following scheme for mixed variational inequalities given as Equation 2.

Algorithm 8

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$\langle \rho Tu_{n+1}, v-u_{n+1}\rangle + \langle u_{n+1}-u_n-\gamma(u_n-u_{n-1}), v-u_{n+1}\rangle \geq 0, \ \forall v \in K.$$

If $\langle Tu, v-u\rangle = 0$, then Algorithm 1 reduces to Algorithm 9.

Algorithm 9

For a given $u_0 \in H$, compute $u_{n+1} \in H$ from the iterative scheme:

$$F(u_{n+1},Tu_{n+1},v)+\langle u_{n+1}-u_n-\gamma(u_n-u_{n-1}), v-u_{n+1}\rangle \geq 0, \ \forall v \in K,$$

which is used for finding the solution of trifunction equilibrium problem of Equation 3.

For suitable and appropriate choice of $F(.,.,.), T$ and spaces, one can define iterative algorithms as special cases of Algorithm 5 and Algorithm 6 to find the solutions to different classes of equilibrium problems and variational inequalities.

We would like to mention that one can study the convergence analysis of Algorithm 5 using the technique of Theorem 1 and Theorem 2. However, for the sake of completeness and to convey the main ideas, we include the main steps of the proof.

Theorem 3

Let $u \in H$ be a solution of Equation 1 and $u_{n+1} \in H$ be an approximate solution obtained from Algorithm 5. If trifunction $F(.,.,.)$ and operator T are monotone, then

$$\|u-u_{n+1}\|^2 \leq \|u-u_n\|^2 - \|u_{n+1}-u_n\|^2. \quad (16)$$

Proof

Let $u \in K$ be a solution of Equation 1. Then, replacing v by u_{n+1} in Equation 1, we have:

$$\rho F(u,u_{n+1}) + \rho\langle Tu, u_{n+1}-u\rangle \geq 0. \quad (17)$$

Let $u_{n+1} \in H$ be the approximate solution obtained from Algorithm 4. Taking $v=u$ in Equation 15, we have:

$$\rho F(u_{n+1},Tu_{n+1},u)+\langle \rho Tu_{n+1}, u-u_{n+1}\rangle + \langle u_{n+1}-u_n, u-u_{n+1}\rangle \geq 0. \quad (18)$$

Adding Equations 17 and 18, we have:

$$\rho[F(u,Tu,u_{n+1})+F(u_n,Tu_n,u)]+\rho\langle Tu-Tu_{n+1}, u_{n+1}-u\rangle + \langle u_{n+1}-u_n, u-u_{n+1}\rangle \geq 0,$$

which implies that

$$\langle u_{n+1}-u_n, u-u_{n+1}\rangle \geq -\rho[F(u_n,Tu_n,u)+F(u,Tu,u_{n+1})] + \rho\langle Tu_{n+1}-Tu, u_{n+1}-u\rangle \geq 0, \quad (19)$$

where we have used the monotonicity of the operator T and the trifunction $F(.,.,.)$.

Using the relation $2\langle u,v\rangle = \|u+v\|^2 - \|u\|^2 - \|v\|^2, \quad \forall u,v \in H$ and from Equation 19, we have:

$\|u-u_{n+1}\|^2 \le \|u-u_n\|^2 - \|u_{n+1}-u_n\|^2$, which is the required result (16).

Theorem 4

Let H be a finite dimensional space. If u_{n+1} is the approximate solution obtained from Algorithm 5 and $u \in K$ is a solution of the problem Equation 1, then $\lim_{n\to\infty} u_n = u$.

Proof

Let $u \in H$ be a solution of Definition 1. Then, we see that the sequence $\{\|u-u_n\|\}$ is non increasing and consequently $\{u_n\}$ is bounded. Also from Equation 16, we have:

$$\sum_{n=0}^{\infty} \|u_{n+1} - u_n\|^2 \le \|u - u_0\|^2 ,$$

which implies that

$$\lim_{n\to\infty} \|u_{n+1} - u_n\| = 0. \tag{20}$$

Let $\hat{u}$ be the cluster point of $\{u_n\}$ and the sub sequence $\{u_{n_j}\}$ of this sequence converges to $\hat{u} \in H$. Replacing u_n by u_{n_j} in Equation 15 and taking the limit $n_j \to \infty$ and using Equation 20, we have:

$$F(\hat{u}, T\hat{u}, v) + \langle T\hat{u}, v - \hat{u} \rangle \ge 0, \qquad \forall v \in K ,$$

which shows $\hat{u}$ solves the mixed equilibrium-variational inequality of Equation 1 and $\|u_{n+1} - \hat{u}\|^2 \le \|u_n - \hat{u}\|^2$.

Thus, it follows from the aforementioned inequality that the sequence $\{u_n\}$ has exactly one cluster point and $\lim_{n\to\infty} u_n = \hat{u}$, the required result.

CONCLUSION

In this paper, we have used the auxiliary principle technique for suggesting and analyzing some explicit and inertial proximal point algorithms for solving the trifunction equilibrium variational inequality problem. We have also discussed the convergence criteria of the proposed new iterative methods under some suitable weaker conditions. In this sense, our results can be viewed as refinement and improvement of the previously known results. Note, this technique does not involve the projection and the resolvent technique. We have also shown that this technique can be used to suggest several iterative methods for solving various classes of equilibrium and variational inequalities problems. Results proved in this paper may inspire further research in this area.

Future directions and research

The problem considered in this paper may inspire further research. One can develop the sensitivity analysis for the problem of Equation 1, which is still an open problem. Secondly, one can also consider the stability analysis of the trifunction equilibrium variational inequalities in a general and unified framework. The problem of comparing the iterative methods for solving the trifunction equilibrium variational inequalities with other technique is an interesting problem for further research. One may be able to explore the applications of these problems in several directions. Using the penalty function technique, one can characterize the problem as a system of variational equalities. This alternative formulation has been used by Noor et al. (1993, 2010, 2011e, 2011f, 2011g) for solving the system of higher-order boundary value problems associated with variational inequalities using the variation of parameters method, variational iteration method and homotopy perturbation and similar methods for finding the approximate solution of the trifunction equilibrium variational inequalities Equation 1. This is still an open problem. It is expected that the technique and ideas of this paper may stimulate further research opportunities in this dynamic and fast growing field.

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