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Effects of heat absorption and chemical reaction on a three dimensional MHD convective flow past a porous plate

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An attempt has been made to investigate the effects of heat sink and chemical reaction on a three dimensional Magneto hydrodynamics (MHD) convective flow with mass transfer of an incompressible viscous electrically conducting fluid past a porous vertical plate with transverse sinusoidal suction velocity. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the main flow. The magnetic Reynolds number is considered to be small that induced magnetic field can be neglected. The governing equations are solved by regular perturbation technique. The expression for velocity field, temperature field, species concentration, current density, the skin friction, Nusselt number and Sherwood number at the plate are obtained in non dimensional forms. The effect of Hartman number, chemical reaction parameter, heat sink parameter on the velocity field, zeroth order skin friction and the amplitude of the first order skin friction, first order Nusselt number and the first order Sherwood number at the plate are discussed graphically. It is seen that chemical reaction and heat sink have significant effects on the flow and on the heat and mass transfer characteristics.

Key words: Three-dimensional convective flow, heat transfer, incompressible viscous fluid, wall shear stress, heat sink.

INTRODUCTION

The investigation of magneto hydrodynamics (MHD) convection with mass transfer problems in presence of transverse magnetic field have attracted the attention of a number of scholars because of its wide application in many branches of science and technology such as geophysics, astrophysics, plasma physics, missile technology, etc. Engineers employ MHD principles in the design of heat exchangers, pumps and flow meters, thermal protection, etc. From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. MHD is also stabilizing a flow against the transition from laminar to turbulent flow and in reduction of turbulent drag and suppression of flow separation. The application of MHD principles in medicine and biology are of paramount interest owing to their significance in biomedical engineering in general and in the treatment of various pathological states in particular. Applications in biomedical engineering include cardiac magnetic resonance imaging (MRI), electro cardio gram (ECG) etc. The principle of dynamo and motor is a classical example of MHD convection.

The problems of above phenomena of MHD convection have been studied by many authors. Ferraro and Plumpton (1966), Cramer and Pai (1973) and Sanyal and Bhattacharya (1992) are some of them. The problem of the convection flows arising in fluids as a result of

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interaction of the force of gravity and density difference caused by simultaneous diffusion of thermal energy and chemical species have been investigated by Bejan and Khair (1985), Raptis and Kafousias (1982) and Ahmed et al. (2005).

The effect of three dimensional flow caused by the periodic suction perpendicular to the main flow when the difference between the wall temperature and free steam temperature gives rise to buoyancy force in the direction of the free steam on heat transfer characteristic was investigated by Ahmed and Sarma (1997), Singh et al. (1998) and Choudhury and Chand (2002). Recently Jain and Gupta (2006) have investigated the effect of transverse sinusiodal injection velocity distribution on three dimensional free convective Couette flow of a various incompressible fluid in slip flow regime under the influence of heat sink. An analytical solution to the problem of the three dimensional free convective flow of an incompressible viscous fluid past a porous vertical plate with transverse sinusoidal suction velocity taking into account the presence of species concentration was obtained by Ahmed et al. (2006).

In many times it has been observed that foreign mass reacts with the fluid and in such a situation chemical reaction plays an important role in chemical industry. Theoretical descriptions of non-linear chemical dynamics have been presented by Epstein and Pojman (1998) and Gray and Scott (1990). The effects of chemical reaction and mass transfer on MHD flow past a semi-infinite plate was analysed by Devi and Kandasamy (2000). The effects of mass transfer, Soret effect and chemical reaction on an oscillatory MHD free convective flow through a porous medium have been investigated by Ahmed and Kalita (2010).

In view of the importance of the combined effect of chemical reaction and heat absorption, it is proposed to study a problem of three dimensional MHD convective flows past a porous vertical infinite plate with chemical reaction and heat absorption. The infinite plate assumption is one such classical idealization of great practical importance. Although the flow over a flat plate is the simplest case of boundary layer development in external flow, yet its significance cannot be undervalued because of its relevance to numerous engineering applications. Several configurations such as flow over airfoils, turbine blades, ship hulls, etc. can initially be estimated as flow past flat plates (Scheme 1). The justification of considering the three dimensional flow is that most of the fluid flows that occur in nature are three dimensional. Of course we have chosen a simple model of a three dimensional flow caused by transverse sinusoidal suction velocity.

The objective of the present work is to investigate the effect of chemical reaction as well as heat sink on a three dimensional convective flow past a porous plate. Our work is a generalization to the work done by Ahmed and Sarma (2010).

BASIC EQUATIONS

The equations governing the steady motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field are:

The equation of continuity: div $\vec{q} = 0$ (1)

The Gauss's law of magnetism: div $\vec{B} = 0$ (2)

The momentum equation:

$$\left(\vec{q}.\vec{\nabla}\right)\vec{q} = -\frac{1}{\rho}\vec{\nabla}p + \frac{\vec{J}\times\vec{B}}{\rho} + \nu\nabla^2\vec{q} + \vec{g} \quad (3)$$

The energy equation:

$$\rho C_p\left[\left(\vec{q}.\vec{\nabla}\right)\bar{T}\right] = k\nabla^2\bar{T} + \varphi + \frac{\vec{J}^2}{\sigma} + Q_0\left(\bar{T}_\infty - \bar{T}\right) \quad (4)$$

The species continuity equation:

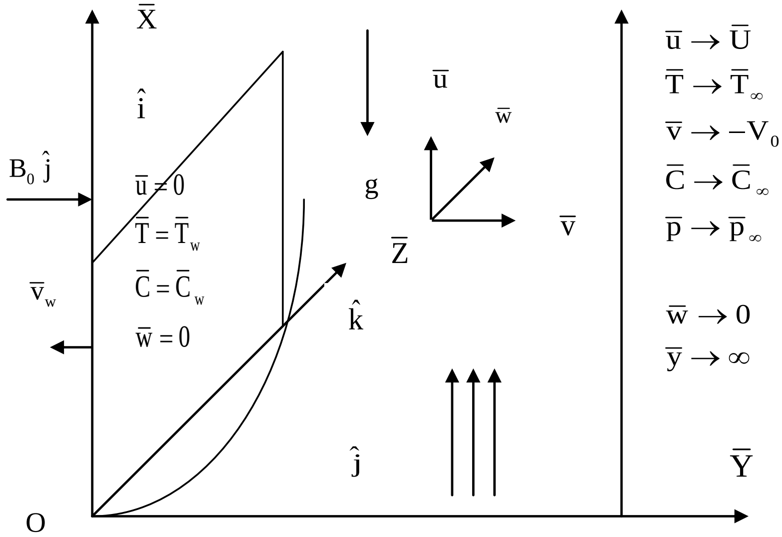
$$\left[\left(\vec{q}.\vec{\nabla}\right)\bar{C}\right] = D_M\nabla^2\bar{C} + D_T\nabla^2\bar{T} + \bar{K}\left(\bar{C}_\infty - \bar{C}\right) \quad (5)$$

The Ohm's law: $\vec{J} = \sigma\left[\vec{E} + \vec{q}\times\vec{B}\right]$ (6)

We now consider the steady convective flow of an incompressible viscous electrically conducting fluid in presence of heat sink taking into account the species concentration and chemical reaction past a vertical porous plate with transverse sinusoidal suction velocity as mentioned earlier by making the following assumptions:

(i) All the fluid properties except the density in the buoyancy term are constant.
(ii) A magnetic field of uniform strength B_0 is applied transversely to the direction of the main steam.
(iii) The magnetic Reynolds number is so small that the induced magnetic field can be neglected.
(iv) The viscous dissipation and magnetic dissipation energy are negligible.
(v) $\bar{T}_w > \bar{T}_\infty$ and $\bar{C}_w > \bar{C}_\infty$.

We introduce a co-ordinate system $\left(\bar{x}, \bar{y}, \bar{z}\right)$ with X-axis vertically upwards along the plate, Y-axis perpendicular to it and directed into the fluid region and Z-axis along the width of the plate. Let $\vec{q} = \bar{u}\,\hat{i} + \bar{v}\,\hat{j} + \bar{w}\,\hat{k}$ be the fluid velocity at the point $\left(\bar{x}, \bar{y}, \bar{z}\right)$ and $B_0\,\hat{j}$ be the applied magnetic field, $\hat{i}, \hat{j}, \hat{k}$ being the unit vectors along +ve X-axis, Y-axis and Z-axis respectively. The suction velocity is taken as follows:



Scheme 1. Flow configuration.

$$\bar{v}_w(\bar{z}) = -V_0\left[1+\varepsilon\cos\frac{\pi\bar{z}}{L}\right] \quad (7)$$

which consists of a basic steady distribution $-V_0$ with a superimposed weak distribution $-\varepsilon V_0 \cos\left(\frac{\pi\bar{z}}{L}\right)$. Since the plate is infinite in length in X-direction, therefore all the quantities except possibly the pressure are assumed to be independent of $\bar{x}$. With the foregoing assumptions and under usual boundary layer and Boussinesq approximation, Equations 1, 3, 4 and 5 are reduced to Equation of continuity:

$$\frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \quad (8)$$

Momentum equations:

$$\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} = g\beta\left(\bar{T}-\bar{T}_\infty\right) + g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right) + \nu\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right) + \frac{\sigma B_0^2}{\rho}\left(\bar{U}-\bar{u}\right) \quad (9)$$

$$\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{v}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}} + \nu\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right) \quad (10)$$

$$\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{z}} + \nu\left(\frac{\partial^2\bar{w}}{\partial\bar{y}^2}+\frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right) - \frac{\sigma B_0^2\,\bar{w}}{\rho} \quad (11)$$

Energy equation:

$$\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{T}}{\partial\bar{z}} = \alpha\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right) + \frac{Q_0\left(\bar{T}_\infty-\bar{T}\right)}{\rho C_p} \quad (12)$$

Species continuity equation:

$$\bar{v}\frac{\partial\bar{C}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{C}}{\partial\bar{z}} = D_M\left(\frac{\partial^2\bar{C}}{\partial\bar{y}^2}+\frac{\partial^2\bar{C}}{\partial\bar{z}^2}\right) + D_T\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right) + \bar{K}\left(\bar{C}_\infty-\bar{C}\right) \quad (13)$$

Equation 2 is satisfied by $\vec{B} = B_0\,\hat{j}$. The symbols are defined in the nomenclature. The relevant boundary conditions are:

at $\bar{y}=0$: $\bar{u}=0$, $\bar{v}=\bar{v}_w$, $\bar{w}=0$, $\bar{T}=\bar{T}_w$, $\bar{C}=\bar{C}_w$ (14)

at $\bar{y}\to\infty$: $\bar{u}=\bar{U}$, $\bar{v}=-V_0$, $\bar{w}=0$, $\bar{T}=\bar{T}_\infty$, $\bar{C}=\bar{C}_\infty$, $\bar{p}=\bar{p}_\infty$ (15)

We introduce the following non-dimensional quantities:

$$\left.\begin{aligned} & y=\frac{\bar{y}}{L},\ z=\frac{\bar{z}}{L},\ u=\frac{\bar{u}}{V_0},\ v=\frac{\bar{v}}{V_0},\ U=\frac{\bar{U}}{V_0},\ w=\frac{\bar{w}}{V_0},\ \theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ Q=\frac{Q_0 L}{\rho C_p V_0}, \\ & \phi=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},\ P_r=\frac{\nu}{\alpha},\ S_c=\frac{\nu}{D_M},\ S_r=\frac{D_T\left(\bar{T}_w-\bar{T}_\infty\right)}{\nu\left(\bar{C}_w-\bar{C}_\infty\right)},\ G_r=\frac{Lg\beta\left(\bar{T}_w-\bar{T}_\infty\right)}{V_0^2},\ K=\frac{\bar{K}L}{V_0} \\ & G_m=\frac{Lg\bar{\beta}\left(\bar{C}_w-\bar{C}_\infty\right)}{V_0^2},\ M=\frac{\sigma B_0^2\,\nu}{\rho V_0^2},\ R_e=\frac{V_0 L}{\nu},\ p=\frac{\bar{p}}{\rho\left(\frac{\nu}{L}\right)^2},\ p_\infty=\frac{\bar{p}_\infty}{\rho\left(\frac{\nu}{L}\right)^2} \end{aligned}\right\} \quad (16)$$

The non-dimensional forms of Equations 8, 9, 10, 11, 12 and 13

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (17)$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = G_r\,\theta + G_m\,\phi + \frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) + M R_e\left(U-u\right) \quad (18)$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{R_e^2}\frac{\partial p}{\partial y} + \frac{1}{R_e}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad (19)$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{R_e^2}\frac{\partial p}{\partial z}+\frac{1}{R_e}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-MR_e w \quad (20)$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{P_r R_e}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)-Q\theta \quad (21)$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{S_c R_e}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}\right)+\frac{S_r}{R_e}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right)-K\phi \quad (22)$$

with relevant boundary conditions:

$$y=0\ :\ u=0\,,\ \bar{v}=-(1+\varepsilon \mathrm{Cos}\,\pi z)\,,\ w=0\,,\ \theta=1\ ,\ \phi=1 \quad (23)$$

$$y\to\infty\ :\ u=U\,,\ \bar{v}=-1\,,\ \bar{w}=0\,,\ \theta=0\ ,\ \phi=0\ ,\ p=p_\infty \quad (24)$$

METHOD OF SOLUTION

We assume the solution of Equations 17 to 22 to be of the form:

$$u=u_0(y)+\varepsilon u_1(y,z)+0(\varepsilon^2) \quad (25)$$

$$v=v_0(y)+\varepsilon v_1(y,z)+0(\varepsilon^2) \quad (26)$$

$$w=w_0(y)+\varepsilon w_1(y,z)+0(\varepsilon^2) \quad (27)$$

$$p=p_0(y)+\varepsilon p_1(y,z)+0(\varepsilon^2) \quad (28)$$

$$\theta=\theta_0(y)+\varepsilon \theta_1(y,z)+0(\varepsilon^2) \quad (29)$$

$$\phi=\phi_0(y)+\varepsilon \phi_1(y,z)+0(\varepsilon^2) \quad (30)$$

with $p_0=p_\infty$, $w_0=0$ (31)

Substituting these in Equations 17 to 22 and equating the harmonic terms and neglecting ε^2 we get the following set of the differential equations:

Zeroth-order equations:

$$\frac{dv_0}{dy}=0 \quad (32)$$

$$v_0\frac{du_0}{dy}=G_r\theta_0+G_m\phi_0+\frac{1}{R_e}\frac{d^2u_0}{dy^2}+MR_e(U-u) \quad (33)$$

$$v_0\frac{d\theta_0}{dy}=\frac{1}{P_r R_e}\frac{d^2\theta_0}{dy^2}-Q\theta_0 \quad (34)$$

$$v_0\frac{d\phi_0}{dy}=\frac{1}{S_c R_e}\frac{d^2\phi_0}{dy^2}+\frac{S_r}{R_e}\frac{d^2\theta_0}{dy^2}-K\phi_0 \quad (35)$$

First-order equations:

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \quad (36)$$

$$-\frac{\partial u_1}{\partial y}+v_1\frac{du_0}{dy}=G_r\theta_1+G_m\phi_1+\frac{1}{R_e}\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)-MR_e u_1 \quad (37)$$

$$-\frac{\partial v_1}{\partial y}=-\frac{1}{R_e^2}\frac{\partial p_1}{\partial y}+\frac{1}{R_e}\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right) \quad (38)$$

$$-\frac{\partial w_1}{\partial y}=-\frac{1}{R_e^2}\frac{\partial p_1}{\partial y}+\frac{1}{R_e}\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-MR_e w_1 \quad (39)$$

$$-\frac{\partial \theta_1}{\partial y}+v_1\frac{d\theta_0}{dy}=\frac{1}{P_r R_e}\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right)-Q\theta_1 \quad (40)$$

$$-\frac{\partial \phi_1}{\partial y}+v_1\frac{d\phi_0}{dy}=\frac{1}{S_c R_e}\left(\frac{\partial^2 \phi_1}{\partial y^2}+\frac{\partial^2 \phi_1}{\partial z^2}\right)+\frac{S_r}{R_e}\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right)-K\phi_1 \quad (41)$$

Subject to boundary conditions:

$$y=0\ :\ u_0=0\,,\ v_0=-1\,,\ \theta_0=1\,,\ \phi_0=1\,,\ u_1=0\,,\ v_1=-\mathrm{Cos}\,\pi z$$
$$w_1=0\ ,\ \theta_1=0\ ,\ \phi_1=0 \quad (42)$$

$$y\to\infty\ :\ u_0=U\ ,\ v_0=-1\,,\ \theta_0=0\ ,\ \phi_0=0\ ,\ u_1=0\ ,\ v_1=0$$
$$,\ w_1=0\ ,\ p_1=0\ ,\ \theta_1=0\ ,\ \phi_1=0 \quad (43)$$

The solution of Equations 32 to 35 under the boundary conditions 42 and 43 are

$$v_0=-1 \quad (44)$$

$$\theta_0=e^{-ay} \quad (45)$$

$$\phi_0=(1-a_1)e^{-by}+a_1e^{-ay} \quad (46)$$

$$u_0=U+A_1e^{-ay}+A_2e^{-b_e y}+(-A_1-A_2-U)e^{-\lambda R_e y} \quad (47)$$

where

$$a=\frac{P_rR_e+\sqrt{P_r^2R_e^2+4P_rR_eQ}}{2}\ ,\ a_1=\frac{-a^2S_rS_c}{a^2-S_cRa-KS_cR_e}\ ,\ \lambda=\frac{1+\sqrt{1+4M}}{2}\ ,$$

$$b=\frac{S_cR_e+\sqrt{S_c^2R_e^2+4KS_cR_e}}{2},$$

$$A_1=\frac{-G_m a_1R_e}{a^2-R_ea-MR_e^2}-\frac{G_rR_e}{a^2-R_ea-MR_e^2}\ ,\ A_2=\frac{-G_m(1-a_1)R_e}{b^2-R_eb-MR_e^2}$$

We shall first consider the Equations 36, 38 and 39 for $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ which are independent of main flow component u_1, temperature field θ_1 and concentration field ϕ_1. The suction velocity $v_w = -(1+\varepsilon \operatorname{Cos} \pi z)$ consists of a uniform distribution -1 with superimposed weak sinusoidal distribution $\varepsilon \operatorname{Cos} \pi z$. Hence the velocity components v, w and p are also separated into mean and small sinusoidal components v_1 , w_1 and p_1. We assume v_1 , w_1 and p_1 to be of the following forms:

$$v_1 = -\pi v_{11}(y) \operatorname{Cos} \pi z \tag{48}$$

$$w_1 = v'_{11}(y) \operatorname{Sin} \pi z \tag{49}$$

$$p_1 = R_e^2\, p_{11}(y) \operatorname{Cos} \pi z \tag{50}$$

On substitution of Equations 48, 49 and 50, Equation 36 is satisfied and Equations 38 and 39 reduce to the following ordinary differential equations

$$v''_{11} + R_e\, v'_{11} - \pi^2 v_{11} = -\frac{R_e\, p'_{11}}{\pi} \tag{51}$$

$$v'''_{11} + R_e\, v''_{11} - (\pi^2 + M R^2) v'_{11} = -R_e\, \pi\, p_{11} \tag{52}$$

with relevant boundary conditions

$$y = 0 \quad : \quad v_{11} = \frac{1}{\pi} \quad , \quad v'_{11} = 0 \tag{53}$$

$$y \to \infty \; : \; v_{11} = 0 \; , \; v'_{11} = 0 \; , \; p_{11} = 0 \tag{54}$$

The solutions of these equations are:

$$v_{11} = \frac{1}{\pi(\bar{\xi} - \xi)} \left[\bar{\xi}\, e^{-\xi y} - \xi\, e^{-\bar{\xi} y} \right] \tag{55}$$

$$p_{11} = \frac{1}{R_e \pi^2 (\bar{\xi} - \xi)} \left[\left(\pi^2 + M R_e^2 + R_e \bar{\xi} - \bar{\xi}^2\right) e^{-\bar{\xi} y} - \left(\pi^2 + M R_e^2 + R_e \xi - \xi^2\right) e^{-\xi y} \right]$$

$$= \frac{1}{R_e \pi^2 (\bar{\xi} - \xi)} \left[\bar{\xi}_1\, e^{-\bar{\xi} y} - \xi_1\, e^{-\xi y} \right] \tag{56}$$

Where

$$\xi = \frac{R_e \lambda + \sqrt{R_e^2 \lambda^2 + 4\pi^2}}{2},$$

$$\bar{\xi} = \frac{R_e \bar{\lambda} + \sqrt{R_e^2 \bar{\lambda}^2 + 4\pi^2}}{2}, \quad \lambda = \frac{1 + \sqrt{1 + 4M}}{2},$$

$$\bar{\lambda} = \frac{1 - \sqrt{1 + 4M}}{2} \quad , \quad \bar{\xi}_1 = \left(\pi^2 + M R_e^2 + R_e \bar{\xi} - \bar{\xi}^2\right) ,$$

$$\xi_1 = \left(\pi^2 + M R_e^2 + R_e \xi - \xi^2\right)$$

Hence the solutions for the velocity components v_1, w_1 and pressure p_1 are as follows:

$$v_1 = \frac{1}{\xi - \bar{\xi}} \left[\bar{\xi}\, e^{-\xi y} - \xi\, e^{-\bar{\xi} y} \right] \operatorname{Cos} \pi z \tag{57}$$

$$w_1 = \frac{\xi \bar{\xi}}{\pi(\xi - \bar{\xi})} \left[e^{-\bar{\xi} y} - e^{-\xi y} \right] \operatorname{Sin} \pi z \tag{58}$$

$$p_1 = \frac{R_e\, \xi \bar{\xi}}{\pi^2 (\bar{\xi} - \xi)} \left[\bar{\xi}_1 e^{-\bar{\xi} y} - \xi_1 e^{-\xi y} \right] \operatorname{Cos} \pi z \tag{59}$$

SOLUTION FOR FIRST ORDER FLOW, CONCENTRATION AND TEMPERATURE FIELD

We now consider Equations 30, 33 and 34. The solutions for velocity component u, temperature field θ and concentration field ϕ are also separated into mean and sinusoidal components u_1 , θ_1 and ϕ_1. To reduce the partial differential Equations 30, 33, 34 into ordinary differential equations, we consider the following forms for u_1 , θ_1 and ϕ_1 .

$$u_1 = u_{11}(y) \operatorname{Cos} \pi z \tag{60}$$

$$\theta_1 = \theta_{11}(y) \operatorname{Cos} \pi z \tag{61}$$

$$\phi_1 = \phi_{11}(y) \operatorname{Cos} \pi z \tag{62}$$

Using the expressions for v_1 , u_1 , θ_1 , ϕ_1 in Equations 37, 40 and 41 we get the following differential equations:

$$u''_{11} + R_e\, u'_{11} - (\pi^2 + M R_e^2) u_{11} = -\pi R_e\, v_{11}\, u'_0 - R_e\, G_r\, \theta_{11} - R_e\, G_m\, \phi_{11} \tag{63}$$

$$\theta''_{11} + P_r\, R_e\, \theta'_{11} - (\pi^2 + P_r\, R_e\, Q)\theta_{11} = -\pi P_r\, R_e\, v_{11}\, \theta'_0 \tag{64}$$

$$\phi''_{11} + S_c\, R_e\, \phi'_{11} - (\pi^2 + K)\phi_{11} = -\pi S_c\, R_e\, v_{11}\, \phi'_0 - S_c\, S_r \left(\theta''_{11} - \pi^2 \theta_{11}\right) \tag{65}$$

with the boundary conditions

$$\left.\begin{aligned} y = 0 \; &: \; u_{11} = 0 \, , \; \theta_{11} = 0 \; , \; \phi_{11} = 0 \\ y \to \infty \; &: u_{11} = 0 \; , \; \theta_{11} = 0 \; , \; \phi_{11} = 0 \end{aligned}\right\} \tag{66}$$

The solutions of Equations 64, 65 and 63 subject to boundary conditions (66) are

$$\theta_{11} = G_0 e^{-hy} + G_1 e^{-(\xi+a)y} + G_2 e^{-(\bar{\xi}+a)y} \quad (67)$$

$$\phi_{11} = H_0 e^{-my} + H_1 e^{-hy} + H_2 e^{-(\xi+a)y} + H_3 e^{-(\bar{\xi}+a)y} + H_4 e^{-(\xi+b_e)y} + H_5 e^{-(\bar{\xi}+b)y} \quad (68)$$

$$u_{11} = M_0 e^{-ny} + M_1 e^{-hy} + M_2 e^{-my} + M_3 e^{-(\xi+a)y} + M_4 e^{-(\bar{\xi}+a)y} + M_5 e^{-(\xi+b)y} + +M_6 e^{-(\bar{\xi}+b)y} + M_7 e^{-(\xi+\lambda R_e)y} + M_8 e^{-(\bar{\xi}+\lambda R_e)y} \quad (69)$$

where

$$G_1 = \frac{a P_r R_e \bar{\xi}}{(\bar{\xi}-\xi)\left\{(\xi+a)^2 - P_r R_e(\xi+a) - (\pi^2 + P_r R_e Q)\right\}},$$

$$G_2 = \frac{-a P_r R_e \xi}{(\bar{\xi}-\xi)\left\{(\bar{\xi}+a)^2 - P_r R_e(\bar{\xi}+a) - (\pi^2 + P_r R_e Q)\right\}}, \; G_0 = -(G_1 + G_2),$$

$$h = \frac{P_r R_e + \sqrt{P_r^2 R_e^2 + 4(\pi^2 + P_r R_e Q)}}{2},$$

$$m = \frac{S_c R_e + \sqrt{S_c^2 R_e^2 + 4(\pi^2 + K)}}{2},$$

$$E_1 = -S_c S_r G_0 (h^2 - \pi^2), \; E_2 = -S_c S_r G_1\left\{(\xi+a)^2 - \pi^2\right\}, \; E_3 = -S_c S_r G_2\left\{(\bar{\xi}+a)^2 - \pi^2\right\},$$

$$B_1 = \frac{(1-a_1) b S_c R_e \xi}{(\bar{\xi}-\xi)}, \; B_2 = \frac{a a_1 S_c R_e \bar{\xi}}{(\bar{\xi}-\xi)}, \; B_3 = \frac{-S_c R_e b \xi (1-a_1)}{(\bar{\xi}-\xi)}, \; B_4 = \frac{-a a_1 S_c R_e \xi}{(\bar{\xi}-\xi)},$$

$$H_1 = \frac{E_1}{h^2 - S_c R_e h - (\pi^2 + K)}, \; H_2 = \frac{B_2 + E_2}{(\xi+a)^2 - S_c R_e(\xi+a) - (\pi^2+K)},$$

$$H_3 = \frac{B_4 + E_3}{(\bar{\xi}+a)^2 - S_c R_e(\bar{\xi}+a) - (\pi^2+K)}, \; H_4 = \frac{B_1}{(\xi+b)^2 - S_c R_e(\xi+b) - (\pi^2+K)},$$

$$H_5 = \frac{B_3}{(\bar{\xi}+b)^2 - S_c R_e(\bar{\xi}+b) - (\pi^2+K)}, \; H_0 = -\sum_{i=1}^{5} H_i$$

$$, \; n = \frac{R_e + \sqrt{R_e^2 + 4(\pi^2 + M R_e^2)}}{2},$$

$$K_1 = \frac{a A_1 R_e \bar{\xi}}{(\xi - \bar{\xi})},$$

$$K_2 = \frac{A_2 b R_e \bar{\xi}}{(\bar{\xi}-\xi)}, \; K_3 = -\frac{\bar{\xi}\lambda R_e^2 (A_1 + A_2 + U)}{(\bar{\xi}-\xi)}, \; K_4 = \frac{-a A_1 R_e \xi}{(\bar{\xi}-\xi)},$$

$$K_5 = -\frac{A_2 b R_e \xi}{(\bar{\xi}-\xi)}, \; K_6 = \lambda R_e^2 (A_1 + A_2 + U)\xi,$$

$$L_1 = R_e(G_r G_0 + G_m H_1), \; L_2 = -G_m H_0 R_e$$

$$L_3 = K_1 - G_r G_1 R_e - G_m H_2 R_e, \; L_4 = K_4 - G_m H_3 R_e - R_e G_r G_2, \; L_5 = K_2 - G_m H_4 R_e$$

$$L_6 = K_5 - G_m H_5 R_e, \; M_1 = \frac{L_1}{h^2 - R_e h - (\pi^2 + M R_e^2)}, \; M_2 = \frac{L_2}{m^2 - R_e m - (\pi^2 + M R_e^2)},$$

$$M_3 = \frac{L_3}{(\xi+a)^2 - R_e(\xi+a) - (\pi^2 + M R_e^2)}, \; M_4 = \frac{L_4}{(\bar{\xi}+a)^2 - R_e(\bar{\xi}+a) - (\pi^2 + M R_e^2)},$$

$$M_5 = \frac{L_5}{(\xi+b)^2 - R_e(\xi+b) - (\pi^2 + M R_e^2)}, \; M_6 = \frac{L_6}{(\bar{\xi}+b)^2 - R_e(\bar{\xi}+b) - (\pi^2 + M R_e^2)},$$

$$M_7 = \frac{K_3}{(\xi + \lambda R_e)^2 - R_e(\xi + \lambda R_e) - (\pi^2 + M R_e^2)},$$

$$M_8 = \frac{K_6}{(\bar{\xi} + \lambda R_e)^2 - R_e(\bar{\xi} + \lambda R_e) - (\pi^2 + M R_e^2)},$$

$$M_0 = -\sum_{i=1}^{8} M_i$$

Skin friction at the plate

The non-dimensional skin-friction at the plate in direction of the free steam is given by

$$\tau = \frac{\mu \left.\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}}{\rho V_0^2} = -\frac{1}{R_e}\left[u_0'(0) + \varepsilon u_{11}'(0) \cos \pi z\right] = \tau_0 + \varepsilon Q_1 \cos \pi z \quad (70)$$

where

$$\tau_0 = -\frac{1}{R_e} u_0'(0) = \frac{a A_1}{R_e} + \frac{b A_2}{R_e} - \lambda(A_1 + A_2 + U) \quad (71)$$

and

$$Q_1 = -\frac{1}{R_e} u_{11}'(0)$$

$$= \frac{1}{R_e}\begin{bmatrix} n M_0 + h M_1 + m M_2 + (\xi+a) M_3 + (\bar{\xi}+a) M_4 + (\xi+b) M_5 \\ +(\bar{\xi}+b) M_6 + (\lambda R_e + \xi) M_7 + (\lambda R_e + \bar{\xi}) M_8 \end{bmatrix} \quad (72)$$

The co-efficient of rate of heat transfer

The heat flux from the plate to the in terms of Nusselt number Nu is given by

$$Nu = -\frac{k}{\rho V_0 C_p (\bar{T}_w - \bar{T}_\infty)}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0} = -\frac{1}{P_r R_e}\left.\frac{\partial \theta}{\partial y}\right]_{y=0} = Nu_0 + \varepsilon Q_2 \cos \pi z \quad (73)$$

Where

$$Nu_0 = -\frac{\theta_0'(0)}{P_r R_e} = \frac{a}{P_r R_e} \quad (74)$$

And

$$Q_2 = -\frac{\theta_{11}'(0)}{P_r R_e} = \frac{1}{P_r R_e}\left[h G_0 + (\xi + a)G_1 + (\bar{\xi} + a)G_2\right] \quad (75)$$

The coefficient of mass transfer

The mass flux at the wall y = 0 in terms of Sherwood number Sh is given by

$$Sh = \frac{-D_M}{V_0(\bar{C}_w - \bar{C}_\infty)}\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)_{y=0} = -\frac{1}{S_c R_e}\frac{\partial \phi}{\partial y}\bigg]_{y=0} = -\frac{1}{S_c R_e}\left[\phi_0'(0) + \varepsilon \phi_{11}' \mathrm{Cos}\pi z\right]$$

$$= Sh_0 + \varepsilon Q_3 \,\mathrm{Cos}\,\pi z \quad (76)$$

Where

$$Sh_0 = -\frac{1}{S_c R_e}\left[b(1 - a_1) + a a_1\right] \quad (77)$$

and

$$Q_3 = -\frac{1}{S_c R_e}\phi_{11}'(0) = \frac{1}{S_c R_e}\begin{bmatrix} m H_0 + h H_1 + (\xi + a)H_2 + (\bar{\xi} + a)H_3 \\ (\xi + b)H_4 + (\bar{\xi} + b)H_5 \end{bmatrix} \quad (78)$$

Current density

The current density $\vec{J}$ is given by

$$\vec{J} = \sigma \vec{q} \times \vec{B} = \sigma B_0\left(-\hat{i}\,\bar{w} + \hat{k}\,\bar{u}\right) \quad (79)$$

The magnitude of $\vec{J}$ is given

by $\left|\vec{J}\right| = \sigma B_0 \sqrt{\bar{w}^2 + \bar{u}^2} = \sigma B_0 V_0 \sqrt{u^2 + w^2}$ (80)

The current density (in magnitude) in non dimensional form is given by:

$$J_c = \frac{\left|\vec{J}\right|}{\sigma B_0 V_0} = \sqrt{u^2 + w^2} = u\sqrt{1 + \left(\frac{w}{u}\right)^2} = u$$

(since $\frac{w}{u} << 1$) (81)

That is, the magnitude of the non dimensional current density is proportional to the boundary layer velocity.

RESULTS AND DISCUSSION

In order to study the effects of heat sink parameter (Q), Reynolds number (R_e), and chemical reaction parameter (K), we have carried out the data tabulations for u, θ_0, ϕ_0, Q_1, Q_2, Q_3 and τ_0 which are respectively the dimensional velocity, zeroth order temperature, zeroth order species concentration, amplitudes of the first order skin friction, Nusselt number and Sherwood number; the zeroth order skin friction at the plate y = 0 and their values are demonstrated in the graphs. Throughout our discussion P_r (Prandtl number) is considered to be equal to 71 which corresponds to air. Since the water vapor is used as a diffusing chemical species of common interest in air therefore the values of S_c is taken to be 0.60 (water vapor). The values of the Grashof number G_r for heat transfer has been chosen as 10 (externally cooled plate) whereas the values of Grashof number G_m for mass transfer is considered to be 15, the free steam velocity is selected to be 1 and the small reference parameter ε is chosen as 0.001 and the remaining parameters namely chemical reaction parameter (K), heat sink parameter (Q), Reynolds number (R_e), Soret number S_r are chosen arbitrarily.

Figures 1, 2 and 3 exhibit the variation of velocity profile u against y for different values of chemical reaction parameter (K), heat sink parameter (Q) and Hartmann number (M). It is seen from these figures that the velocity quickly increases up to some thin layer of the liquid adjacent to the plate and after this, fluid velocity decreases asymptotically towards 1 as $y \to \infty$; that is, in the free steam. This shows that the buoyancy effects (due to concentration and temperature differences) are significant near the hot plate.

It is observed from Figure 1 that the fluid motion is retarded (that is, the fluid velocity decreases) on account of chemical reaction. This shows that the consumption of chemical species leads to fall in the concentration field which in turn diminishes the buoyancy effects due to concentration gradients. Consequently, the flow field is decelerated. It is also inferred from Figures 2 and 3 that the heat sink parameter (Q) as well as Hartmann number (M) impedes the fluid motion. In other words, fluid motion is retarded due to application of transverse magnetic field. This phenomenon clearly agrees with the fact that Lorentz force that appears due to interaction of the magnetic field and fluid velocity resists the fluid motion.

Figure 4 demonstrates the variation of zeroth order fluid temperature θ_0 against y under the effect of heat sink parameter (Q). It is clear from this figure that zeroth order fluid temperature θ_0 asymptotically falls from 1 to zero as $y \to \infty$. The same figure further indicates that the heat sink parameter results in a steady decrease in the zeroth order fluid temperature.

The variation of zeroth order species concentration ϕ_0 versus y under the influences of heat sink parameter (Q) and chemical reaction parameter (K) have been presented in Figures 5 and 6. These figures show that

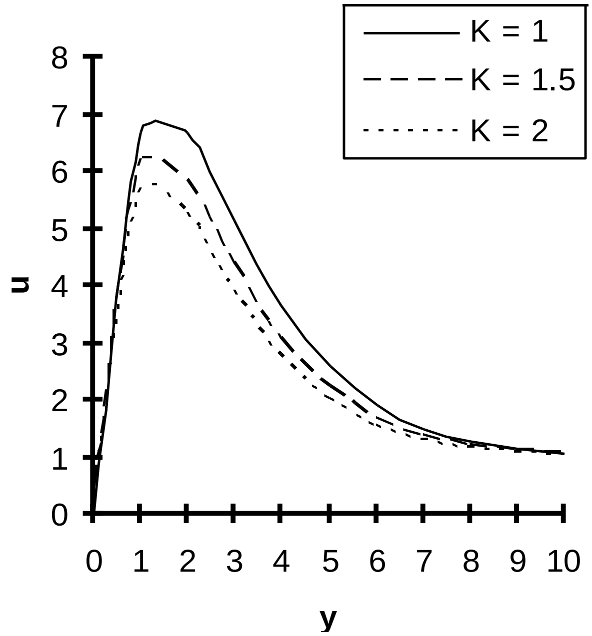


Figure 1. Velocity distribution versus y for K when Q =1, M = 1, R_e = 0.5, S_r = 0.5.

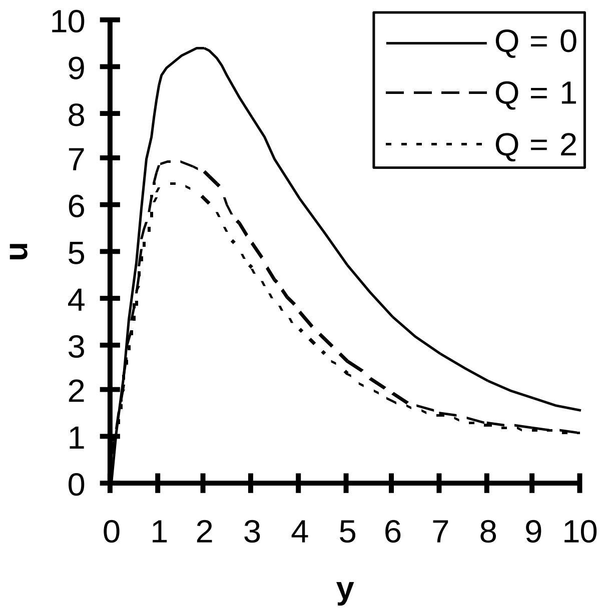


Figure 2. Velocity distribution versus y for Q when K =1, M = 1, R_e = 0.5, S_r = 0.5.

zeroth order concentration of the fluid fall under the effect of heat sink parameter (Q) and chemical reaction parameter (K). Moreover, it is noticed from these figure that ϕ_o asymptotically decreases from maximum value ϕ_o = 1 to its minimum value ϕ_o = 0 as one moves far away the plate ($y \rightarrow \infty$).

Figures 7, 8 and 9 depict the variation of amplitude of the perturbed part of skin-friction Q_1 versus Reynolds number R_e. From these figures we observe that magnetic field effect as well as heat sink effect causes Q_1 to decrease whereas Q_1 increases for the increasing values of chemical reaction parameter. There is an

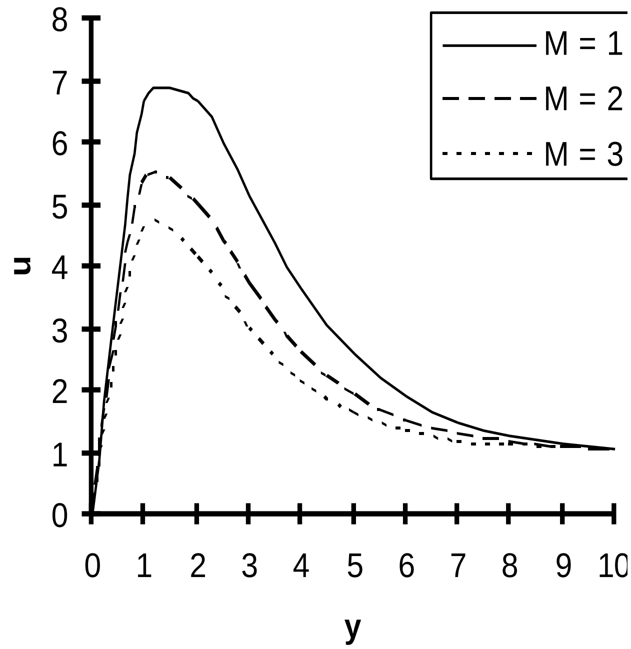


Figure 3. Velocity distribution versus y for M when Q =1, K = 1, R_e =0 .5, S_r = 0.5.

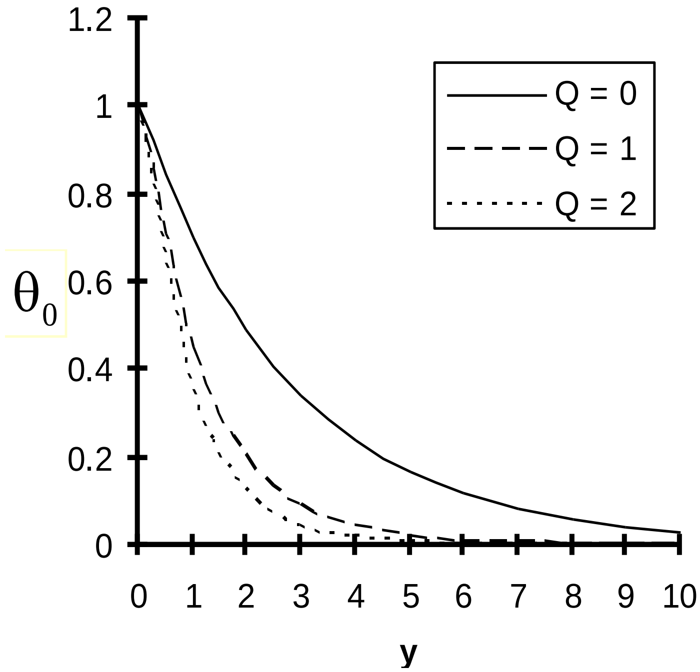


Figure 4. Zeroth order temperature distribution versus y for Q when R_e = 0.5.

indication from these figures that Q_1 falls as R_e increases. That is for low viscosity Q_1 is not significantly affected by heat sink parameter (Q), chemical reaction parameter (K) or by Hartmann number (M).

The influence of heat sink parameter (Q) on the amplitude of Q_2 of the perturbed part of the Nusselt number is displayed in Figure 10. It is noticed from the figure that an increase in the value of Reynolds number

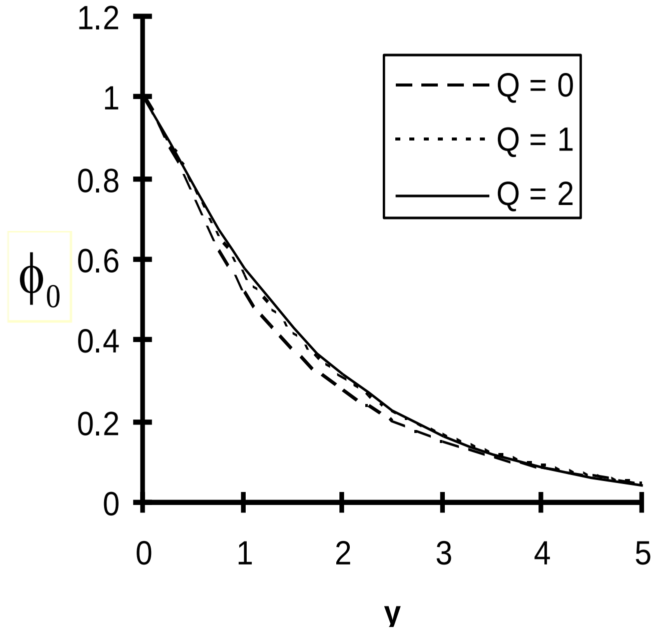


Figure 5. Zeroth order concentration profile versus y for Q when K = 1, R_e = 0.5, S_r = 0.5.

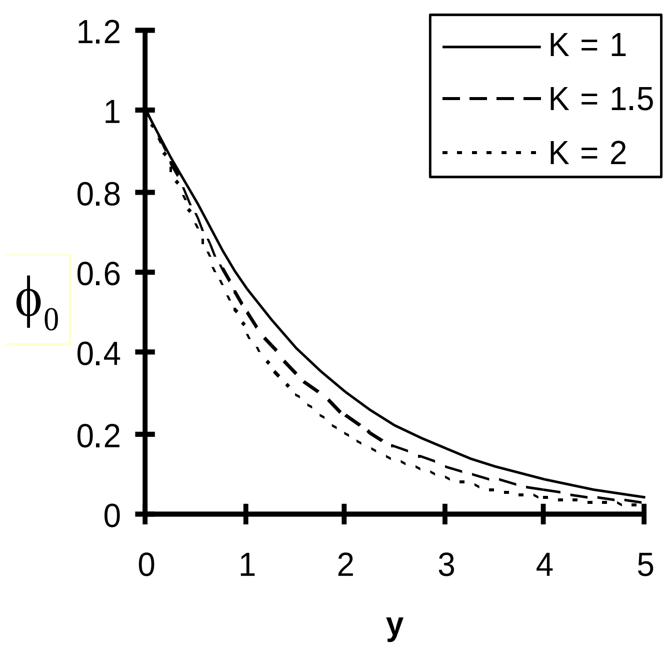


Figure 6. Zeroth order concentration profile versus y for K when Q = 1, R_e = 0.5, S_r = 0.5.

(R_e) or heat sink parameter (Q) causes Q_2 to increase; that is, Q_2 drops due to high viscosity or low strength of heat sink.

Figures 11 and 12 exhibits the change in behaviour of amplitude, of perturbed part, and of the Sherwood number Q_3 under the influence of the Reynolds number R_e, the chemical reaction parameter (K) and

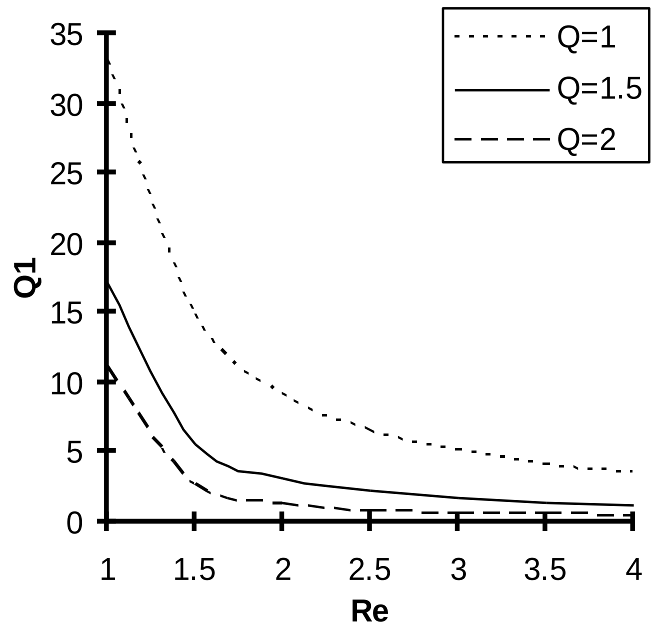


Figure 7. The amplitude Q_1 of the first order skin friction versus R_e for K=1, M=1, S_r=0.5.

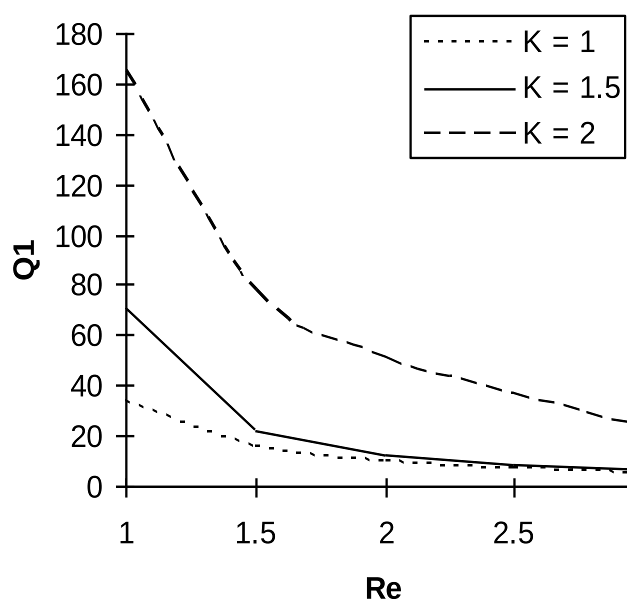


Figure 8. The amplitude Q_1 of the first order skin friction versus R_e for Q=1, M=1, S_r= 0.5.

heat sink parameter (Q). These figures show that Q_3 is increased due to chemical reaction effect where as there is a steady decline in Q_3 when heat sink parameter (Q) is increased.

The variation of the zeroth order skin friction τ_0 at the plate y = 0 under the influence of chemical reaction parameter (K), heat sink parameter (Q) and Hartmann

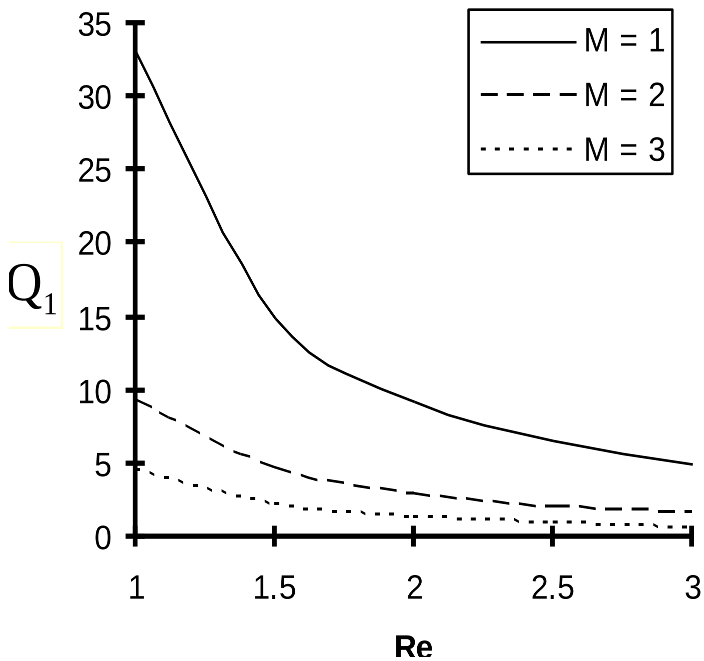


Figure 9. The amplitude Q_1 of the first order skin friction versus R_e for K = 1, Q = 1, S_r = 0.5.

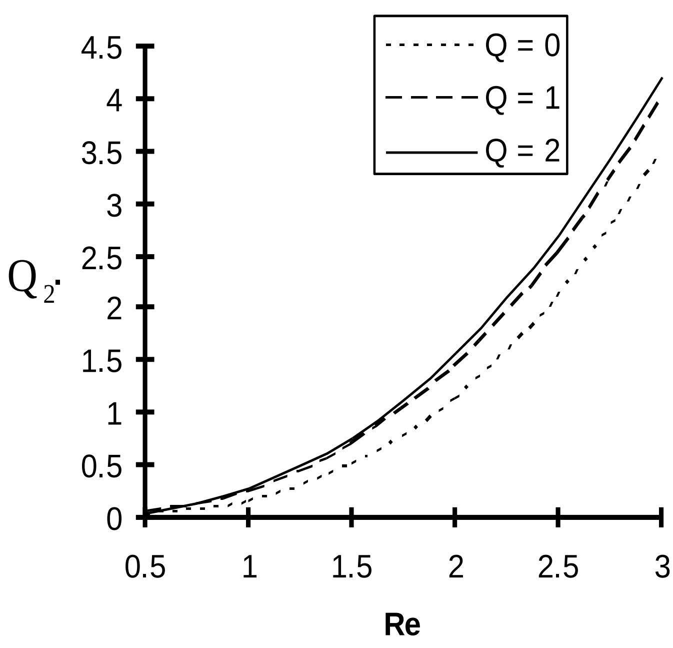


Figure 10. The amplitude Q_2 of the first order Nusselt number versus R_e for K = 1, S_r = 0.5.

number (M) are presented respectively in Figures 13, 14 and 15. It is noticed from these figures that the magnitude of viscous drag at the plate decreases due to the chemical reaction parameter (K), heat sink (Q) and

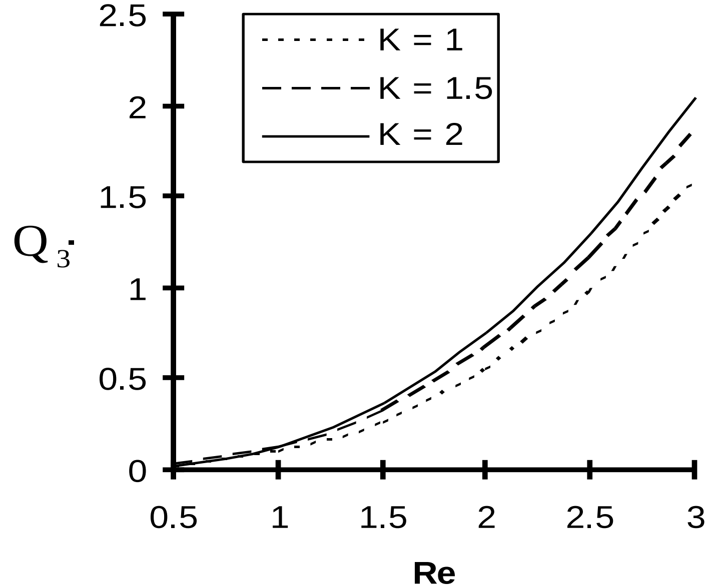


Figure 11. The amplitude Q_3 of the first order Sherwood number versus R_e for Q=1, S_r=0.5.

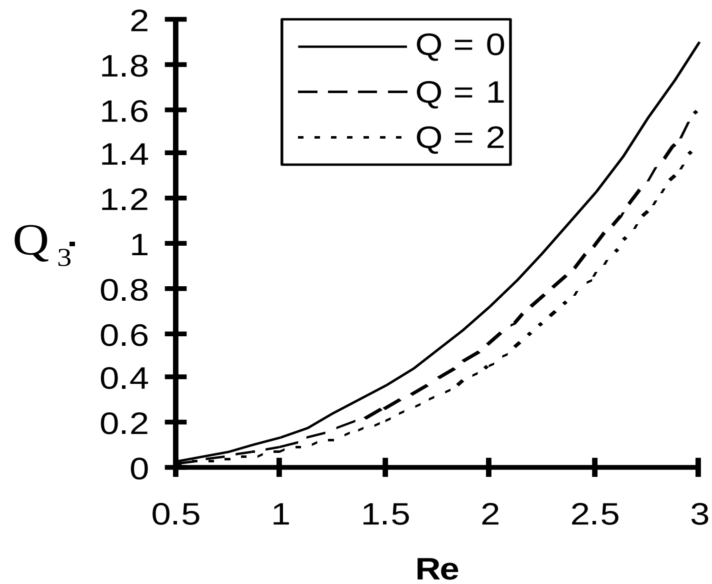


Figure 12. The amplitude Q_3 of the first order Sherwood number versus R_e for Q =1, S_r=0.5.

magnetic field (M).

Conclusion

1. The chemical reaction, heat sink and magnetic field lead the fluid motion to retard. Thus the chemically reacting fluid motion may be controlled with the application of heat sink and magnetic field.
2. The heat sink results in a steady decrease in the fluid temperature. Hence the fluid temperature may be controlled by using a suitable heat sink.

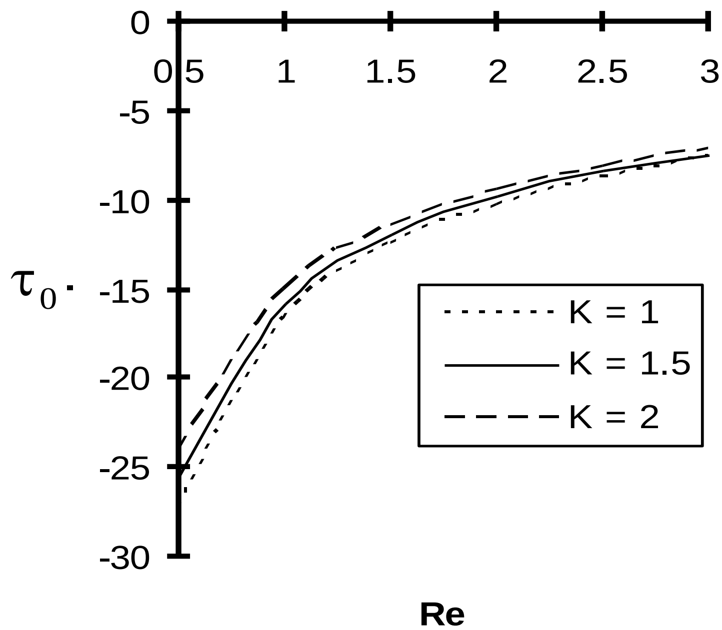


Figure 13. The zeroth order skin friction τ_0 at the plate versus R_e for Q = 1, M=1, S_r=0.5.

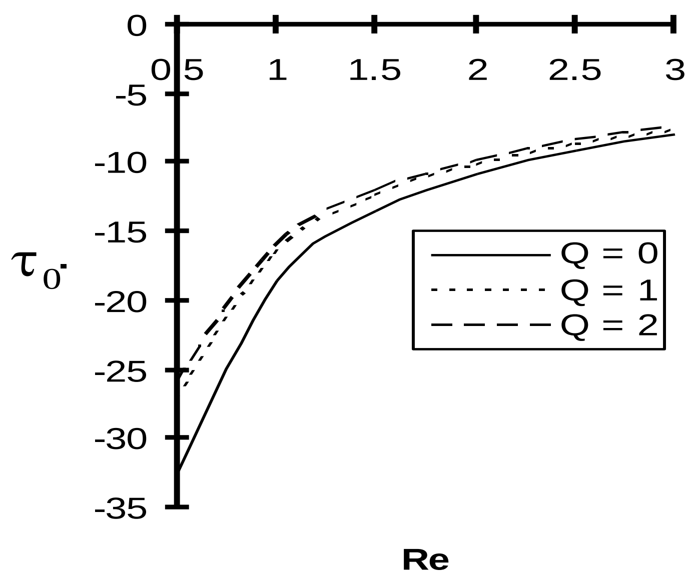


Figure 14. The zeroth order skin friction τ_0 at the plate versus R_e for Q = 1, M=1, S_r=0.5.

3. The concentration of the fluid rises under the effect of heat sink whereas it falls due to the effect of chemical reaction.
4. Magnitude of the first order skin friction increases due to chemical reaction effect and it decreases under the effects of absorption heat sink and the applied transverse magnetic field.
5. The first order Nusselt number drops due to high viscosity or low strength heat sink.
6. The heat absorbing sink leads the first order Sherwood

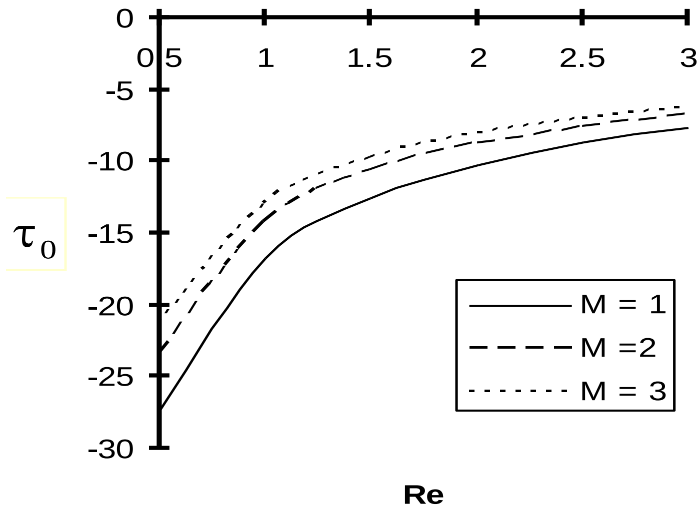


Figure 15. The zeroth order skin friction τ_0 at the plate versus R_e for Q = 1, K=1, S_r=0.5.

number to fall but it rises under the effect of chemical reaction parameter.
7. Magnitude of the zeroth order skin friction diminishes due to chemical reaction effect, magnetic field as well as the heat sink.

Nomenclature: $\vec{B}$**,** magnetic induction vector; B_0**,** strength of applied magnetic field; $\bar{C}_\infty$**,** species concentration in the free stream; $\bar{C}_w$**,** species concentration at the plate; $\bar{C}_p$**,** specific heat at constant pressure; D_M**,** chemical molecular diffusivity; D_T**,** chemical thermal diffusivity; $\vec{E}$**,** electric field; $\vec{g}$**,** gravitational acceleration; **g,** acceleration due to gravity; G_r**,** Grashof number for heat transfer; G_m**,** Grashof number for mass transfer; $\vec{J}$**,** electric current density; **k,** thermal conductivity; $\bar{K}$**,** first order chemical reaction; **K,** chemical reaction parameter; **L,** wave length of the periodic suction; **M,** Hartmann number; $\bar{p}$ **,** pressure; $\bar{p}_\infty$**,** pressure in the free steam; **p,** non dimensional pressure; p_∞**,** non dimensional pressure in the free steam; $\bar{q}$**,** velocity vector; $\bar{Q}$**,** first order heat sink; **Q,** non dimensional first order heat sink; R_e**,** Reynolds number; S_r**,** Soret number; P_r**,** Prandtl number; S_c**,** Schmidt number; $\bar{T}$**,** temperature in the boundary layer; $\bar{T}_w$**,** temperature at the plate; $\bar{T}_\infty$**,** fluid temperature at the free steam; $\bar{U}$**,** free steam velocity; U**,** non dimensional free steam velocity; $(\bar{u}, \bar{v}, \bar{w})$**,** components of the fluid velocity; (u, v, w)**,** non dimensional components of the fluid velocity; V_0**,** mean suction velocity; $(\bar{x}, \bar{y}, \bar{z})$**,** coordinate system; $\hat{i}, \hat{j}, \hat{k}$ **,** unit vectors in the increasing direction of $\bar{x}, \bar{y}, \bar{z}$; $\vec{J} \times \vec{B}$**,** Lorentz force per unit volume; α**,** thermal diffusivity; β**,** coefficient of volume expansion for heat transfer; $\bar{\beta}$**,** coefficient of volume expansion for mass transfer; σ**,** electrical conductivity; ν**,** kinematic viscosity; ρ**,** density of the fluid; ε**,** small reference parameter; θ**,** non dimensional temperature; ϕ**,** non dimensional concentration; φ**,** viscous dissipation of energy per unit volume; μ**,** coefficient of viscosity.

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