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Full Length Research Paper

Suppression of the Buoyoncy-driven Instability of plasma by Larmor radius effects

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Using the magnetohydrodynamic equations with ion gyro-radius corrections, the gravity-driven instability of plasma is analyzed here for the mode which has non zero projection of the wave number along the unperturbed magnetic field. Employing the normal mode technique, the stability analysis had been carried out through the eigen value solution. The suppression of the buoyancy-driven instability in an exponentially stratified semi-infinite plasma layer by the finite Larmor radius effects is shown here.

Key words: Instability, buoyancy-driven, magnetic field, normal mode, finite Larmor radius.

INTRODUCTION

The effect of finite Lamor radius (FLR) on the magnetohydrodynamic (MHD) instability which are interchange mode in mirror machine, heliotron stellarators and Rayleigh-Taylor (RT) instability in spread-region of the ionosphere and astrophysical situations e.g. sunspots, solar corona and stellar atmosphere are well known in literature. The FLR effects exhibit themselves in the fluid equations in the form of a magnetic viscosity. Several researchers, namely, Roberts and Taylor (1962), Jukes (1964), Melchior and Popovich (1968) have examined the FLR effects on different problems of plasma instabilities.

Davidson and Volk (1968) have considered the FLR effects on the growth of the firehose instability for the wave vector parallel to the study magnetic field. Several authors (Ariel and Bhatia, 1969, 1970; Srivastava, 1974; Ogbona and Bhatia, 1984; Chhonkar and Bhatia, 1985) have studied the influence of FLR effects on the RT instability of stratified plasma in a horizontal magnetic field in conjunction with other operative forces such as due to rotation, Hall currents, compressibility and frictional effects with neutrals. All these studies have been carried out for the transverse mode of propagation. The RT instability in plasma had been examined from different points of view in the past. Among several authors, Hassam and Huba (1988) and Winske (1989) have done considerable theoretical and numerical work on the RT instability in a novel regime wherein the plasma is characterized by being in an intermediate stage of magnetization, the ion being unmagnetized while the electrons are strongly magnetized. Hassam and Huba (1990) have studied the nonlinear evolution of the unmagnetized ion RT instability. Hassam (1992) had also investigated the non linear stabilization of the RT instability of the magnetized plasma by external velocity shear as such a study is of relevance to tokomaks and to magnetized plasma confinement. The suppression of the RT instability by convection mass flow in plasma having smooth density gradient has been demonstrated by Budko and Liberman (1992). Book (1996) investigated the suppression of the RT instability through assertion. Huba (1996) discussed the finite Larmor radius MHD of the RT instability. Recently Prasad (1997) has examined the RT instability in dusty plasma.

The polarization in the usual RT mode of instability has zero projection of the wave number on the unperturbed magnetic field. It would be of interest to examine the

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other alternate polarization, a mode which has non zero projection of the wave number along the unperturbed magnetic field. The FLR effects on the gravity-driven instability in a stratified semi-infinite plasma layer are investigated here. The mode of instability considered is not longitudinal since the longitudinal mode is a much weaker instability than the transverse mode because it requires a large amount of magnetic field line bending and thus it is of no practical interest generally speaking. The buoyancy-driven instability is analyzed here for incompressible inviscid ideally conducting plasma using the MHD equations with ion gyro-radius corrections due to Roberts and Taylor (1962). The FLR effects like viscosity, lead to a higher order differential equation and the stability analysis is carried out through the eigen-value solution.

PERTURBATION EQUATION

Consider the motion of an incompressible ideally conducting inviscid plasma of varying density in a variable magnetic field $\vec{H}$. Let $\delta\rho$, $\delta\bar{p}\left(\delta P_{ij}\right)$, $\vec{h}=\left(h_x,h_y,h_z\right)$ and $\vec{v}=\left(u,v,w\right)$ be the perturbations respectively in density ρ, stress tensor P_{ij}, magnetic field $\vec{H}$ and velocity produced by a small disturbance in the plasma. The linearized perturbation equations are

$$\rho\frac{\partial\vec{v}}{\partial t}=-\nabla\delta p+\vec{g}\,\delta\rho+\left(\nabla\times\vec{h}\right)\times\vec{H}+\left(\nabla\times\vec{H}\right)\times\vec{h} \qquad (1)$$

$$\frac{\partial\vec{h}}{\partial t}=\nabla\times\left(\vec{v}\times\vec{H}\right) \qquad (1)$$

$$\frac{\partial}{\partial t}\left(\delta\rho\right)=-\left(\vec{v}.\nabla\right)\rho \qquad (2)$$

$$\nabla.\vec{v}=0,\ \nabla.\vec{h}=0 \qquad (3)$$

We assume that the magnetic field $\vec{H}$ is horizontal along x-axis and is stratified along the vertical that $\vec{H}=\left(H_0\left(z\right),0,0\right)$. It is also assumed that the applied magnetic field is small so that the induced electrical field may be neglected.

As we are interested in examining the mode of instability which has non-zero projection of the wave number along the unperturbed magnetic field, we assume that all the perturbed quantities depend on the space co-ordinates and time t as

$$F\left(z\right)\exp\left(ikx+nt\right) \qquad (4)$$

where $F\left(z\right)$ is some function of z, k is the wave number along the direction of the unperturbed magnetic field that is, along x-axis, and n (where n may be complex) is the rate at which the system departs away from equilibrium.

The components of the stress tensor P_{ij} with ion gyro-radius corrections have been given by Roberts and Taylor (1962). Substituting these components in Equation (1) and using the expression [Equation (5)] in Equation (1) to (4) we get, on writing

$$D\equiv\frac{d}{dz}$$

$$\rho nu=-ik\delta p+h_zDH_0-2ik\nu(\rho Dv+vD\rho \qquad (5)$$

$$\rho nv=-2\rho\nu(D^2+k^2)w+\left(D\rho\right)\left(Dw\right)+\rho\nu D^2w+ikH_0h_y \qquad (6)$$

$$\rho nw=-D\delta p+2k^2\nu\rho v-\left(D\rho\right)\left(Dv\right)-\rho\nu D^2v+H_0\left(ikh_z-Dh_x\right)-h_xDH_0+\frac{gw\left(D\rho\right)}{n} \qquad \ldots(8)$$

$$nh_x=ikH_0u-wDH_0 \qquad \ldots(9)$$

$$nh_y=ikH_0v \qquad \ldots(10)$$

$$nh_z=ikH_0w \qquad \ldots(11)$$

$$n\delta\rho+w\left(D\rho\right)=0 \qquad \ldots(12)$$

$$iku+Dw=0,\quad ikh_x+Dh_z=0 \qquad \ldots(13)$$

where $\rho\nu=\frac{NT}{4w_H}$, w_H being the ion gyration frequency while N and T are respectively the number density and the temperature of the particles.

By eliminating of h_y from Equation (7) and (10) one gets v in terms of w, Dw and D^2w. Since Equation (8) contains a term in D^2v, elimination of v and other variables leads to a fourth order differential equation in w. Before obtaining this equation explicitly, we specify the variations of $\rho\left(z\right)$ and $H_0\left(z\right)$. One can choose any functions of z for these variations but if one choose similar functions of z for these variations so that the ratio $\frac{H_0^{\,2}\left(z\right)}{\rho\left(z\right)}$ is constant throughout the medium, the governing differential equation with constant coefficients. We, therefore, assume that both $\rho\left(z\right)$ and $H_0\left(z\right)$ are stratified exponentially along the vertical that is,

$$\rho(z)=\rho_0\exp(\beta z) \qquad \ldots(14)$$

$$H_0^2(z) = H_1^2 \exp(\beta z) \quad \text{...(15)}$$

where β, the stratification parameter, is a constant and $\frac{H_1^2}{\rho_0} = V^2$ is the square of the Alfvên velocity. The differential equation in *w*, on using the variation given by Equations (14) and (15) is therefore,

$$n^2k^2v^2\left[D^4w+2\beta D^3w+\left(4k^2-\beta^2\right)D^2w+\left(4\beta k^2-3\beta^3\right)Dw+4k^2\left(\beta^2+k^2\right)w\right]$$
$$-\left(n^2+k^2V^2\right)^2\left[D^2w+\beta Dw-k^2w\right]-g\beta k^2\left(n^2+k^2V^2\right)w=0 \quad \text{...(16)}$$

The FLR effects, like viscosity, lead to a differential equation of higher order, increasing the order by two. The considered plasma is assumed to be semi-infinite, infinitely extending along the horizontal directions and confined between two planes of depth d. The confining bounding planes may be rigid or free. Here we consider the case of two rigid boundaries at $z=0$ and $z=1$ (after non-dimensionalizing the various quantities in terms of depth d of the layer). At the rigid boundaries, the normal components of velocity w must vanish and no slip condition must be satisfied so that we must have

$$w = 0 \text{ and } Dw = 0 \text{ at } z = 0 \text{ and } z = 1 \quad \text{...(17)}$$

Needless to say that one can solve for free boundaries also but in that case the second boundary condition would be $D^2w=0$ and the solution would eventually lead to a dispersion relation different from one for the rigid boundaries. One may, however, mention here that the change in the boundary conditions would not be expected to materially affect the nature of the influence of the FLR effects on the considered stability problem.

THE EIGENVALUE SOLUTION

We now obtain the eigenvalue solution of Equation (16) subject to the boundary condition [Equation (17)]. The solution is

$$W = \sum_{i=1}^{4} C_i e^{m_i z} \quad \text{...(18)}$$

where C_i (i=1 to 4) are the constants and m_i (i=1 to 4) are the roots of the auxiliary equation. On applying the four boundary conditions [Equation (17)] to the solution Equation (18) we get the dispersion relation as follows:

$$\left(e^{m_2}-e^{m_1}\right)\left[\left(m_1-m_3\right)\left(m_4e^{m_4}-m_1e^{m_1}\right)+\left(m_4-m_1\right)\left(m_3e^{m_3}-m_1e^{m_1}\right)\right]$$
$$+\left(e^{m_3}-e^{m_1}\right)\left[\left(m_2-m_1\right)\left(m_4e^{m_4}-m_1e^{m_1}\right)+\left(m_1-m_4\right)\left(m_2e^{m_2}-m_1e^{m_1}\right)\right]$$
$$+\left(e^{m_4}-e^{m_1}\right)\left[\left(m_3-m_1\right)\left(m_2e^{m_2}-m_1e^{m_1}\right)+\left(m_1-m_2\right)\left(m_3e^{m_3}-m_1e^{m_1}\right)\right]=0 \quad \text{...(19)}$$

The roots m_1 to m_4 of the auxiliary equation are obtained by Brown's method of solving a quartic equation. The values of m_1 to m_4 are given by

$$m_{1,2} = \frac{1}{2}\left[\left(A_1-\beta\right) \pm \sqrt{\left(\beta - A_1\right)^2 + 4A_2 - 2S_1}\right] \quad \text{...(20)}$$

$$m_{3,4} = \frac{1}{2}\left[-\left(A_1+\beta\right) \pm \sqrt{\left(\beta + A_1\right)^2 - 4A_2 - 2S_1}\right] \quad \text{...(21)}$$

where A_1 and A_2 are given by

$$A_1 = \frac{1}{2}\frac{\beta}{A_2}\left[4k^2-3\beta^2-S_1-\frac{1}{n^2k^2v^2}\left(n^2+k^2V^2\right)^3\right], \text{ if } A_2 \neq 0 \quad \text{...(22)}$$

$$A_1 = \left[S_1+2\beta^2-4k^2-S_1+\frac{1}{n^2k^2v^2}\left(n^2+k^2V^2\right)^2\right]^{\frac{1}{2}}, \text{ if } A_2 = 0 \quad \text{...(23)}$$

$$A_2 = \left[\frac{S_1^2}{4}-4k^2\left(\beta^2+k^2\right)-\frac{1}{n^2v^2}\left(n^2+k^2V^2\right)\left(n^2+k^2V^2+g\beta\right)\right] \quad \text{...(24)}$$

and S_1 is the largest root of the cubic equation.

$$S_1^3 - b_2S_1^2 + b_1S_1 + b_0 = 0 \quad \text{...(25)}$$

The coefficients b_2, b_1 and b_0 in Equation (25) are

$$b_2 = 4k^2 - \beta^2 - \frac{1}{n^2k^2v^2}\left(n^2+k^2V^2\right)^2 \quad \text{...(26)}$$

$$b_1 = -\frac{1}{n^2k^2v^2}\left[n^2v^2k^2\left(8\beta^2k^2+16k^4+6\beta^4\right)+\left(n^2+k^2V^2\right)^2\left(2\beta^2+4k^2\right)+4g\beta k^2\left(n^2+k^2V^2\right)\right] \quad \text{... (27)}$$

$$b_0 = \frac{1}{n^2k^4v^4}\left[\begin{aligned}&n^4k^4v^4\left(16\beta^2k^4-8k^2\beta^4-9\beta^6+64k^6\right)-n^2k^2v^2\left(n^2+k^2V^2\right)^2\left(16\beta^2k^2+3\beta^4\right)\\&+g\beta k^2\left(n^2+k^2V^2\right)\left(16k^2-8\beta^2\right)-4g\beta k^2\left(n^2+k^2V^2\right)^3-\left(4k^2+\beta^2\right)\left(n^2+k^2V^2\right)^4\end{aligned}\right] \quad \text{...(28)}$$

The root S_1 of the cubic Equation (25) is given by

$$S_1 = \frac{1}{3}b_2 + \left[\left(P^3+Q^2\right)^{\frac{1}{2}}-Q\right]^{\frac{1}{3}} - \left[\left(P^3+Q^2\right)^{\frac{1}{2}}+Q\right]^{\frac{1}{3}} \quad \text{...(29)}$$

where P and Q are given by

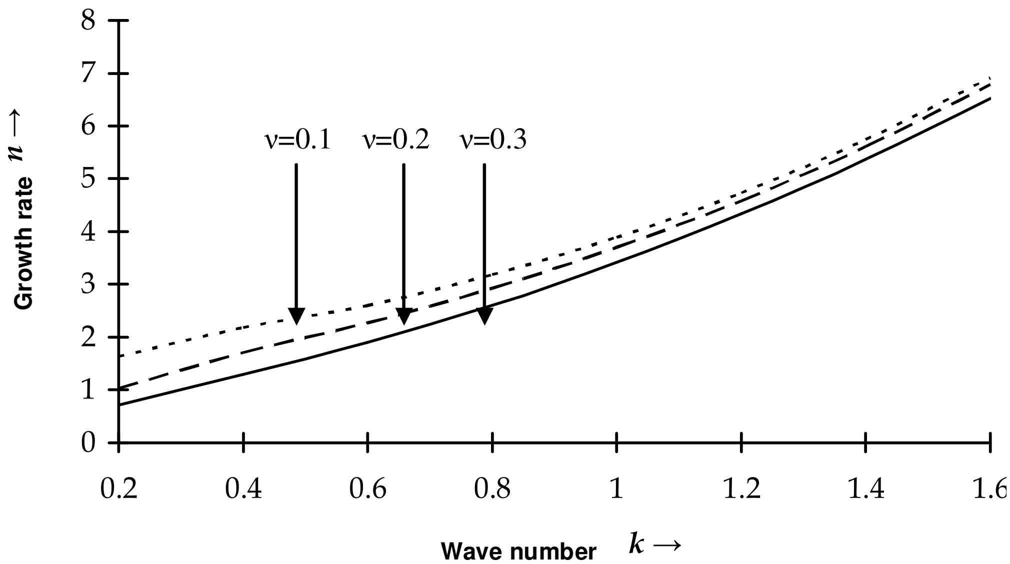

Figure 1. The variation of growth rate n with the wave number k for FLR ν = 0.1, 0.2 and 0.3.

$$P=\frac{1}{n^4k^4\nu^4}\left[\begin{array}{l} n^4k^4\nu^4\left\{\frac{16}{3}k^2\left(\beta^2-k^2\right)-2\beta^4\right\}-\left(n^2+k^2V^2\right)^4 \\ +n^2k^2\nu^2\left\{\left(n^2+k^2V^2\right)^2\left(\frac{8}{3}k^2-\frac{5}{3}\beta^2\right)-\frac{4}{3}g\beta k^2\left(n^2+k^2V^2\right)\right\} \end{array}\right] \quad \ldots(30)$$

$$Q=\frac{1}{2n^4k^6\nu^6}\left[\begin{array}{l} n^6k^6\nu^6\left\{-\frac{8}{27}\beta^2k^2+\frac{16}{3}\beta^4k^2-\frac{32}{3}\beta^2k^4-\frac{16}{27}k^4-\frac{1}{27}\beta^4-11\beta^6+\frac{256}{3}k^6\right\} \\ +n^4k^4\nu^4\left\{\begin{array}{l}\left(n^2+k^2V^2\right)^2\left(8\beta^2k^2+\frac{8}{28}k^2-16k^4-\frac{2}{27}\beta^2-\frac{20}{3}\beta^4\right) \\ +g\beta k^2\left(n^2+k^2V^2\right)\left(\frac{64}{3}k^2-\frac{4}{3}\beta^2k^2-8\right)\end{array}\right\} \\ +n^2k^2\nu^2\left\{\left(n^2+k^2V^2\right)^4\left(-\frac{11}{3}\beta^2+\frac{8}{3}k^2-\frac{1}{27}\right)-\frac{16}{3}g\beta k^2\left(n^2+k^2V^2\right)^3-\left(n^2+k^2V^2\right)^6\right\} \end{array}\right] \quad \ldots(31)$$

DISCUSSION

The dispersion relation [Equation (19)] is quite complex as it involves terms of the form $\exp(m_1)$ etc. and the expressions for the roots m_1 to m_4 [Equations (20) and (21)] involve n, k, ν and V^2. We have therefore solved Equation (19) numerically using MATHEMATICA 6.0 to examine, qualitatively, the influence of FLR on the gravity-driven instability of plasma. The numerical calculations are presented in Figure 1, where we plot the growth rate n against wave number k for ν = 0.1, 0.2 and 0.3 when $V^2 = 1$ and $\beta = 0.1$.

From Figure 1, we observe that the growth rate n decreases on increasing ν for a fixed value of the wave number k. The effect of the FLR is therefore stabilizing in the buoyancy-driven instability of plasma as in the case of Rayleigh-Taylor instability problem. The present study thus clearly demonstrates the suppression of the buoyancy-driven instability of semi-infinite plasma by finite Larmor radius effects.

Nomenclature: $\vec{g}$ (0, 0, -g), Acceleration due to gravity, $\vec{k}$, wave number; p, fluid pressure; t, time; $\vec{v}(u, \text{v}, w)$, fluid velocity; $\vec{H}\left(H_0(z), 0, 0\right)$, magnetic field; $\vec{h}\left(h_x, h_y, h_z\right)$, perturbation in magnetic field; P_{ij}, stress tensor; n, growth rate (a complex number); N, number density; T, temperature of the particle; w_H, Ion gyration frequency; x, y, z, space co-ordinates.

Greek letters: ν, kinematic viscosity; ρ, density; β, stratification constant; V^2, square of the Alfven velocity.

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