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*Full Length Research Paper*

# Newtonian heating and magnetohydrodynamic effects in flow of a Jeffery fluid over a radially stretching surface

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**The combined effects of Newtonian heating and magnetohydrodynamics (MHD) in a flow of a Jeffery fluid are analyzed in stagnation point flow over a radially stretching surface. The governing equations are modeled by invoking boundary layer analysis. The computed solution by a homotopy approach is valid in the spatial domain. Graphical results for the velocity and temperature fields are displayed and discussed. The local Nusselt number for various values of embedding parameters is shown. It is noted that the magnetic field retards the flow, whereas Newtonian heating acts as a boosting agent in order to increase the temperature of the fluid.**

**Key words:** Radially stretching surface, Newtonian heating, Jeffery fluid, magnetohydrodynamics (MHD).

## INTRODUCTION

The flows of non-Newtonian fluids are significant in many industrial and engineering applications. Certain paints, salt solutions, molten polymers, ketchup, custard, toothpaste, starch suspensions, paints, blood at low shear rate and shampoo are few examples of the non-Newtonian fluids. Such fluids in view of diverse characteristics cannot be examined by using single constitutive relationship. These fluids have been classified into three types, namely, differential, rate and integral. A vast amount of literature is available on the flows of non-Newtonian fluids. However, the rate type fluids amongst these are not given proper attention. Recently, various researchers studied the flows of rate type fluids under different flow aspects including suction/ injection at the boundaries, magnetohydrodynamics (MHD), heat and mass transport process, thermal-diffusion and diffusion-thermo effects, thermal radiations, etc (Fetecau et al., 2010a, b; Wang and Tan, 2011; Hayat and Awais, 2011; Hayat et al., 2010a).

Fluid motion in the region of a stagnation point exists on all moving solid bodies. The role of stagnation point is important, because the separation streamlines passing through them describe different flow regions. Thus, problems studying the stagnation point flow over a stretching surface are a classic problem in fluid mechanics. Initially, Hiemenz (1911) presented the steady flow in the neighborhood of a stagnation point. Later, the seminal work of Hiemenz was extended by various researchers. For instance, Attia (2007) presented the axisymmetric stagnation point flow towards a stretching surface in the presence of a uniform magnetic field with heat generation. Unsteady stagnation point flow over a plate moving along the direction of flow impingement has been studied by Zhong and Fang (2011). Hayat et al. (2011a) investigated the effects of mass transfer on the stagnation point flow of an upper-convected Maxwell (UCM) fluid. Effects of suction/ blowing on steady boundary layer stagnation point flow and heat transfer towards a shrinking sheet with thermal radiation has been investigated by Bhattacharyya and Layek (2011). Bachok et al. (2011) analyzed the flow in the region of stagnation point towards a stretching sheet with homogeneous-heterogeneous reactions effects. Rosali et al. (2011) studied stagnation point flow and

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heat transfer over a stretching/shrinking sheet in a porous medium. The boundary layer of an unsteady incom-pressible stagnation-point flow with mass transfer has been analyzed by Fang et al. (2011). Recently, stagnation point flow of a Burgers' fluid over a stretching surface was examined by Hayat et al. (2011b).

It is known that heat transfer is concerned with the exchange of thermal energy from one physical system to another. Merkin (1994) pointed out four common heating processes specifying the wall-to-ambient temperature distributions. These are (1) constant or prescribed wall temperature (CWT), (2) constant or prescribed surface heat flux (CHF), (3) conjugate conditions, where heat is supplied through a bounding surface of finite thickness and finite heat capacity, and (4) Newtonian heating (NH), where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and which is usually termed conjugate convective flow. Generally, the boundary conditions 1 and 2 were used in modeling the convection boundary layer flow problems. However, recently, Newtonian heating conditions 4 have been used by researchers in view of their practical applications in several engineering devices, for instance in a heat exchanger where the conduction in a solid tube wall is greatly influenced by the convection in the fluid flowing over it. Further, for conjugate heat transfer around fins where the conduction within the fin and the convection in the fluid surrounding it must be simultaneously analyzed in order to obtain vital design information and also in convection flows set up when the bounding surfaces absorb heat by solar radiation. Free convection flow above a nearly horizontal surface in a porous medium subject to Newtonian heating has been studied by Lesnic et al. (2004). Unsteady free convection flow past an impulsively started vertical surface in the presence of Newtonian heating was addressed by Chaudhary and Jain (2006). Forced convection boundary layer flow at a forward stagnation point with Newtonian heating was studied by Salleh et al. (2009). Recently, Niu et al. (2010) analyzed the stability of thermal convection of an Oldroyd-B fluid in a porous medium with Newtonian heating.

In this paper, a new dimension is added by including Newtonian heating effects over radially stretching surface. To the best of our knowledge, no such attempt has been presented. A Jeffery fluid model (Kothandapani and Srinivas, 2008; Hayat et al., 2011c) has been selected here, because it has a different rheology than a viscous fluid. The homotopy analysis method (HAM) (Liao, 2004; Rashidi et al., 2009, Hayat et al., 2010b, Hayat et al., 2011d) is used for the solutions development. It is due to the reason that the HAM does not depends upon any small/large physical parameters in the problem. It is usually noted that perturbation approximations are valid only for nonlinear problems with weak nonlinearity, but when nonlinearity is strong, then perturbation approximations of the nonlinear problems often break down. In such cases, HAM provides a simple way to ensure the convergence of the series solution so that one can obtain accurate enough approximations even in the nonlinear problems. Further, it provides great freedom for selecting auxiliary linear operator, so that one can efficiently finds the approximate solution of a nonlinear problem which is correct up to 6 or 7 decimal places. Various graphical and numerical results are presented to analyze the flow analysis. Finally, various outcomes are presented to summarize the flow analysis.

## MATHEMATICAL ANALYSIS

Two dimensional stagnation point flow of Jeffery fluid over a radially stretching surface is considered. The surface coincides with the plane $z=0$, whereas the fluid occupies the region $z\geq 0.$ A uniform magnetic field of strength $B_0$ is applied along the $y-$axis. The velocity distribution (Attia, 2007) in the flow close to the stagnation point is given by $U_e(r)=ar,\ W_e(z)=-2az$ and the velocity of the stretching sheet is $U_w(r)=cr$, where $a$ and $c$ are the positive constants and r is the radial direction Figure 1.

The constitutive relationships in a Jeffery fluid (Kothandapani and Srinivas, 2008; Hayat et al., 2011c) are:

$$\mathbf{T}=-p\mathbf{I}+\mathbf{S}, \tag{1}$$

$$\mathbf{S}=\frac{\mu}{1+\lambda_1}\left(\dot{r}+\lambda_2\ddot{r}\right), \tag{2}$$

where $p$ denotes the pressure, $\mathbf{I}$ the identity tensor, $\mu$ the dynamic viscosity, $\lambda_1$ the ratio of relaxation and retardation times and $\lambda_2$ is the retardation time. The quantities $\dot{r}$ and $\ddot{r}$ are defined by

$$\dot{r}=\nabla\mathbf{V}+\left(\nabla\mathbf{V}\right)^T, \tag{3}$$

$$\ddot{r}=\frac{d}{dt}(\dot{r})=\frac{\partial}{\partial t}(\dot{r})+(\mathbf{V}\cdot\nabla)\dot{r}. \tag{4}$$

Using the equation of continuity and motion, the resulting boundary layer equations are

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{5}$$

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=\frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial z^2}+\lambda_2\left(u\frac{\partial^3 u}{\partial r\partial z^2}+w\frac{\partial^3 u}{\partial z^3}+\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial r\partial z}+\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\right)\right] +\frac{\sigma B_0^2}{\rho}\left(u_e-u\right)+u_e\frac{du_e}{dr}, \tag{6}$$

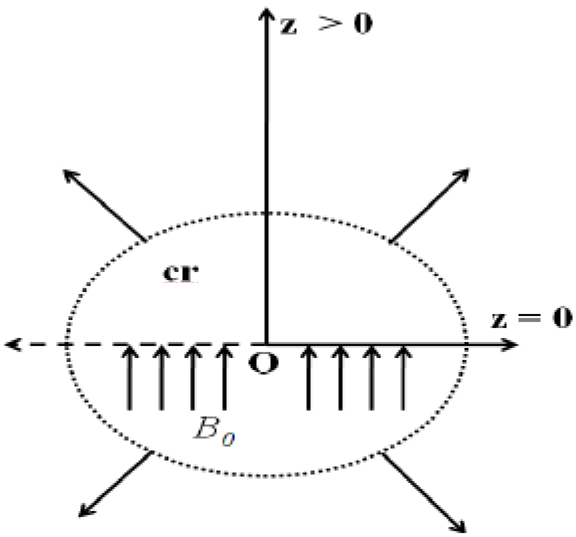


**Figure 1.** Physical model.

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial z^2}\right), \tag{7}$$

with the conditions

$$u=U_w(r)=cr,\ w=0,\frac{\partial T}{\partial z}=-h_sT \text{ at } z=0,$$
$$u\to U_e(r)=ar,\quad T\to T_\infty \text{ as } z\to\infty, \tag{8}$$

where $u$ and $w$ are the velocity components along the radial $(r)$ and axial $(z)$ directions, respectively, $T$ is the temperature of the fluid, $c_p$ is the specific heat, $k$ is the thermal conductivity, $\rho$ is the density, $\nu$ is the kinematic viscosity, $h_s$ is the heat transfer parameter and $T_\infty$ is the ambient temperature. The following transformations,

$$\eta=\sqrt{\frac{c}{\nu}}z,\ u=crf'(\eta),\ w=-2\sqrt{c\nu}\,f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_\infty}, \tag{9}$$

satisfy the continuity equation identically and Equations 6 to 8 are reduced as follows:

$$f'''+\beta\left(f''^2-ff''''-2ff'''''\right)+(1+\lambda_1)\begin{bmatrix}M^2(A-f')+A^2\\+2ff''-f'^2\end{bmatrix}=0, \tag{10}$$

$$\theta''+2\Pr f\theta'=0, \tag{11}$$

$$f'(\eta)=1,\ f(\eta)=0,\ \theta'(\eta)=-\gamma(1+\theta(\eta)) \quad \text{at} \quad \eta=0,$$
$$f'(\eta)=A,\ \theta(\eta)=0 \quad \text{as} \quad \eta\to\infty, \tag{12}$$

where $\beta$ denotes the Deborah number, $M$ the Hartman number, $A$ the ratio of free stream velocity to the stretching velocity, $\Pr$ the Prandtl number and $\gamma$ the conjugate parameter for Newtonian heating. These are defined as:

$$\beta=\lambda_2c,\ M^2=\frac{\sigma B_0^2}{\rho c},\ A=\frac{a}{c},\ \Pr=\frac{\nu}{\alpha},\ \gamma=h_s\sqrt{\frac{\nu}{c}}. \tag{13}$$

The local Nusselt number $Nu_r$ is defined as:

$$Nu_r=\frac{rq_w}{k(T-T_\infty)}, \tag{14}$$

where the heat flux is:

$$q_w=-k\left(\frac{\partial T}{\partial z}\right)_{z=0}, \tag{15}$$

$k$ the thermal conductivity. In dimensionless form we get:

$$(R_{e_r})^{-1/2}Nu_r=\gamma\left(1+\frac{1}{\theta(0)}\right), \tag{16}$$

where $R_{er}=(ar^2/\nu)$ denotes the local Reynolds number.

## RESULTS AND DISCUSSION

Equations 10 and 11 were solved in combination with the boundary conditions of Equation 12 by employing the HAM (Liao, 2004; Rashidi et al., 2009; Hayat et al., 2010b, 2011d). Here $\hbar_f$ and $\hbar_\theta$ are selected as auxiliary parameters for the functions $f$ and $\theta$, respectively in order to adjust and control the convergence of the obtained solutions. We have plotted the $\hbar$-curves in Figures 2 and 3 to determine the permissible values of involved auxiliary parameters. The ranges for admissible values of $\hbar_f$ and $\hbar_\theta$ are $-0.9\le\hbar_f\le-0.3$ and $-1.0\le\hbar_\theta\le-0.2$, respectively. The series solutions converge in the whole region of $\eta$ $(0<\eta<\infty)$ for $\hbar_f=-0.5=\hbar_\theta$.

Table 1 shows that the convergent solution is obtained at the 15th order of approximation. The behaviors of various parameters on the velocity and temperature profiles are addressed as shown in Figures 4 to 9. Figure 4 delineates that an increase in $A$ yields an increase in velocity and the boundary layer thickness $(0\le A<1)$. It is also noted that the boundary layer thickness vanishes when $A=1.0$. Furthermore, when the free stream

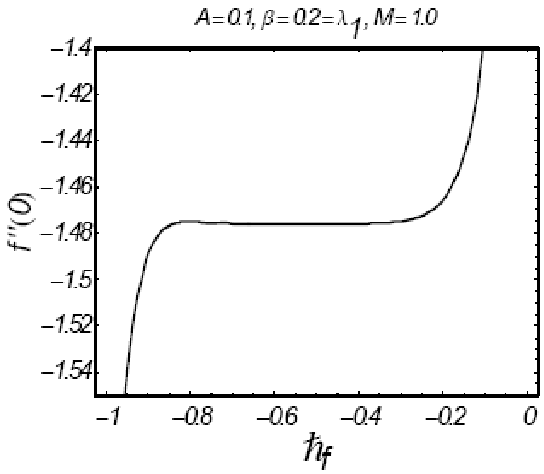


**Figure 2.** $\hbar$ -curve for $f(\eta)$.

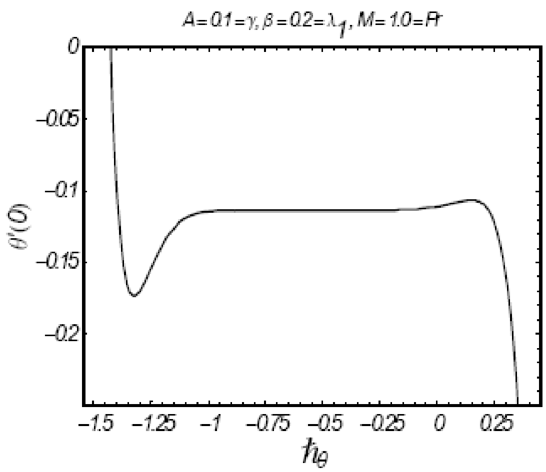


**Figure 3.** $\hbar$ -curve for $\theta(\eta)$.

**Table 1.** Convergence of the homotopy solutions for different orders of approximation when $A = 0.1 = \gamma$, $M = 1.0$, $\beta = 0.2 = \lambda_1$ and $\Pr = 1.0$.

| Order of approximation | $-f''(0)$ | $-\theta'(0)$ |
|---|---|---|
| 1 | 1.3230000 | 0.1120370 |
| 5 | 1.4727158 | 0.1134814 |
| 10 | 1.4759301 | 0.1135826 |
| 15 | 1.4759632 | 1.1135742 |
| 25 | 1.4759635 | 0.1135734 |
| 35 | 1.4759635 | 0.1135734 |

velocity is greater than the velocity of the stretching sheet, that is, $A > 1$, the velocity increases and the boundary layer thickness decreases by increasing $A$.

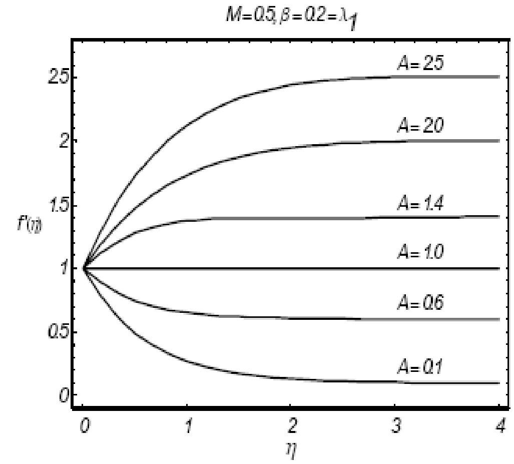


**Figure 4.** Influence of $A$ on $f'$.

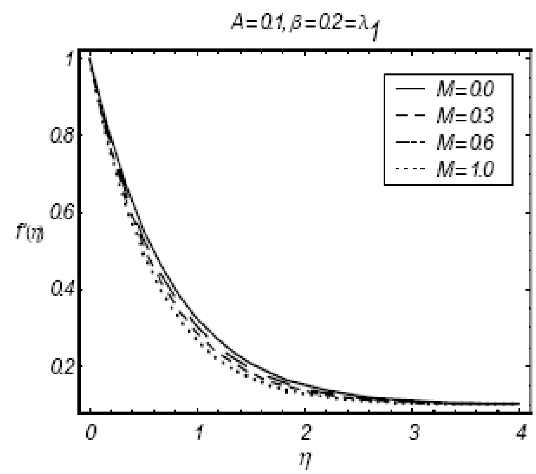


**Figure 5.** Influence of $M$ on $f'$.

Physically, the larger values of $A$ accompany with the higher free stream velocity results into an increase in the fluid motion. The effects of $M$ on $f'$ are plotted as shown in Figure 5. It has been noticed that the magnetic field retards the flow. Physically, when magnetic field is applied to any fluid, then the apparent viscosity of the fluid increases to the point of becoming a viscoelastic solid. It is of great interest that the yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of an electromagnet, which gives rise to many possible control-based applications, including MHD power generation, electromagnetic casting of metals, MHD ion propulsion, etc. The influence of the Deborah number $\beta$ on $f'$ is as shown in Figure 6. It is observed that the

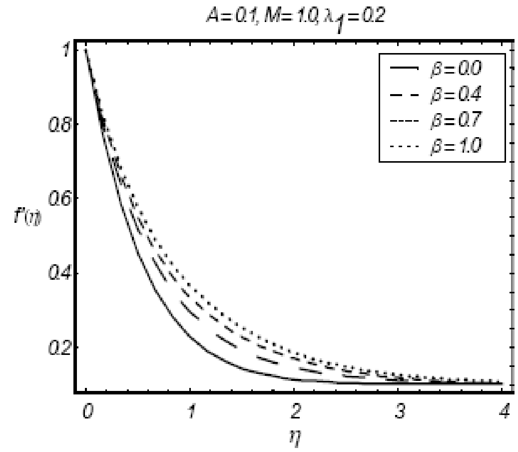


**Figure 6.** Influence of $\beta$ on $f'$.

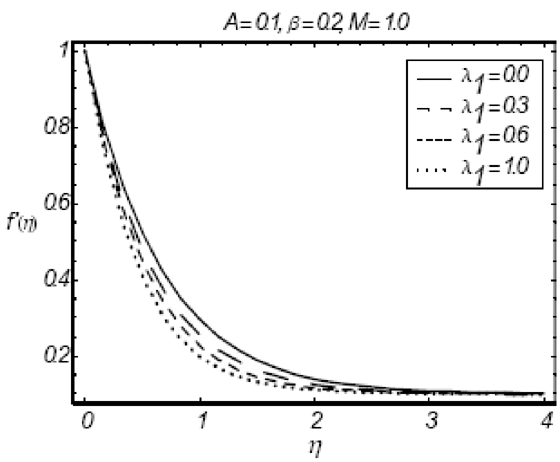


**Figure 7.** Influence of $\lambda_1$ on $f'$.

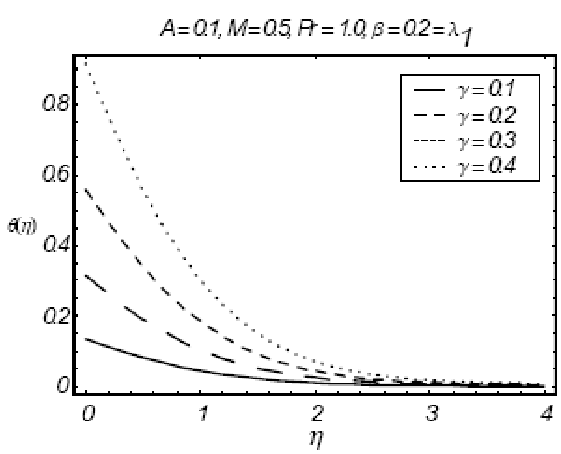


**Figure 8.** Influence of $\gamma$ on $\theta$.

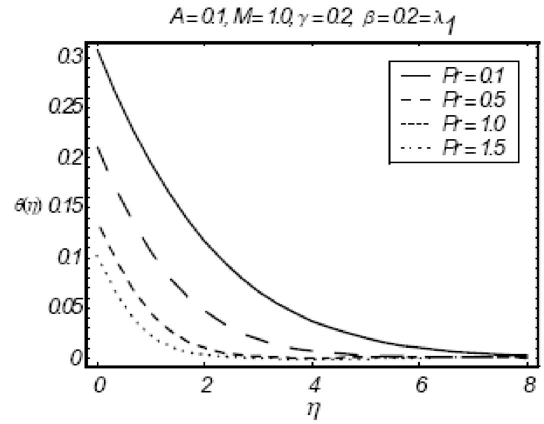


**Figure 9.** Influence of $\Pr$ on $\theta$.

velocity of the fluid and the associated boundary layer thickness increase with an increase in $\beta$. Since the Deborah number $\beta$ is dependent upon $\lambda_2$ (retardation time), physically larger retardation time of any material makes it less viscous resulting in an increase in its motion. Figures 7 portrays the effects of $\lambda_1$ on $f'$. It is seen that an increase in $\lambda_1$ being a viscoelastic parameter retards the flow. Figures 8 and 9 show the effects of $\gamma$ and $\Pr$ on the dimensionless temperature $\theta(\eta)$. The influence of $\gamma$ on the temperature is observed as shown in Figures 8. A significant deviation in the temperature profiles are observed for large values of $\gamma$. Further, the temperature and the thermal boundary layer thickness are increasing functions of $\gamma$. An increase in $\gamma$ results in an increase in the heat transfer rate from the surface which raises the temperature. An increase in $\Pr$ on the temperature is as shown in Figure 9. The definition of $\Pr$ $(=\upsilon/\alpha)$ indicates that a large Prandtl number has relatively lower thermal conductivity. Therefore, a reduction in the thermal boundary layer thickness is noticed for large values of $\Pr$. Interestingly, variations in the temperature profiles are more prominent for the smaller values of $\Pr$ when compared with larger values.

Table 2 depicts a comparison of the present results with the already published work in the limiting sense (Attia, 2007). It is noted that the present results are in a good agreement with the published results (Attia, 2007). Numerical values of the local Nusselt number for various values of embedding parameters are displayed in Table 3. It is noticed that the local Nusselt number is a decreasing function of $\lambda_1$ and $M$. However, an increase in $A$, $\beta$ and $\Pr$ causes an increase in the magnitude of the local Nusselt number.

**Table 2.** Comparison of the values of $f''(0)$ in the limiting case when $\beta = 0.0 = \lambda_1$.

| A | Attia (2007) | | Present results | |
|---|---|---|---|---|
| | **M=0.0** | **M=1.0** | **M=0.0** | **M=1.0** |
| 0.1 | -1.1246 | -1.4334 | -1.124601 | -1.433473 |
| 0.5 | -0.7534 | -0.9002 | -0.753297 | -0.899974 |
| 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 1.1 | 0.1821 | 0.2070 | 0.181935 | 0.206817 |
| 1.5 | 1.0009 | 1.1157 | 0.993184 | 1.094581 |

**Table 3.** Local Nusselt number for various values of embedding parameters.

| $A$ | $\beta$ | $\lambda$ | $M$ | Pr | $(R_{e_r})^{-1/2} Nu_r$ |
|---|---|---|---|---|---|
| 0.0 | 0.2 | 0.2 | 1.0 | 1.0 | 0.78756 |
| 0.1 | - | - | - | - | 0.83673 |
| 0.2 | - | - | - | - | 0.87829 |
| 0.3 | - | - | - | - | 0.91558 |
| 0.1 | 0.0 | - | - | - | 0.80931 |
| - | 0.2 | - | - | - | 0.83673 |
| - | 0.4 | - | - | - | 0.85830 |
| - | 0.2 | 0.0 | - | - | 0.85818 |
| - | - | 0.2 | - | - | 0.83673 |
| - | - | 0.4 | - | - | 0.81797 |
| - | - | 0.2 | 0.5 | - | 0.85865 |
| - | - | - | 1.0 | - | 0.83673 |
| - | - | - | 1.5 | - | 0.81762 |
| - | - | - | 1.0 | 0.5 | 0.54296 |
| - | - | - | - | 1.0 | 0.83675 |
| - | - | - | - | 1.5 | 1.08175 |

## Conclusions

Axisymmetric stagnation point flow of a Jeffery fluid is considered. The flow is induced by a radially stretching surface. The concept of Newtonian heating is analyzed. The solutions of highly nonlinear differential equations are computed by using an efficient analytic approach, namely, HAM. The final outcomes are as follow:

1. Magnetic field retards the flow.
2. Newtonian heating acts as a boosting agent in order to enhance the temperature of the fluid.
3. Velocity of the fluid can be boosted by increasing its retarding time.
4. Prandtl number should be increased in order to control the thermal boundary layer.
5. Constructed tables show that the solution is convergent and already published results can be obtained in the limiting sense.

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