

*Full Length Research Paper*

# Level studies of $^{71}\text{Ge}$ via $^{71}\text{Ga}(\text{P}, \text{n}\gamma)^{71}\text{Ge}$ reaction and density of discrete levels in $^{71}\text{Ge}$

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The excited states of  $^{71}\text{Ge}$  have been investigated via the  $^{71}\text{Ga}(\text{P}, \text{n}\gamma)^{71}\text{Ge}$  reaction with the proton beam energies from 2.5 to 4.3 MeV. The parameters of the nuclear level density formula have been determined from the extensive and complete level scheme for  $^{71}\text{Ge}$ . The Bethe formula for the back-shifted Fermi gas model and the constant temperature model are compared with the experimental level densities.

**Key words:** Level schemes of  $^{71}\text{Ge}$ ,  $^{71}\text{Ga}(\text{P}, \text{n}\gamma)^{71}\text{Ge}$  reaction, angular distribution, Fermi gas model.

## INTRODUCTION

Information about the  $^{71}\text{Ge}$  nucleus has been obtained from experimental studies by  $\beta$ -decay (Meyer et al., 1990),  $(\text{P}, \text{n}\gamma)$  (Malan et al., 1974),  $(\alpha, \text{n}\gamma)$  reactions (Eberth et al., 1976) as well as neutron transfer  $(\text{P}, \text{d})$  (Fournier et al., 1973) and  $(\text{d}, \text{P})$  reaction studies (Bieszk et al., 1977). On the other hand, in all statistical theories, the nuclear level density is the most characteristic quantity and plays a major role in the study of nuclear structure. Most experimental data on nuclear level density have been analyzed with analytical functions of the level density.

The Fermi gas model has often been used in the study of statistical treatment of nuclear properties. The philosophy of the model is to replace the complicated nucleon-nucleon interaction by an average potential.

In this work, we have provided additional experimental information about existing level structure of  $^{71}\text{Ge}$  through the  $^{71}\text{Ga}(\text{P}, \text{n}\gamma)^{71}\text{Ge}$  reaction and then measured the spin values and the multipole mixing ratios from the angular distributions of de-excitation  $\gamma$ -rays. Finally, we have determined nuclear level density parameters of the Bethe formula and constant temperature model for the  $^{71}\text{Ge}$  nucleus.

A thick self-supporting pellet of spectroscopic ally pure natural Ga was used as a target. The proton beam of 2.5 to 4.3 MeV energies (Cyclotron Laboratory at Panjab University, India) was used for bombardment to excite the levels of  $^{71}\text{Ge}$  through the  $^{71}\text{Ga}(\text{P}, \text{n}\gamma)^{71}\text{Ge}$  reaction (Q value is = - 1.017 MeV). The target was placed at an angle of  $45^\circ$  with respect to the beam direction and was thick enough to stop incident protons. The angular distributions were measured at 0, 30, 45, 55, 75 and  $90^\circ$ . The  $\gamma$ -rays were detected with a  $70\text{ cm}^3\text{J}$  coaxial HPGe detector with a resolution of 1.9 keV for the 1332 keV  $\gamma$ -ray of  $^{60}\text{Co}$ . The excitation functions of various  $\gamma$ -rays have been measured at  $55^\circ$  in the range of 2.5 to 4.3 MeV beam energies to ascertain that the channel of the compound decay is dominant as compared to the coulomb excitation at the incident proton energy of 4.3 MeV. The other details of the experimental procedure are given in our previous publications (Behkami et al., 2009; Kakavand and Singh, 2002; Razavi and Kakavand, 2011).

## DATA ANALYSIS

The gamma-ray spectra were analyzed using the computer code PEAKFIT (Singh et al., 1994). A typical gamma-ray spectrum at  $90^\circ$  for incident proton energy of 4.3 MeV is given in our previous publication (Kakavand and Singh, 2002). The excitation functions of all the observed gamma-rays were analyzed carefully as a function of energy and those from the  $(\text{P}, \text{n}\gamma)$  reaction were easily identified with a characteristic rise above

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their threshold energy. The relative branching ratios used for further analysis are the weighted average of the respective values at 4.0 and 4.3 MeV bombarding energies.

The extraction of multipole mixing ratios of the observed transitions and the assignment of spin values to the excited levels were made from the  $\chi^2$ -fitting of angular distribution data at 4.3 MeV proton beam energy. The optical model parameter sets given by Perey and Perey (1976), which are based on the results of Perey (1963) for protons and Wilmore and Hodgson (1964) for neutrons, were used to calculate the transmission coefficients. Besides the observed neutron channel, all known (P,  $P'\gamma$ ) channels and (P,  $n\gamma$ ) channels were included as competing channels. The Moldauer was also taken into account with the fluctuation correction (Rotbard et al., 1983). The typical experimental angular distributions of some of the observed transitions together with theoretical curves for different possible spins of the levels and the respective  $\chi^2$ -fitting are given in our previous publication (Kakavand and Singh, 2002). The 0.1% confidence limit was used to exclude unacceptable spins and  $\delta$  values. The coefficients  $A_2$  and  $A_4$  from the polynomial fits to the experimental distribution along with the multipole mixing ratios ( $\delta$ ) are given in Table 1.

## STATISTICAL FORMULA

The statistical considerations are based on the analysis of level densities. The nuclear temperature  $T$  can be defined by the nuclear level density  $\rho(E)$  (Ericson, 1959)

$$\frac{1}{T} = \frac{d}{dE} \ln \rho(E) \quad (1)$$

The integration yields the constant temperature Fermi gas formula

$$\rho(E) = \frac{1}{T} \exp\left(\frac{E - E_0}{T}\right) \quad (2)$$

The nuclear temperature  $T$  and the ground state back shift  $E_0$  can be determined using the experimental data on the level density. The Bethe formula of the level density (Bethe, 1937) for the back-shifted Fermi gas model (Dilg et al., 1973; Egidy et al., 1988) can be written as

$$\rho(E) = \frac{\exp(2\sqrt{a(E - E_1)})}{12\sqrt{2}\sigma a^{1/4}(E - E_1)^{5/4}} \quad (3)$$

In this case, the level density parameter  $a$  and the ground

state back shift  $E_1$  is obtained by a fit to the experimental results. The distribution of spins  $J$  is determined; Bethe, 1937) by the spin cut-off parameter  $\sigma^2$

$$f(J) = \exp\left(\frac{-J^2}{2\sigma^2}\right) - \exp\left(\frac{-(J+1)^2}{2\sigma^2}\right) \approx \frac{2J+1}{2\sigma^2} \exp\left[-\left(J+\frac{1}{2}\right)^2 / 2\sigma^2\right] \quad (4)$$

With this spin distribution, the spin-dependent level density is

$$\rho(E, J) = \rho(E) f(J) \quad (5)$$

where  $\sigma^2$  is related to an effective moment of inertia  $I_{eff}$  and to the nuclear temperature  $T$  (Ericson, 1959; Dilg et al., 1973)

$$\sigma^2 = \frac{I_{eff} T}{\hbar^2} \quad (6)$$

The nuclear moment of inertia for a rigid body is  $I_{Rigid} = (2/5)MR^2$  (where  $M = A$ , the amu nuclear mass;  $R = 1.25A^{1/3}$  fm, the nuclear radius) resulting in (Behkami et al., 2009)

$$\sigma^2 = 0.0150 A^{5/3} T \quad (7)$$

Gilbert and Cameron (1965) calculated the spin cut-off parameter for the Bethe formula with the reduced moment of inertia,

$$\sigma^2 = 0.0888 A^{2/3} \sqrt{a(E - E_1)} \quad (8)$$

## FIT OF THE LEVEL DENSITY FORMULA

Each of the two level density formulas has two free parameters. It may be obtained by fitting the measured level schemes experimentally. We have applied these formulas to the measured level scheme for  $^{71}\text{Ge}$  reported in Table 1. Our best fit values obtained using the Bethe formula are the level density parameter  $a = 5.38 \text{ MeV}^{-1}$  and the back shift  $E_1 = -2.196 \text{ MeV}$ . The results obtained using the constant temperature formula are  $T = 1.689 \text{ MeV}$  and the back shift  $E_0 = -4.4134 \text{ MeV}$ .

The accumulated levels  $N(E)$  as a function of energy are plotted in Figures 1 and 2. The examination of these

**Table 1.** Level energies and the results of the angular distribution measurements in  $^{71}\text{Ge}$ .

Transition	Gamma rays [keV]	$J_i^\pi \rightarrow J_f^\pi$	Multipole mixing ratios, s	$A_2$	$A_4$
525.1→198.4	326.7	$\frac{5^+}{2} \rightarrow \frac{9^+}{2}$	$0.10^{+0.05}_{-0.02}$	$-0.009(5)^*$	0.010(5)
589.8→198.4	391.3	$\frac{7^+}{2} \rightarrow \frac{9^+}{2}$	$-0.11^{+0.03}_{-0.02}$	0.004(6)	-0.017(6)
708.0→0	708.0	$\frac{3^-}{2} \rightarrow \frac{1^-}{2}$	$-0.61^{+0.78}_{-2.2}$	-0.115(12)	0.060(12)
747.1→175.0	572.1	$\frac{5^-}{2} \rightarrow \frac{5^-}{2}$	$-0.3^{+0.05}_{-0.08}$	0.026(22)	-0.027(22)
831.1→525.1	306.0	$\frac{3^-}{2} \rightarrow \frac{5^+}{2}$	$0.1^{+0.05}_{-0.02}$ or $-11.7^{+3.4}_{-4.4}$	-0.003(20)	-0.016(20)
831.1→0	831.1	$\frac{3^-}{2} \rightarrow \frac{1^-}{2}$	$-0.5 \pm 0.1$	-0.114(13)	0.032(13)
1026.6→499.8	526.6	$\frac{5^-}{2} \rightarrow \frac{3^-}{2}$	$-0.75^{+0.14}_{-0.12}$	-0.173(13)	-0.006(13)
1026.6→0	1026.6	$\frac{5^-}{2} \rightarrow \frac{1^-}{2}$	$12.5 \pm 0.25$	0.359(75)	-0.237(75)
1096.0→175.0	921.0	$\frac{3^-}{2} \rightarrow \frac{5^-}{2}$	$-1.0^{+0.57}_{-0.80}$	-0.326(25)	0.093(25)
1212.2→499.8	712.5	$\frac{5^-}{2} \rightarrow \frac{3^-}{2}$	$-0.49^{+0.09}_{-0.26}$ $-1.54 \pm 0.2$	0.037(56)	-0.060(64)
1298.6→0	1298.6	$\frac{3^-}{2} \rightarrow \frac{1^-}{2}$	$-0.61^{+0.74}_{-1.8}$	-0.154(20)	0.090(30)
1377.8→1096.0	281.8	$\frac{5^-}{2} \rightarrow \frac{3^-}{2}$	$2.9 \pm 0.1$	0.103(950)	0.005(950)
1406.5→175.0	1231.6	$\frac{5^-}{2} \rightarrow \frac{5^-}{2}$	$0.49^{+0.15}_{-0.11}$	0.078(98)	0.011(103)
1406.5→499.8	906.3	$\frac{5^-}{2} \rightarrow \frac{3^-}{2}$	$-0.2 \pm 0.1$	-0.095(128)	0.002(139)
1422.1→175.0	1247.1	$\frac{9^-}{2} \rightarrow \frac{5^-}{2}$	$0.1 \pm 0.07$	0.230(63)	-0.009(63)
1558.8→525.1	1033.7	$\frac{5^+}{2} \rightarrow \frac{5^+}{2}$	$0.48^{+0.26}_{-0.18}$	0.072(39)	0.005(44)
1566.1→589.8	975.5	$\frac{9^+}{2} \rightarrow \frac{7^+}{2}$	$-1.54^{+0.22}_{-0.16}$	-0.177(98)	-0.004(108)
1743.3→808.0	935.6	$\frac{3^-}{2} \rightarrow \frac{1^-}{2}$	$-0.5^{+0.4}_{-0.8}$	-0.055(129)	0.005(143)
1743.3→0	1743.3	$\frac{3^-}{2} \rightarrow \frac{1^-}{2}$	$-11.59^{+4.3}_{-3.5}$	0.015(30)	-0.002(31)

\*The numbers in the parenthesis in the last two columns indicate the uncertainties in the deduced values of the coefficients.

figures shows that the agreement between the theory and experiment is very good and both formulas fit the measured level scheme equally well.

Furthermore, the spin cut-off parameter  $\sigma^2$  has been obtained by fitting the known spin distribution,  $N(J)$  of Table 1 with the theoretical expression (4) shown in

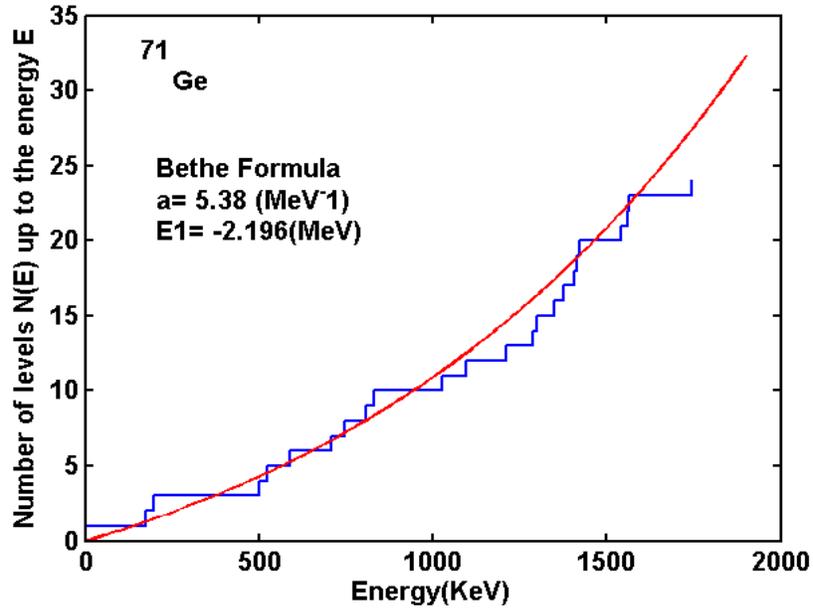


Figure 1. Plot of the number of levels  $N(E)$  up to the energy  $E$  for  $^{71}\text{Ge}$  together with the fitted curve calculated by the Bethe formula.

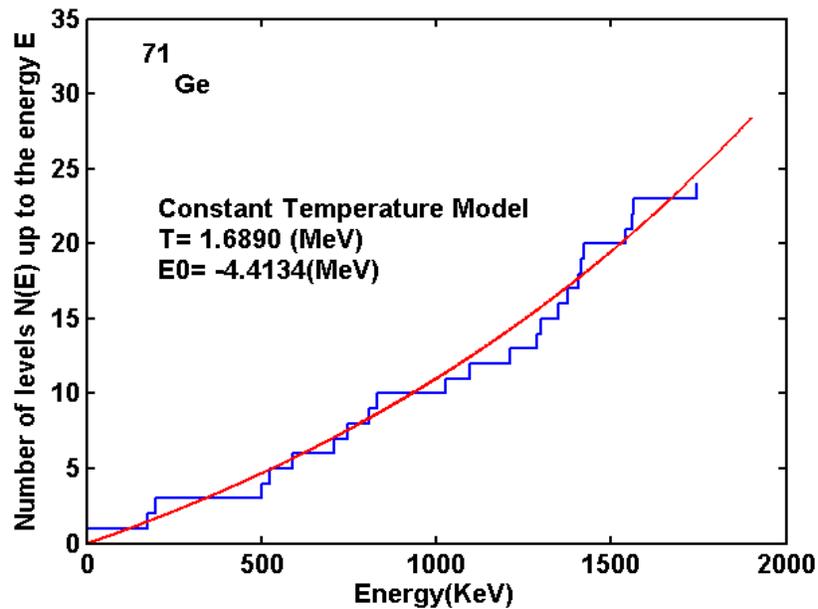
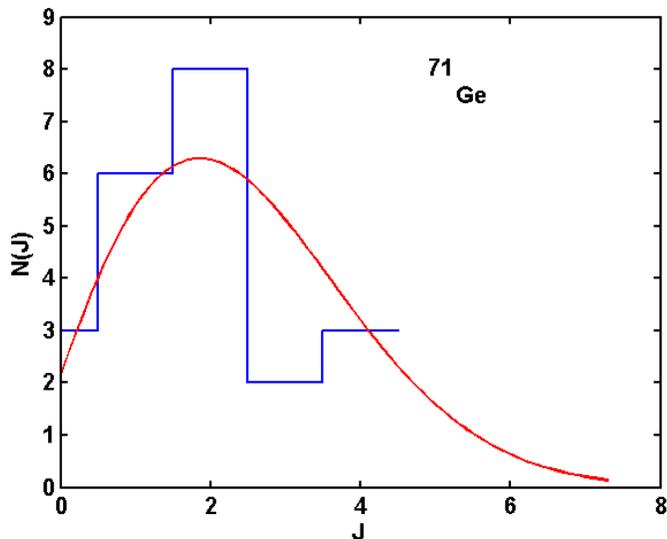


Figure 2. Plot of the number of levels  $N(E)$  up to the energy  $E$  for  $^{71}\text{Ge}$ , together with the fitted curve calculated by the constant temperature model.

Figure 3. Our best fit value of this parameter is  $\sigma^2 = 5.540$ . This deduced value is different from its corresponding rigid body value of  $\sigma^2 = 11.857$ . This finding is opposite to the claim made by some authors that the spin cut-off parameter reduces to its rigid body value at lower energies.

## Conclusion

The purpose of the present study was to provide additional experimental information on the existing level structure of  $^{71}\text{Ge}$  through  $(P, n\gamma)$  reaction. We have measured the  $\gamma$ -ray energies, branching ratios and



**Figure 3.** The spin distribution of low-lying states. The histogram is the experimental data (data from Table 1). The curve is the description by the statistical distribution with  $\sigma^2 = 5.54$ .

multipole mixing ratios of various transitions in  $^{71}\text{Ge}$ . The level energies and spins measured in the present study are in general agreement with previous measurements.

The complete and extensive nuclear level scheme of  $^{71}\text{Ge}$  provides a sufficient basis for statistical interpretations of low energy nuclear level schemes using various tests of statistical theories. The level density near the ground state is well reproduced by the Bethe formula as well as by the constant temperature formula if two parameters are fitted. The energy dependence of the level density predicted by the microscopic theory is in good agreement with the experiment.

The spin cut-off parameter of  $^{71}\text{Ge}$  has been determined from the analysis of the experimental data on spins of low-lying states given in Table 1. It is not confirmed with its corresponding rigid body value.

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