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Full Length Research Paper

A novel algorithm for solving fuzzy differential inclusions based on reachable set

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In this paper, a numerical method for solving n th-order fuzzy differential inclusions with fuzzy initial conditions is considered. A scheme based on the classical Taylor method is discussed in full details, which is followed by a complete error analysis. The method is illustrated by solving some linear and nonlinear fuzzy initial value problems.

Key words: Differential inclusions, approximation, fuzzy initial value problem.

INTRODUCTION

Knowledge about dynamic systems modeled by differential equations is often incomplete or vague. For example, parameter values, functional relationships, or initial conditions, well-know methods for solving initial value problems analytically or numerically can only be used to find selected system behavior, such as, fixing unknown parameters to some plausible values. However, in this way it is not possible to characterize the hole set of system behaviors compatible with our partial knowledge. We may set the fuzzy input somehow transformed into the fuzzy output by corresponding crisp systems, thereby, motivating us to refer to such systems as fuzzy input-fuzzy output (FIFO) systems. Here, we are going to "operationalize" our approach, that is, we are going to propose a method for computing approximation of the solution for a fuzzy differential equation using numerical methods. Since finding this set of solutions analytically does only work with trivial examples, a numerical approach seems to be the only way for "solving" such problems. The topics of fuzzy differential equations which attracted growing interest for some time, in particular in relation to fuzzy control, have been rapidly developed in recent years. The concept of fuzzy derivative was first introduced (Chang and Zadeh, 1972). It was followed up by Dubois and Prade (1982), who defined and used the extension principle. The fuzzy differential equation and the initial value problem were regularly treated by Kaleva (1990) treated the numerical methods. Recently Hullermeier (1990) suggested a different formulation of FIVP based on a family of differential inclusions at each α-level, $0 \le \alpha \le 1$, $x'(t) \in [f(t,x(t))]_\alpha$, $x(0) \in [x_0]_\alpha$. Where $[f(.,.)]_\alpha : [0,T] \times \Re^n \to \kappa_c^n$ and κ_c^n is the space of nonempty convex compact subsets of $\Re^n$. Our approach shows that the solution has the property that $diam(Supp\, x(t)) \to 0$ as $t \to \infty$.

This paper is organized as follows: some basic definitions and results on fuzzy numbers and definition of a fuzzy derivative, which have been discussed by Ma et al. (1999) and Diamond (2000) are given. We also defined the problem, which is a fuzzy initial value problem showing that numerical solution is the main interest of this work, and the numerical method for fuzzy differential equation was discussed. Finally the proposed algorithm is illustrated by solving some examples followed by a conclusion.

PRELIMINARIES

Denote by κ^n the set of all nonempty compact subset of R^n and by κ_c^n the subset of κ^n consisting of nonempty convex compact sets. Recall that $\rho(x, A) = \min_{a \in A} \|x - \alpha\|$, is the distance of a point $x \in \Re^n$ from $A \in \kappa^n$ and that the Hausdorff separation

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$\rho(A,B)$ of $A,B \in \kappa^n$ is defined as $\rho(A,B) = \max_{a\in A} \rho(a,B)$.

Note that the notation is consistent, since $\rho(a,B) = \rho(\{a\},B)$. Now, ρ is not a metric. In fact, $\rho(A,B) = 0$ if and only if $A \subseteq B$: The Hausdorff metric d_H on κ^n is defined by $d_H(A,B) = \max\{\rho(A,B), \rho(B,A)\}$, and (κ^n, d_H) is a complete metric space. An open ε-neighborhood of $A \in \kappa^n$ is the set $N(A,\varepsilon) = \{x \in \Re^n : \rho(x,A) < \varepsilon\} = A + \varepsilon B^n$, where B^n is the open unit ball in $\Re^n$. A mapping $F : \Re^n \to \kappa^n$ is upper semi-continuous at x_0 if for all $\varepsilon > 0$ there exit $\delta = \delta(\varepsilon, x_0)$ such that $F(x) \subset N(F(x_0, \varepsilon)) = F(x_0 + \varepsilon B^n)$, for all $x \in N(x_0, \delta)$. Let D^n denote the set of upper semi-continuous normal fuzzy sets on $\Re^n$ with compact support. That is, $u \in D^n$, then $u : \Re^n \to [0,1]$ is upper semi-continuous, $Supp(u) = \overline{\{x \in \Re^n : u(x) > 0\}}$ is compact and there exists at least one $\xi \in Supp(u)$ for which $u(\xi) = 1$. The α-level set of u, $0 < \alpha \le 1$ is $[u]_\alpha = \{x \in \Re^n : u(x) \ge \alpha\}$.

Clearly, for $\alpha \le \beta$, $[u]_\alpha \supseteq [u]_\beta$. The level sets are nonempty from normality and compact by usc and compact support. The metric d_H is defined on D^n as $d_\infty(u,v) = \sup\{d_H([u]_\alpha, [v]_\alpha) : 0 \le \alpha \le 1\}, u, v \in D^n$, and (D^n, ∞) is a complete metric space. Denote by E^n the subset of fuzzy convex elements of D^n. The metric space (E^n, d_∞) is also complete (Diamond and Kloeden, 1994). Let $I = [0,T]$ be a finite interval, $y_0 \in \Re^n$ and G be a map from $I \times \Re^n$ into the set of all subsets of R^n, one must find an absolutely continuous function $x(.)$ on I such that, we have:

$$\begin{cases} x'(t) \in G(t, x(t)); & for\, almost\; all\; t \in I, \\ x(0) = y_0 \in Y_0 \subset \Re^n. \end{cases} \tag{1}$$

Recall that a continuous function $x : I \to Y \subseteq \Re^n$ is said to be absolutely continuous if there exist a locally integrable function v such that $\int_t^s v(\alpha)d\alpha = x(s) - x(t)$, for all $t, s \in I$. The differential inclusion Equation 1 is said to have a solution $x(t)$ on I if $x(.)$ is absolutely continuous, $x(0) = y_0$ and $x(.)$ satisfies the inclusion a.e. in I. Let $\sum(y_0, \tau)$, be the reachable set that is, $\sum(y_0,\tau) := \{x : I \to \Re^n \,|\, x\, is\, solution\, of\, (1)\} \subset C(I)$ a nd $A(y_0, \tau) = \{x(\tau) : x(.) \in \sum(y_0, \tau)\}$ be the attainable set that is, the set of all points $x(.)$ that are ends of trajectories of Equation 1. Obviously $A(y_0, \tau),\ 0 < \tau < T$ is a compact subset of R^n. As a rule, the set $\sum(y_0, \tau)$ consists of more than one element that is we have a bundle of trajectories. We use a finite difference scheme together with suitable selection procedures resulting in a sequence of grid functions $y_0, y_1, \cdots, y_N$ say, on a uniform grid $0 = t_0 \le t_1 \le \cdots \le t_N = T$ with step size $h = \frac{T - t_0}{N} = t_i - t_{i-1}, i = 1, 2, \cdots, N$. In the Taylor method of order p for Equation 1, we have:

$$y_0 \in Y_0 \subset \Re^n,\ y_{i+1} \in y_i + hT(t_i, y_i),\ i = 0, \cdots, N-1 \tag{2}$$

where

$$T(t_i, y_i) = \sum_{j=0}^{p-1} \frac{h^j}{(j+1)!} G^{(j)}(t_i, y_i).$$

Definition 1. The fuzzy number $X \in E^n$ is called pyramidal if its α-level sets n-dimensional rectangles for $0 \le r \le 1$.

A FUZZY INITIAL VALUE PROBLEM

Let $f : I \times E^n \to E^n$ and consider the fuzzy initial value problem (FIVP) as follows:

$$\begin{cases} x'(t) = f(t, x(t)), & t \in I = [0,T], \\ x(0) = Y_0 \in E^n, \end{cases} \tag{3}$$

interpreted as a family of differential inclusions. Set $[f(t,x)]_\alpha = F(t,x;r)$ and identify the FIVP with the family of differential inclusions.

$$\begin{cases} x'_\alpha(t) = F(t, x_r(t); r), & t \in I = [0,T], \\ x_r(0) = y_0 \in [Y_0]_\alpha, \end{cases} \tag{4}$$

where $F : \Omega \times [0,1] \to \kappa_c^n$ and Ω is an open subset of $I \times E^n$ containing $(0,[Y_0]_\alpha), r \in [0,1]$, Denote the set of all solution of Equation 4 on I by $\sum([Y_0]_\alpha, T)$ and the attainable set by $A_\alpha([Y_0]_\alpha, T) = \{x(T) : x(.) \in \sum_\alpha([Y_0]_\alpha, T)\}$. Let $Z_T(R^n) = \{x(.) \in C([0,T]; R^n) : x_0'(.) \in L^\infty([0,T]; R^n)\}$.

The following Theorems 1 and 2 are consequence a fuzzy initial value problem, likewise the definition earlier (Blasi and Myiak, 1985).

Theorem 1

Let $Y_0 \in E^n$ and let Ω be an open set $\Re \times \Re^n$ containing $\{0\} \times Supp(Y_0)$. Suppose that $f : \Omega \to E^n$ is USC and write $F(t,x;r) = [f(t,x)]_\alpha \in \kappa_c^n$ for all $(t,x,r) \in \Re^{n+1} \times [0,1]$. Let the boundedness assumption, hold for all $y_0 \in Supp(Y_0)$ and the inclusion, $x'(t) \in F(t,x;0), x(0) \in Supp(Y_0)$.

Then the attainable sets $A_\alpha([Y_0]_\alpha, T), \alpha \in [0,1]$, of the family of inclusions of Equation 4 are the level sets of a fuzzy set $A(Y_0,T) \in D^n$. The solution sets $\sum_\alpha([Y_0]_\alpha, T)$ of Equation 4 are the level sets of a fuzzy set $\sum(Y_0,T)$ defined on $Z_T(\Re^n)$.

In this part of the work we prove that the solution of Equation 4 is unique for each $y_0 \in \sup p(Y_0) \subset \Re^n$. Under some conditions suppose $x(t) := z(t,y_0)$ is a solution of Equation 4 for each $y_0 \in Supp(Y_0)$. The authors (Vorobiev and Seikala, 2002) constructed n families of α-parameterized interval-valued mappings $g^k(\alpha) : I \to [g_1^k(t,r), g_2^k(t,r)]$ in the following way:

$$\begin{cases} g_1^k(t,r) = \min\{z^k(t,y_0) : y_0 \in [Y_0]_\alpha\}, \\ g_1^k(t,r) = \max\{z^k(t,y_0) : y_0 \in [Y_0]_\alpha\}, \alpha \in [0,1], k = 1, \cdots, n. \end{cases} \tag{5}$$

where $z(t,y_0) = (z^1(t,y_0), \cdots, z^n(t,y_0)) \in \Re^n$.

The minimum vector and maximum vector of $z(t,y_0)$ is, respectively, $z_{\min}(t,y_0) = (g_1^1(t,\alpha), \cdots, g_1^n(t,\alpha))$, and $z_{\max}(t,y_0) = (g_2^1(t,\alpha), \cdots, g_2^n(t,\alpha))$, where obviously $z(t,y_0) \in [z_{\min}(t,y_0), z_{\max}(t,y_0)]$. Let $X : I \to E^n$ be a fuzzy process then we have:

$$[x(t)]_\alpha = \times_{k=1}^n = [g_1^k(t,r), g_2^k(t,r)], \tag{6}$$

where $\times$ denotes the usual set-theoretical Cartesian product, (Vorobiev and Seikala, 2002), then $\{(z^1(t,y_0), \cdots, z^n(t,y_0)) : z^i(t,y_0) \in [g_1^i(t,r), g_2^i(t,r)], i = 1, \cdots, n\} = [X(t)]_\alpha, \alpha \in [0,1]$, hence the convex hull of corners of n-dimensional rectangles is $[X(t)]_\alpha$ for any $\alpha \in [0,1]$.

Let $F(t, x_\alpha(t), r) = [f(t,x(t))]_\alpha \in \kappa_c^n, x_r(t) \in [X(t)]_\alpha, \alpha \in [0,1]$ and $f = (f^1, f^2, \cdots, f^n)^t,\ f_1 = (f_1^1, \cdots, f_1^n)^t, f_2 = (f_2^1, \cdots, f_2^n)^t$.

We construct n families of α-parameterized interval-valued mappings $f^k : I \times E^n \to [f^k(t, x_\alpha(t); \alpha), f^k(t, x_\alpha(t); \alpha)]$ in the following way:

$$\begin{cases} f_1^k(t,x_\alpha(t);\alpha) = f^k(t,U_{\min}^k;\alpha) = \min\{f^k(t,U) : U \in [X(t)]_\alpha\}, \\ f_2^k(t,x_\alpha(t);\alpha) = f^k(t,U_{\max}^k;\alpha) = \max\{f^k(t,U) : U \in [X(t)]_\alpha\}, k = 1, \cdots, n.\ \alpha \in [0,1]. \end{cases} \tag{7}$$

Obviously, the fuzzy set valued function $F : \Omega \times [0,1] \to \kappa_c^n$ is as follows:

$$F(t,x_\alpha(t),\alpha) = \times_{k=1}^n [f_1^k(t,x_\alpha(t);\alpha)] = [f_1(t,x_\alpha(t),r), f_2(t,x_\alpha(t),\alpha)] \in \kappa_c^n; x_\alpha(t) \in [X(t)]_\alpha, 0 \le \alpha \le 1, \tag{8}$$

this means that

$$\{(f^1(t,x_\alpha(t);\alpha), \cdots, f^n(t,x_\alpha(t);\alpha)) | f^i(t,x_\alpha(t);\alpha) \in [f_1^i(t,x_\alpha(t);\alpha), f_2^i(t,x_\alpha(t);\alpha);\alpha], i = 1, \cdots, n\}$$
$$= F(t, x_\alpha(t), \alpha),\ \alpha \in [0,1], \tag{9}$$

and also

$[Y_0]_\alpha = \times_{k=1}^n [y_1^k(0,\alpha),\ y_2^k(0,\alpha)]$, is the surface of n-dimensional rectangles for any $\alpha \in [0,1]$, where $Y_1(0,\alpha) = (y_1^1(0,\alpha), \cdots, y_1^n(0,\alpha)),\ Y_2(0,\alpha) = (y_2^1(0,\alpha), \cdots, y_2^n(0,\alpha))$. Now the problem of Equation 4 is transformed to the equation that follows:

$$x_\alpha'(t) \in [f_1(t,x_\alpha(t),\alpha), f_2(t,x_\alpha(t),\alpha)],\ x_\alpha(0) \in [Y_1(0,\alpha), Y_2(0,\alpha)] = [Y_0]_\alpha,\ for all\ \alpha \in [0,1]. \tag{10}$$

$x_\alpha(0)$ chosen randomly then the selection of $x_\alpha'(t)$ is random too that the set of all selections are formed the reachable set hence Equation 10 converted to a crisp differential inclusions for any $\alpha \in [0,1]$. The problem of Equation 10 might be stiff differential equation, then by considering the fact that $\Re^n$ be equipped with the scalar product $\langle .,. \rangle$ and the corresponding induced norm $\|.\|$, we have the following theorem.

Theorem 2

Suppose f^k is satisfied in one-sided Lipschitz condition, that is $\forall U', U'' \in \Re^n, \exists L_k > 0$, such that $\left| f^k(t,U') - f^k(t,U'') \right| \le L_k \|U' - U''\|,\ for\, k = 1,\cdots,n$. Then the problem of Equation 10 has a unique solution.

Proof

Let $U' = (u_1', \cdots, u_n')^t$ and $U'' = (u_1'', \cdots, u_n'')^t$. Then

$$\langle f(t,U') - f(t,U''), U' - U'' \rangle = \left(f^1(t,U') - f^1(t,U'')\right)(u_1' - u_1'') + \cdots + \left(f^n(t,U') - f^n(t,U'')\right)(u_n' - u_n'')$$
$$\le \left|f^1(t,U') - f^1(t,U'')\right| \left|u_1' - u_1''\right| + \cdots + \left|f^n(t,U') - f^n(t,U'')\right| \left|u_n' - u_n''\right| \le (L_1 + \cdots + L_n) \|U' - U''\|^2$$

$$\le nL \|U' - U''\|^2,$$

where $L = \max\{L_1, \cdots, L_n\}$. The proof is completed.

TAYLOR METHOD OF ORDER P

Let $x_\alpha(t_i) \cong y_i(\alpha)$ for all $\alpha \in [0,1]$, then the Taylor method of order p for approximating the reachable set of problem of Equation 10 is proposed as follows:

$$y_0(\alpha) \in [Y_0]_\alpha \subset \Re^n,$$
$$y_{i+1}(\alpha) \in \bigcup_{s_\alpha \in [Y(\alpha)]_\alpha} \left(s_\alpha + h\phi_G(\alpha_i, s_\alpha; \alpha)\right), i = 0, \cdots, N-1, \forall \alpha \in [0,1], \tag{11}$$

where

$$\phi_G(\alpha_i, s_\alpha; \alpha) = \sum_{j=0}^{p-1} \frac{h^j}{(j+1)!} G^{(j)}(t_i, s_\alpha; \alpha),$$

and

$$[Y(t_i)]_\alpha = \times_{k=1}^n \left[y_1^k(t_i,\alpha), y_2^k(t_i,\alpha)\right], i - 0, \cdots, N-1,\ \alpha \in [0,1].$$

Lemma 1

Let a sequence of numbers $\{W_n\}_{n=0}^N$ satisfy $|W_{n+1}| \le A|W_n| + B,\ 0 \le n \le N-1$, for some given positive constants A and B. Then $|W_n| \le A^n |W_0| + B \dfrac{A_n - 1}{A_1},\ 0 \le n \le N$ (Ma et al., 1999).

Theorem 3

Let $F \in C^{p+1}(\Omega)$ in Equation 4 be a compact convex valued mapping such that satisfies Lipschitz condition in x with Lipschitz constant $L > 0$ and x_α be a solution of Equation 4 then $\lim_{h \to 0} y_N(\alpha) = x_\alpha(T)$, for any $\alpha \in [0,1]$.

Proof

Let $x_\alpha(t_i + 1) = \bigcup_{\bar{x}_\alpha \in [X(t_i)]_\alpha} \left(\bar{x}_\alpha + h\phi_F(t_i, \bar{x}_\alpha; \alpha)\right), a.e.$ And $y_{i+1}(\alpha) = \bigcup_{\bar{y}_\alpha \in [Y(t_i)]_\alpha} \left(\bar{y}_\alpha + h\phi_F(t_i, \bar{y}_\alpha; \alpha)\right) + O(h^{p+1})$, it is enough we prove $\lim_{h \to 0} \left\| \bar{y}_{i+1}(\alpha) - \bar{x}_\alpha(t_{i+1}) \right\| = 0,\ i = 0, \cdots, N-1,\ \alpha \in [0,1]$.
Since
$\bar{y}_{i+1}(\alpha) = \bar{y}_i(\alpha) + h\phi_F\left(t_i, \bar{y}_i(\alpha); \alpha\right) + O(h^{p+1})$, and
$\bar{x}_\alpha(t_{i+1}) \approx \bar{x}_\alpha(t_i) + h\phi_F(t_i, \bar{x}_\alpha(t_i); \alpha)$, where
$\phi_F(t, u; \alpha) = \sum_{j=0}^{p-1} \dfrac{h^j}{(j+1)!} F^j(t, u; \alpha)$. Then
$\left\| \bar{y}_{i+1}(\alpha) - \bar{x}_\alpha(t_{i+1}) \right\| \le \left\| \bar{y}_i(\alpha) - \bar{x}_\alpha(t_i) \right\| (1 + Lh) + O(h^{p+1})$. By using the Lemma 1 for all t_i in particular at T proof is completed as $\left\| \bar{y}_N(\alpha) - \bar{x}_\alpha(T) \right\| \le \dfrac{1}{L} O(h^p)\left(e^{LT} - 1\right)$.

EXAMPLES

Example 1

Consider a fuzzy differential inclusions with constant coefficients

$$\begin{cases} \dfrac{dx_1(t,\alpha)}{dt} \in 3x_1(t,\alpha) - 2x_2(t,\alpha), 0 \le t \le 0.01, \\ \dfrac{dx_2(t,\alpha)}{dt} \in 2x_1(t,\alpha) - x_2(t,\alpha), \end{cases} \tag{12}$$

as an initial value for the fuzzy initial-value problem of

Table 1. The distance between reachable set and its approximations.

H	4th Taylor	2nd Taylor	Euler
0.01	2.8725e-011	3.3661e-006	0.0789
0.005	1.8211e-012	8.571e-007	0.0716
0.0025	1.1083e-013	2.1886e-007	0.0710
0.00125	7.0672e-015	5.3138e-008	0.0694
0.000625	9.9301e-016	1.3425e-008	0.0611

Equation 12 we take a number $Y_0 \in E^2$ such that $[Y_0]_\alpha = \{(x_1(0,\alpha), x_2(0,\alpha)) \in \Re^2 : x_1(0;\alpha) \in [\alpha-1, 1-\alpha]; x_2(0,\alpha) \in [0.5+0.5\alpha, 1.5-0.5\alpha]\}, \alpha \in [0,1].$ where

$$\sum_\alpha([Y_0]_\alpha, t) = \begin{pmatrix} x_1(t,\alpha) \\ x_2(t,\alpha) \end{pmatrix} - \begin{pmatrix} e^t[x_1(0,\alpha) + 2t(x_1(0,\alpha) - x_2(0,\alpha))] \\ e^t[x_2(0,\alpha) + 2t(x_1(0,\alpha) - x_2(0,\alpha))] \end{pmatrix} \quad (13)$$

And $(x_1(0,\alpha), x_2(0,\alpha)) \in [Y_0]_\alpha$. Let $C = \bigcup_\alpha ([Y_0]_\alpha, T)$ and B be the approximation of C which is obtained by numerical methods. In Table 1 we compare $d_\infty(C,B)$ for Taylor method of order two and four and Euler method.

Example 2

Consider the following fuzzy differential inclusions:

$$x_1'(t,\alpha) \in -x_2(t,\alpha) + 0.1x_1(t,\alpha)(9 - x_1(t,\alpha)^2 - x_2(t,\alpha)^2) + w(\alpha),$$
$$x_2'(t,\alpha) \in -x_1(t,\alpha) + 0.1x_2(t,\alpha)(9 - x_1(t,\alpha)^2 - x_2(t,\alpha)^2) + w(\alpha),$$

we take a number $Y_0 \in E^2$ such that

$$[Y_0]_\alpha = \{(x_1(0,\alpha), x_2(0,\alpha)) \in \Re^2 : x_1(0,\alpha) \in [\alpha-1, 1-\alpha]; x_2(0,\alpha) \in [0.5+0.5\alpha, 1.5-0.5\alpha]\}, \alpha \in [0,1].$$

and

$$w(\alpha) \in [\alpha - 1, 1 - \alpha].$$

Then

$$f_1(t, x_\alpha(t), \alpha) = \begin{pmatrix} -x_2(t,\alpha) + 0.1x_1(t,\alpha)(9 - x_1(t,\alpha)^2 - x_2(t,\alpha)^2) + \alpha - 1 \\ -x_1(t,\alpha) + 0.1x_2(t,\alpha)(9 - x_1(t,\alpha)^2 - x_2(t,\alpha)^2) + \alpha - 1 \end{pmatrix},$$

and

$$f_2(t, x_\alpha(t), \alpha) = \begin{pmatrix} -x_2(t,\alpha) + 0.1x_1(t,\alpha)(9 - x_1(t,\alpha)^2 - x_2(t,\alpha)^2) + \alpha - 1 \\ -x_1(t,\alpha) + 0.1x_2(t,\alpha)(9 - x_1(t,\alpha)^2 - x_2(t,\alpha)^2) + \alpha - 1 \end{pmatrix}.$$

For example if

$$f(t, x_\alpha(t), \alpha) = \frac{f_1(t, x_\alpha(t), \alpha) + f_2(t, x_\alpha(t), \alpha)}{2},$$

chosen from $[f_1(t, x_\alpha(t), \alpha), f_2(t, x_\alpha(t), \alpha)]$.

Conclusion

The method presented in this paper for approximation reachable set is based on pyramidal fuzzy numbers. The α-level sets of this fuzzy numbers are n-dimensional rectangles that the convex hull of the corners of rectangles or the set of all points on them (in n-dimensional space) forms the reachable set. In this paper each point (n-dimensional vectors) which belongs to reachable set is approximated by using the Taylor method of order two and four, which in contrast with the Euler method has a higher order of convergence.

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