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Full Length Research Paper

A class of exact solutions of second grade fluid

A. M. Siddiqui[1], M. KamranAlam[2]*, S. Islam[3], M. T. Rahim[2] and Manzoor Elahi[3]

[1]Department of Mathematics Pennsylvania State University, York Campus, 1031 Edgecombe Avenue, New York, PA 17403, USA.
[2]Department of Mathematics, National University of Computer and Emerging Sciences, Peshawar, Pakistan.
[3]COMSATS Institute of Information Technology, Islamabad 44000, Pakistan.

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This paper describes a class of exact solutions of the equations of motion for an unsteady, incompressible non-Newtonian second grade fluid in the Cartesian co-ordinates. The exact solutions of non-linear equations governing the flow of second grade fluids are obtained through generalized separation variables method by assuming certain form of stream functions. The expressions for velocity profile and vorticity stream function are constructed.

Key words: Exact solutions, vorticity functions, second grade fluids, separation of variables.

INTRODUCTION

The nature of Navier-Stocks equations is nonlinear, so it is very complicated to find the exact solutions except a few particular cases available in literature. This is, in general, due to the non-linearites, which occur in the inertial part. The exact solutions of the equations of motion have some physical meaning and it can be used as a check against complicated numerical codes that have been developed for much more complex flows. By taking vorticity to be proportional to the stream functions perturbed by a uniform stream, Taylor (1923) investigated that the non-linear convective term vanished and found an exact solution by representing doubly infinite array of vertices. Kovasnay (1948) showed that the non-linarites in the Navier-Stokes equations are self-canceling and found an exact solution which represents the motion behind a two dimensional grid. Wang (1996) was also able to linearize the non-linear part of Navier-Stocks equations and showed the results of Taylor (1923) and Kovasznay (1948) as special case in his work. Similar results were obtained by Lin and Tobak (1986) and Hui (1987) in which the non-linear inertial part canceled automatically.

Turning to the non-linearity in non-Newtonian fluids, namely fluids of second grade, these problems become even difficult to solve because the non-linarites not only occur in the inertial part but also in the viscous part of these equations. For this reason, inverse methods become attractive in the study of non-Newtonian second grade fluids. Rajagopal (1980) investigated that, in the equations of motion of a second grade fluids, the non-linear convective term vanish for the specific problems. Rajagopal and Gupta (1981) also found a class of exact solutions to the equations of motion of second grade fluids in which the non-linearities are self-canceling and showed a subclass of the solutions obtained by Wang (1996) for the Navier-Stokes equations. Kaloni and Huschlit (1984), Siddiqui and Kaloni (1984) and Siddiqui (1986) found solutions for second grade fluids for steady case by considering certain forms of stream functions. Ting (1963) and Rajagopal (1982) obtained solutions for unsteady flows. Detail discussions on the exact solutions of the Navier-Stocks equations and equations of motion for the non-Newtonian fluids are given by Hamdan (1998), Siddiqui et al. (2006), Kamel and Hamdan (2006), Labropulu (2000) and Islam et al. (2008).

Different types of solutions are in general described in the form of finite sums, (Polyanin, 2001; Polyanin et al., 2004). In present paper we extended the work of Polyanin (2001) and Polyanin et al. (2004) by using the separation of variables method for an incompressible, second grade fluid.

GOVERNING EQUATIONS

The constitutive equation of an incompressible fluid of second grade is of the form (Rivlin and Ericksen, 1979)

*Corresponding author. E-mail: kamranalam2005@yahoo.com.

$$\mathbf{T} = -pI + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1{}^2 \quad (1)$$

where $\mathbf{T}$ is the Cauchy stress tensor, p denotes the pressure, I is the identity tensor, μ , α_1 *and* α_2 are measurable material constants. They denote, respectively, the viscosity, elasticity and cross-viscosity. These material constants can be determined from viscometric flows of any real fluid. $\mathbf{A}_1$*and* $\mathbf{A}_2$ are Rivlin-Ericksen tensors (Rivlin and Ericksen, 1979) and they denote, respectively, the rate of strain and acceleration. Where $\mathbf{A}_1$ *and* $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = (grad \quad \mathbf{v}\ ) + (grad \quad \mathbf{v})^T \quad (2)$$

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1 \ (grad \quad \mathbf{v}) + (grad \quad \mathbf{v})^T \mathbf{A}_1 \quad (3)$$

Here v is the velocity, ∇ is the grade operator and $\frac{d}{dt}$ the material time derivative.

The basic equations governing the motion of an incompressible fluid are

$$div \quad \mathbf{v} = 0 \quad (4)$$

$$\rho \frac{d\mathbf{v}}{dt} = div\mathbf{T} + \rho\chi \quad (5)$$

where ρ is the density and χ the body force.

Inserting (1) in (5) and making use of (2) and (3) we obtain the following vector equation

$$grad\left[\frac{1}{2}\rho \ |\mathbf{v}|^2 + p - \alpha_1\left(\mathbf{v}.\nabla^2\mathbf{v} + \frac{1}{4}|\mathbf{A}_1|^2\right)\right]$$
$$\alpha_1\left[\nabla^2\mathbf{v}_t + \nabla^2(\nabla\times\mathbf{v})\times\mathbf{v}\right] + (\alpha_1 + \alpha_2) \ div\mathbf{A}_1{}^2$$
$$+\rho\left[\mathbf{v}_t - \mathbf{v}\times(\nabla \times\mathbf{v})\right] = \mu\nabla^2\mathbf{v} +$$
$$+ \rho\chi \quad (6)$$

in which ∇^2 is the Laplacian operator, $\mathbf{v}_t = \partial V/\partial t$, and $|\mathbf{A}_1|$ is the usual norm of matrix $\mathbf{A}_1$.

If this model is required to be compatible with thermodynamics, then the material constants must meet the restrictions (Dunn and Fosdick, 1974; Fosdick and Rajagopal, 1979)

$$\mu \geq 0, \ \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0 \quad (7)$$

On the other hand, experimental results of tested fluids of second-grade showed that $\alpha_1 < 0$ and $\alpha_1 + \alpha_2 \neq 0$ which contradicts the aforementioned conditions and implies that they are unstable. This controversy is discussed in detail in Rajagopal (1995). However, in this paper we will discuss only the case, where $\alpha_1 > 0$.

Let us consider the motion of an unsteady incompressible second grade fluid in which the velocity field is of the form

$$\mathbf{v}(x,y,z,t) = \left[u_1(x,y,z,t) \ , \ u_2(x,y,z,t) \ ,0\right] \quad (8)$$

where u_1 *and* u_2 are the velocity components in the x and y-directions, respectively. Putting Equation (8) in (4) and (6) and making use of the assumption (7) we obtain, in the absence of body forces, the following equations

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} = 0 \ , \quad (9)$$

$$\frac{\partial \hat{p}}{\partial x} + \rho\left[\frac{\partial u_1}{\partial t} - u_2\omega\right] = \left(\mu + \alpha_1\frac{\partial}{\partial t}\right)\nabla^2 u_1 - \alpha_1 u_2\nabla^2\omega \quad (10)$$

$$\frac{\partial \hat{p}}{\partial y} + \rho\left[\frac{\partial u_2}{\partial t} + u_1\omega\right] = \left(\mu + \alpha_1\frac{\partial}{\partial t}\right)\nabla^2 u_2 + \alpha_1 u_1\nabla^2\omega \quad (11)$$

$$\omega = \frac{\partial u_1}{\partial x} - \frac{\partial u_1}{\partial y} \quad (12)$$

$$\hat{p} = p + \frac{1}{2}\rho\left(u_1{}^2 + u_2{}^2\right) -$$
$$\alpha_1\left(u_1\nabla^2 u_1 + u_2\nabla^2 u_2 + \frac{1}{4}|\mathbf{A}_1|^2\right) , \quad (13)$$

$$|\mathbf{A}_1|^2 = 4\left(\frac{\partial u_1}{\partial x}\right)^2 + 4\left(\frac{\partial u_2}{\partial y}\right)^2 + 2\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right)^2 \quad (14)$$

On setting $\alpha_1 = 0$ in (10) and (11) and considering only steady case we recover the equation for Newtonian fluid (Berker, 1963). Eliminating pressure in (10) and (11), by applying the integribility condition $\frac{\partial^2\hat{p}}{\partial x\partial y} = \frac{\partial^2\hat{p}}{\partial y\partial x}$ we get the vorticity equation

$$\rho\left[\frac{\partial\omega}{\partial t} + \left(u_1\frac{\partial}{\partial x} + u_2\frac{\partial}{\partial y}\right)\omega\right] = \left(\mu \quad \alpha_1\frac{\partial}{\partial t}\right)\nabla^2\omega + \alpha_1\left[\left(u_1\frac{\partial}{\partial x} + u_2\frac{\partial}{\partial y}\right)\nabla^2\omega\right] \tag{15}$$

Introducing the stream function

$$u_1 = \frac{\partial\psi}{\partial y}, \quad u_2 = -\frac{\partial\psi}{\partial x}, \tag{16}$$

We see that the continuity Equation (9) is satisfied identically and (13) in (12) yields. The following compatibility equation in terms of stream function

$$\rho\left[\frac{\partial}{\partial t}\nabla^2\psi - \{\psi, \nabla^2\psi\}\right] = \left(\mu + \alpha_1\frac{\partial}{\partial t}\right)\nabla^4\psi - \alpha_1\{\psi, \nabla^2\psi\} \tag{17}$$

in which $\nabla^4 = \nabla^2 . \nabla^2$ and the vorticity vector and poison bracket in terms of stream function are respectively given by

$$\omega = -\nabla^2 \psi, \tag{18}$$

$$\{\psi, \nabla^2\psi\} = \left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\nabla^2\psi \tag{19}$$

On defining $\upsilon = \frac{\mu}{\rho}$ and $\alpha = \frac{\alpha_1}{\rho}$, Equation (17) can be written as:

$$\frac{\partial}{\partial t}\nabla^2\psi - \{\psi, \nabla^2\psi\} = \left(\upsilon + \alpha\frac{\partial}{\partial t}\right)\nabla^4\psi - \alpha\{\psi, \nabla^4\psi\} \tag{20}$$

EXACT SOLUTIONS WITH GENERALIZED SEPARATION OF VARIABLES METHOD

The exact solutions of the second-grade flow equation in terms of stream function are obtained by generalized (incomplete) separation of variables method. Different types of solutions are in general described in the form of finite sums, as (Polyanin, 2001; Polyanin et al., 2004)

$$\psi(x,y,t) = \sum_{k=1}^{n} f_k(x) \; g_k(y,t), \tag{21}$$

or

$$\psi(x,y,t) = \sum_{k=1}^{n} f_k(x, t) \; g_k(y), \tag{22}$$

Here the unknown functions f_k and g_k are arbitrary which should be chosen in such a way that they satisfy Equation (18). We specializes these functions by prescribing one set of the functions depending on coordinate [for example, $f_k(x)$] in the following simple forms:

$$f_k(x) = x^k, \; f_k(x) = e^{\lambda_k x}, \; f_k(x) = \sin(\alpha_k x), \; f_k(x) = \cos(\beta_k x),$$

and their linear combinations in order to find exact solutions to Equation (18). Here λ_k, α_k and β_k $(k = 1, 2, 3, ..., n)$ are arbitrary parameters. The other set of functions g_k is determined by solving corresponding nonlinear equations.

Steady-state solutions

Here the possible exact solutions of non-linear equations governing the flow of second grade fluids are obtained through generalized separation variables method by assuming certain form of stream functions. The expressions for velocity profile and vorticity stream function are constructed in the following cases.

Case (1)

$$\psi(x,y) = (Ax + B)e^{-\lambda y} \; ax + C; \quad a = \frac{\upsilon\lambda}{1-\alpha\lambda^2},$$

$$u_1 = -\lambda(Ax + B)e^{-\lambda y}, \quad u_2 = -\lambda A e^{-\lambda y} - a,$$

$$\omega = -\lambda^2(Ax + B)e^{-\lambda y},$$

where A, B, C are constants of integration and α, λ are arbitrary parameters. Figure 1 shows the behavior of the aforementioned stream function.

Case (2)

$$\psi(x,y) = [A\sin(\beta x) + B\cos(\beta x)]\, e^{-\lambda y} + ax + C;$$

$$a = \frac{\upsilon(\lambda^2 - \beta^2)}{[1 - \alpha\lambda(\lambda^2 - \beta^2)]},$$

$$u_1 = -\lambda[A\sin(\beta x) + B\cos(\beta x)]\, e^{-\lambda y},$$

$$u_2 = -\beta[A\sin(\beta x) + B\cos(\beta x)]^2\, e^{-2\lambda y} - a,$$

$$\omega = (\lambda^2 - \beta^2)[A\sin(\beta x) + B\cos(\beta x)]\, e^{-\lambda y},$$

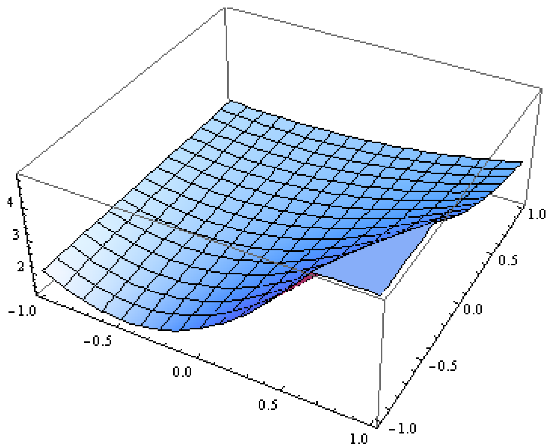

Figure 1. The streamline flow for A = B = C = 1, λ = υ = 1, and α = 0.5.

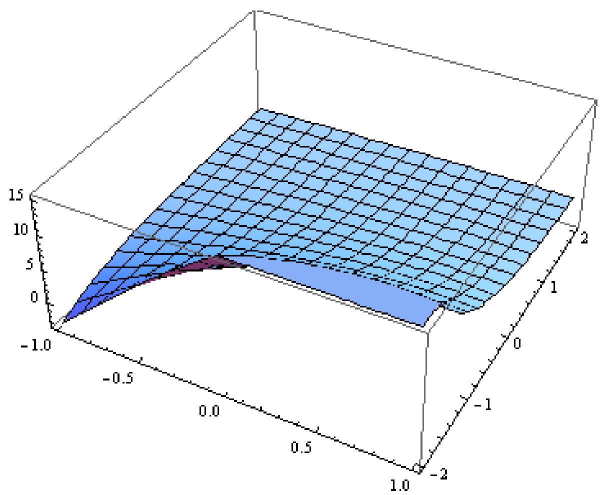

Figure 2. The streamline flow for A = B = 2, C = 1, λ = β = 1, υ = 2, α = 0.5.

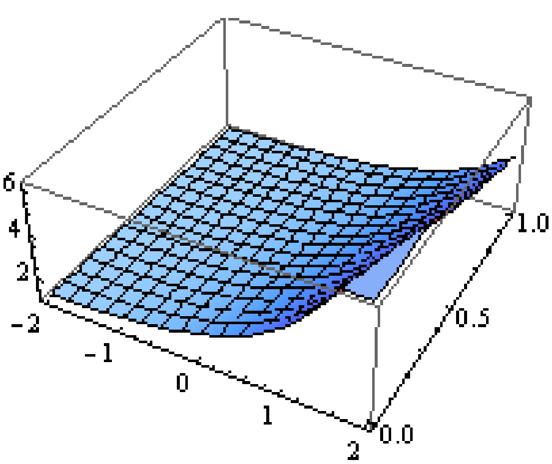

Figure 3. The streamline flow for A = B = C = 1, β = λ = υ = 1, α = 0.3.

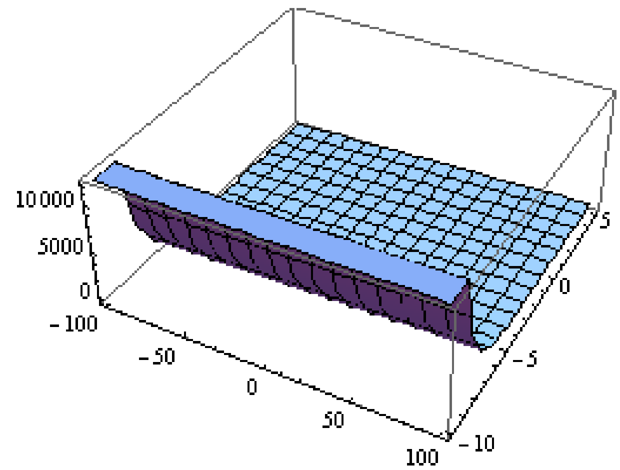

Figure 4. The streamline flow for $A = B = C = 1, \lambda = 2, \upsilon = 0.5, \alpha = 0.3$.

Here A, B, C are constants of integration and α, λ *and* β represents arbitrary parameters. The behavior of the aforementioned stream function is presented in Figure 2.

Case (3)

$$\psi(x,y) = \left[A\sinh(\beta x) + B\cosh(\beta x)\right] e^{-\lambda y} + ax + C;$$

$$a = \frac{\upsilon(\lambda^2 - \beta^2)}{\left[1 - \alpha\lambda(\lambda^2 - \beta^2)\right]},$$

$$u_1 = -\lambda\left[A\sinh(\beta x) + B\cosh(\beta x)\right] e^{-\lambda y},$$

$$u_2 = -\beta\left[A\sinh(\beta x) + B\cosh(\beta x)\right]^2 e^{-\lambda y} - a,$$

$$\omega = -(\lambda^2 + \beta^2)\left[A\sinh(\beta x) + B\cosh(\beta x)\right] e^{-\lambda y},$$

where A, B, C are constants of integration and α, λ *and* β are arbitrary parameters. Figure 3 shows the behavior of the aforementioned stream function.

Case (4)

$$\psi(x,y) = Ae^{-\lambda x} + Be^{-\lambda y} + a(x-y) + C;\ a = \frac{\upsilon\lambda}{1-\alpha\lambda^2},$$

$$u_1 = -\lambda Be^{-\lambda y} - a,$$

$$u_2 = -\lambda Ae^{-\lambda y} - a,$$

$$\omega = -\lambda^2\left(Ae^{-\lambda x} + Be^{-\lambda y}\right),$$

where A, B, C are constants of integration and α, λ are arbitrary parameters. Figure 4 shows the behavior of the aforementioned stream function.

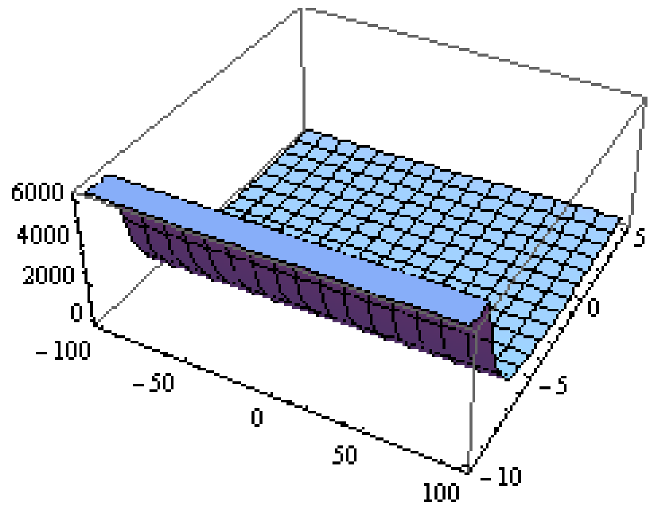

Figure 5. The streamline flow for A =1, B =2, C =1, λ = υ = 1, α = 0.5.

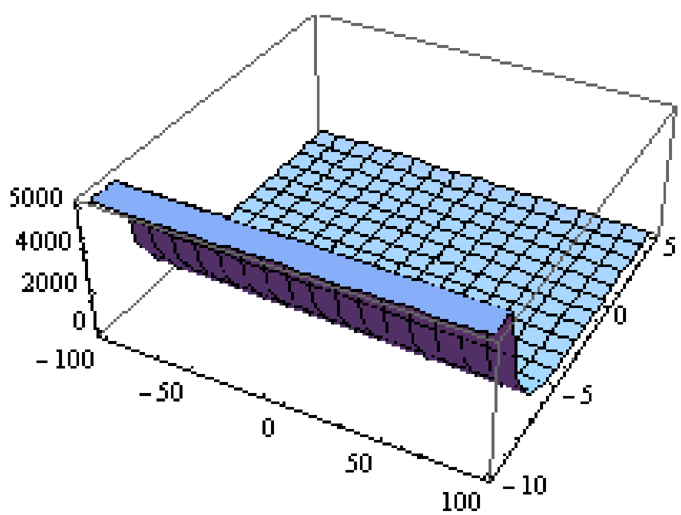

Figure 6. The streamline flow for A =B = C=λ = υ =1 and α = 0.5.

Case (5)

$$\psi(x,y) = Ae^{\lambda x} + Be^{-\lambda y} + a(x+y) + C \;;\; a = \frac{\upsilon\lambda}{1 - \alpha\lambda^2},$$

$$u_1 = -\lambda Be^{\lambda y} + a,$$

$$u_2 = -\lambda Ae^{\lambda y} - a,$$

$$\omega = -\lambda^2 \left(Ae^{\lambda x} + Be^{\lambda y} \right),$$

where A, B, C are constants of integration and α, λ are arbitrary parameters. Figure 5 shows the behavior of the aforementioned stream function.

Case *(6)*

$$\psi(x,y) = Ae^{\lambda x} + Be^{-\lambda y} - a(x-y) + C \;;\; a = \frac{\upsilon\lambda}{1 - \alpha\lambda^2},$$

$$u_1 = \lambda Be^{\lambda y} + a,$$

$$u_2 = -\lambda Ae^{\lambda y} + a,$$

$$\omega = -\lambda^2 \left(Ae^{\lambda x} + Be^{\lambda y} \right),$$

where A, B, C are constants of integration and α, λ are arbitrary parameters. Figure 6 shows the behavior of the aforementioned stream function.

Case (7)

$$\psi(x,y) = Ae^{-\lambda x} + Be^{\lambda y} - a(x+y) + C \;;\; a = \frac{\upsilon\lambda}{1 - \alpha\lambda^2},$$

$$u_1 = \lambda Be^{\lambda y} + a,$$

$$u_2 = -\lambda Ae^{\lambda y} + a,$$

$$\omega = -\lambda^2 \left(Ae^{\lambda x} + Be^{\lambda y} \right),$$

where A, B, C are constants of integration and α, λ are arbitrary parameters. Figure 7 shows the behavior of the aforementioned stream function.

Case (8)

$$\psi(y) = C_1 y^3 + C_2 y^2 + C_3 y + C_4,$$

$$u_1 = 3C_1 y^2 + 2C_2 y + C_3, \quad u_2 = 0, \quad \omega = -6C_1 y - 2C_2,$$

Case (9)

$$\psi(x,y) = C_1 x^2 + C_2 x + C_3 y^2 + C_4 y + C_5,$$

$$u_1 = 2C_3 y + C_4,$$

$$u_2 = -2C_1 x - C_2,$$

$$\omega = -2(C_1 + C_3),$$

where C_1, C_2, C_3 *and* C_4 are constants of integration. Figure 9 shows the behavior of the aforementioned stream function.

Case (10)

$$\psi(x,y) = C_1 e^{-\lambda y} + C_2 y^2 + C_3 y + ax + C_4 \;;\; a = \frac{\upsilon\lambda}{1 - \alpha\lambda^2},$$

$$u_1 = -\lambda C_1 e^{-\lambda y} + 2C_2 y + C_3,$$

$$u_2 = -a, \quad \omega = -\lambda^2 C_1 e^{-\lambda y} - 2C_2,$$

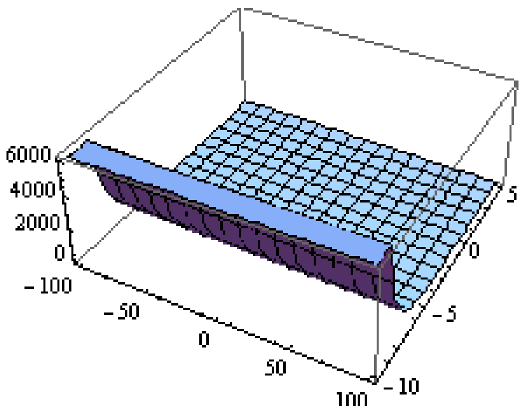

Figure 7. The streamline flow for A = B = C = λ = υ = 1, α = 0.5.

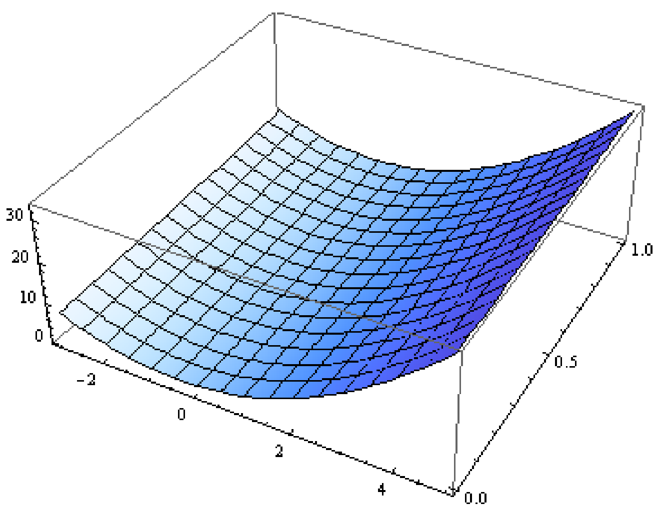

Figure 9. The streamline flow for the constant $C_{1=}$ C_3 = C_3 = C_4 = 1.

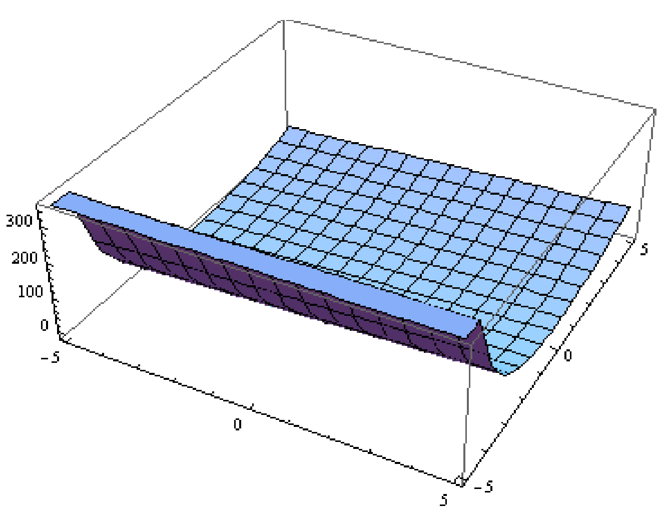

Figure 10. The streamline flow for the $C_{1=}$ C_3 = C_3 =2, C_4 =1, λ = υ = 1, α = 0.5.

where C_1, C_2, C_3 *and* C_4 are constants of integration and α, λ are arbitrary parameters. Figure 10 shows the behavior of the aforementioned stream function.

Case (11)

$$\psi(x,y) = A(kx + \lambda y)^3 + B(kx + \lambda y)^2 + C(kx + \lambda y) + D,$$
$$u_1 = 3A\lambda(kx + \lambda y)^2 + 2B(kx + \lambda y) + C\lambda,$$
$$u_2 = -3Ak(kx + \lambda y)^2 - 2Bk(kx + \lambda y) - Ck,$$
$$\omega = -6A(k^2+\lambda^2)(kx + \lambda y) - 2B(k^2+\lambda^2),$$

where A, B, C *and* D are constants of integration and α is arbitrary parameters. Figure 11 shows the behavior of the aforementioned stream function.

Case (12)

$$\psi(x,y) = Ae^{-\lambda(y+kx)} + B(y + kx)^2 + C(y + kx) + ax + D;$$
$$a = \frac{\upsilon\lambda(k^2+1)}{1-\alpha\lambda^2(k^2+1)},$$
$$u_1 = -\lambda Ae^{-\lambda(y+kx)} + 2B(y + kx) + C,$$
$$u_2 = k\lambda Ae^{-\lambda(y+kx)} - 2Bk(y + kx) - Ck - a,$$
$$\omega = \lambda^2(k^2+1)Ae^{-\lambda(y+kx)} - 2B(k^2+1),$$

where A, B, C *and* D are constants of integration and α, λ are arbitrary parameters. Figure 12 shows the behavior of the aforementioned stream function.

Case (13)

$$\psi(x,y) = Ae^{\lambda y+\beta x} + Be^{\gamma y} + ay + bx + C; \; a = \frac{\upsilon\lambda(k^2+1)}{1-\alpha\lambda^2(k^2+1)},$$
$$u_1 = \lambda Ae^{\lambda y+\beta x} + \gamma Be^{\gamma y} + a,$$
$$u_2 = -\beta Ae^{\lambda y+\beta x} - b,$$
$$\omega = -(\beta^2+\lambda^2)Ae^{\lambda y+\beta x} - \gamma^2 Be^{\gamma y},$$

where A, B, C are constants of integration and α, λ, γ *and* β are arbitrary parameters. Figure 13 shows the behavior of the aforementioned stream function.

Conclusion

All the solutions were reduced to viscous solutions given in Polyanin (2001) and Polyanin et al. (2004).

$\psi(x,y)$ is a solution of the Equation (20), then $-\psi(x,y)$, $\psi(C_1x+C_2, C_1y+C_3)+C_4$ and

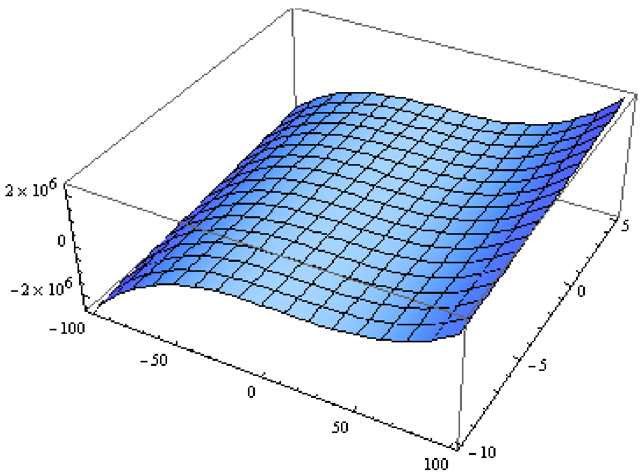

Figure 11. The streamline flow for A = B = C =2, D =1, and $\lambda = \kappa = 1$.

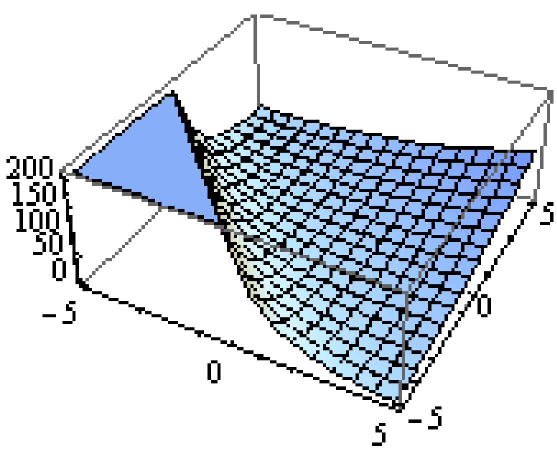

Figure 12. The streamline flow for A = B = C = D =1, k $=\lambda = \upsilon = 1$.

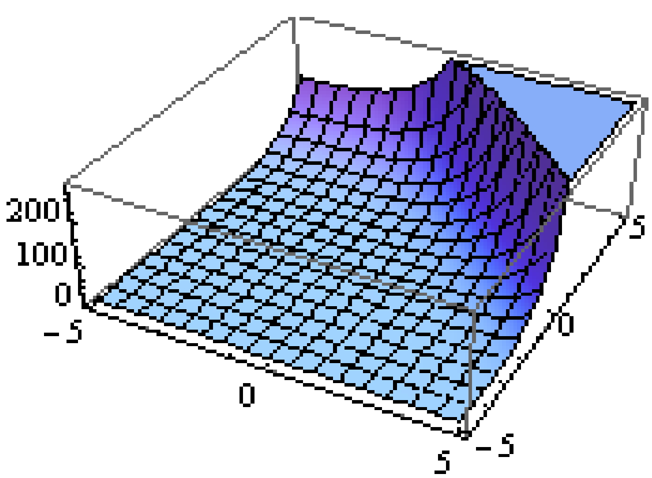

Figure 13. The streamline flow for A = B = C =1, k $=\upsilon = \lambda = b = 1$, $\gamma = \beta = 2$ and $\alpha = 0.5$.

$\psi(x\cos\beta + y\sin\beta, -x\sin\beta + y\cos\beta)$ are also solutions of the equation. For the creeping flow case in the solutions (1 to 7), (11) and (12) $a = -\dfrac{\upsilon}{\alpha\lambda}$ and in (13) $a = -\dfrac{\upsilon}{\alpha\gamma}$, $b = -\dfrac{\upsilon}{\lambda\gamma}$.

In this paper, the exact solutions of non-linear equation governing the flow of second grade fluid in the steady case are obtained through generalized separation of variables method by assuming certain form of the stream functions. The expressions for the stream line, velocity and vorticity distributions are constructed in each case. The physical interpretations of the results are given with the help of several graphs. Figures 1 to 13 for the stream functions are plotted for different values of the integration constants A, B, C and D and for arbitrary parameters α, β, γ and λ respectively. Our results are strongly depends on the non-Newtonian parameter α_1. It is obtained that increase in α_1 leads to decrease in vorticity and vice versa.

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