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Full Length Research Paper

Some classification results on totally umbilical proper slant and hemi-slant submanifolds of a nearly Kenmotsu manifold

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This paper is concerned with the study of slant and hemi-slant submanifolds of a nearly Kenmotsu manifold. We prove that every totally umbilical proper slant submanifold of a nearly Kenmotsu manifold is totally geodesic and derive the integrability conditions of involved distributions in the definition of a hemi-slant submanifold. Finally, we obtain a classification theorem on a totally umbilical hemi-slant submanifold of a nearly Kenmotsu manifold.

Key words: Slant submanifold, hemi-slant submanifold, totally umbilical, minimal submanifold, nearly Kenmotsu manifold.

INTRODUCTION

Kenmotsu (1972) introduced a class of almost contact Riemannian manifolds known as Kenmotsu manifolds. Mihai et al. (1976) generalized the notion of Kenmotsu manifold. In fact they introduced the structure of $f-$ Kenmotsu manifold. Several authors studied the semi-invariant submanifold of Kenmotsu manifolds and $f-$ Kenmotsu manifolds, a generalized version of Kenmotsu manifold. It is therefore worthwhile to study the slant and hemi-slant submanifolds of a nearly Kenmotsu manifold. Recently, we have studied the semi-invariant submanifolds of a nearly Kenmotsu manifold (Khan et al., 2007). A nearly Kenmotsu structure on an almost contact metric manifold is given by a slightly weaker condition than that of a Kenmotsu manifold.

Slant immersion in complex geometry were defined by Chen (1990) as a natural generalization of both holomorphic and totally real immersions. Many authors have studied such slant immersions in almost Hermitian manifolds. Lotta (1996) introduced the notion of slant immersion of a Riemannian manifold into an almost contact metric manifold. Later on, slant submanifolds in Sasakian manifolds have been studied by Cabrerizo et al. (2000). Moreover, Papaghiuc (1994) introduced a class of submanifolds in an almost Hermitian manifold, called the *semi-slant* submanifolds, such that the class of proper CR-submanifolds and the class of slant submanifolds appear as particular cases in the class of semi-slant submanifolds. Recently, Carriazo (2002) defined and studied bi-slant immersion in almost Hermitian manifold and termed it as an anti-slant submanifold. However, the term anti-slant may suggest that the submanifold has no slant part. Hence, Sahin (2009) studied and define the notion of hemi-slant submanifolds. The purpose of the present paper is to study the slant and hemi-slant submanifolds of a nearly Kenmotsu manifold.

Subsequently, we review basic formulas and definitions for a nearly Kenmotsu manifold and their sub-manifolds, which we shall use later. Afterwards, we recall the definition and some basic properties of slant

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submanifolds. Also, we prove that a totally umbilical proper slant submanifold of a nearly Kenmotsu manifold is totally geodesic. This study then dealt with hemi-slant submanifolds of a nearly Kenmotsu manifold. Here, we obtain the integrability conditions of the distributions of a hemi-slant submanifold and classify all totally umbilical hemi-slant submanifolds of a nearly Kenmotsu manifold.

PRELIMINARIES

Let $\overline{M}$ be (2m+1)-dimensional almost contact metric manifold together with a metric tensor g , a tensor field ϕ of type $(1,1)$, a vector field ξ , a 1-form η on $\overline{M}$ which satisfy (Blair, 1976).

$$\phi^2 = -I + \eta \otimes \xi, \phi\xi = 0, \eta(\phi) = 0, \eta(\xi) = 1, \eta(X) = g(X,\xi) \quad (2.1)$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), g(\phi X, Y) = -g(X, \phi Y) \quad (2.2)$$

for any vector field X, Y on M . If in addition to above relations,

$$(\overline{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X \quad (2.3)$$

then it is called Kenmotsu manifold. We also have on a Kenmotsu manifold $\overline{M}$

$$\overline{\nabla}_X \xi = X - \eta(X)\xi.$$

If in the tensorial equation (2.3) the right hand side is zero then the manifolds have a *cosyplectic structure*, that is for a cosymplectic manifold we have $(\overline{\nabla}_X \phi)Y = 0$ and from this relation we get $\overline{\nabla}_X \xi = 0$.

The almost contact metric manifold $\overline{M}$ is called a nearly Kenmotsu manifold if it satisfy the following condition

$$\left.\begin{aligned} &(i)\ (\overline{\nabla}_X \phi)X = -\eta(X)\phi X,\\ &(ii)\ (\overline{\nabla}_X \phi)Y + (\overline{\nabla}_Y \phi)X = -\eta(X)\phi Y - \eta(Y)\phi X \end{aligned}\right\} \quad (2.4)$$

for all $X, Y \in T\overline{M}$.

If the tensorial relation (2.4) is identically vanish then the manifold is called *nearly cosymplectic*. Hence, the nearly cosymplectic structure equations are:

(i) $(\overline{\nabla}_X \phi)Y + (\overline{\nabla}_Y \phi)X = 0$ or equivalently $(\overline{\nabla}_X \phi)X = 0$ and

(ii) $\overline{\nabla}_X \xi = 0,$ for any $X, Y \in T\overline{M}.$

Now, let M be a submanifold immersed in $\overline{M}$. The Riemannian metric induced on M is denoted by the same symbol g . Let TM and $T^{\perp}M$ be the Lie algebra of vector fields tangential to M and normal to M respectively and ∇ be the induced Levi-Civita connections on M , then the Gauss and Weingarten formulas are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y), \quad (2.5)$$

$$\overline{\nabla}_X V = -A_V X + \nabla_X^{\perp} V \quad (2.6)$$

For any $X, Y \in TM$ and $V \in T^{\perp}M,$ where $\nabla^{\perp}$ is the connection on the normal bundle $T^{\perp}M$, h is the second fundamental form and A_V is the Weingarten map associated with V as

$$g(A_V X, Y) = g(h(X,Y), V) \quad (2.7)$$

For $X \in TM$, we write

$$\phi X = TX + NX, \quad (2.8)$$

where $TX \in TM$ and $NX \in T^{\perp}M.$ Similarly, for $V \in T^{\perp}M,$ we have

$$\phi V = tV + nV, \quad (2.9)$$

where tV (resp. nV) is the tangential component (resp. normal component) of $\phi V.$

Now, for any $X, Y \in TM$, let us denote the tangential and normal parts of $(\overline{\nabla}_X \phi)Y$ by $P_X Y$ and $Q_X Y$, respectively. Then we decompose

$$(\overline{\nabla}_X \phi)Y = P_X Y + Q_X Y \quad (2.10)$$

Thus, by an easy computation, we obtain the following formulae

$$P_X Y = (\overline{\nabla}_X T)Y - A_{NY} X - th(X,Y) \quad (2.11)$$

$$Q_X Y = (\overline{\nabla}_X N)Y + h(X,TY) - nh(X,Y) \quad (2.12)$$

Similarly, for any $V \in T^{\perp}M$, denoting tangential and normal parts of $(\overline{\nabla}_X \phi)V$ by $P_X V$ and $Q_X V$

respectively, we obtain

$$P_X V = (\overline{\nabla}_X t)V + TA_V X - A_{nV} X \quad (2.13)$$

$$Q_X V = (\overline{\nabla}_X n)V + h(tV, X) + NA_V X \quad (2.14)$$

The covariant derivative of the morphisms T, N, t and n are defined respectively as

$$(\overline{\nabla}_X T)Y = \nabla_X TY - T\nabla_X Y \quad (2.15)$$

$$(\overline{\nabla}_X N)Y = \nabla_X^{\perp} NY - N\nabla_X Y \quad (2.16)$$

$$(\overline{\nabla}_X t)V = \nabla_X tV - t\nabla_X^{\perp} V \quad (2.17)$$

$$(\overline{\nabla}_X n)V = \nabla_X^{\perp} nV - n\nabla_X^{\perp} V \quad (2.18)$$

for all $X, Y \in TM$ and $V \in T^{\perp}M$.

A submanifold M of an almost contact metric manifold $\overline{M}$ is said to be totally umbilical if

$$h(X, Y) = g(X, Y)H, \quad (2.19)$$

where H is the mean curvature vector. If $h(X, Y) = 0$ for any $X, Y \in TM$, then M is said to be *totally geodesic* and *minimal* if $H = 0$.

SLANT SUBMANIFOLDS OF A NEARLY KENMOTSU MANIFOLD

Throughout the section we consider M as a proper slant submanifold of a nearly Kenmotsu manifold. For any $x \in M$ and $X \in T_x M$ if the vectors X and ξ are linearly independent, the angle $\theta(X) \in \left[0, \frac{\pi}{2}\right]$ between $\phi(X)$ and $T_x M$ is well defined. If $\theta(X)$ does not depend on the choice of $x \in M$ and $X \in T_x M$, we say that M is *slant* in $\overline{M}$. The constant angle θ is then called the *slant angle* of M in $\overline{M}$. The anti-invariant submanifold of an almost contact metric manifold is slant submanifold with slant angle $\theta = \frac{\pi}{2}$ and an invariant submanifold is slant submanifold with slant angle $\theta = 0$. If the slant angle θ neither zero nor $\frac{\pi}{2}$, then the slant submanifold is called a *proper slant* submanifold. If M is a slant submanifold of an almost contact manifold then the tangent bundle TM of M is decomposed as

$$TM = D_\theta \oplus < \xi > \quad (3.0)$$

where $< \xi >$ denotes the distribution spanned by the structure vector field ξ and D_θ is the complementary distribution of $< \xi >$ in TM, known as the *slant distribution* on M. For a proper slant submanifold M of an almost contact manifold $\overline{M}$ with a slant angle θ, Lotta (1996) proved that

$$T^2 X = -\cos^2\theta[X - \eta(X)\xi] \quad (3.1)$$

for any $X \in TM$. Recently, Cabrerizo et al. (2000) extended the above result into a characterization for a slant submanifold in a contact metric manifold. In fact, they obtained the following crucial theorem.

For a slant submanifold M of an almost contact metric manifold $\overline{M}$, the normal bundle $T^{\perp}M$ of M is decomposed as

$$T^{\perp}M = N(TM) \oplus \mu,$$

where μ is the invariant normal subbundle with respect to ϕ orthogonal to $N(TM)$.

Theorem 3.1.

Let $\mathbf{M}$ be a submanifold of an almost contact metric manifold $\overline{\mathbf{M}}$ such that $\xi \in \mathbf{TM}$ (Cabrerizo et al., 2000). Then, $\mathbf{M}$ is slant if and only if there exists a constant $\lambda \in [0,1]$ such that

$$\mathrm{T}^2 = \lambda(-\mathrm{I} + \eta \otimes \xi). \quad (3.2)$$

Furthermore, if θ is the slant angle of $\mathbf{M}$, then it verifies that $\lambda = -\cos^2\theta$.

The following relations are the consequences of the above theorem

$$g(TX, TX) = \cos^2\theta[g(X, Y) - \eta(X)\eta(Y)] \quad (3.3)$$

$$g(NX, NY) = \sin^2\theta[g(X, Y) - \eta(X)\eta(Y)] \quad (3.4)$$

for any $X, Y \in TM$.

In the following two theorems, we assume M as a totally umbilical proper slant submanifold of a nearly Kenmotsu manifold $\overline{M}$.

Theorem 3.2.

Let $\mathbf{M}$ be a totally umbilical proper slant submanifold of a nearly Kenmotsu manifold $\overline{\mathbf{M}}$, then $\mathbf{H} \in \boldsymbol{\mu}$ if and only if $\mathbf{M}$ has a nearly cosymplectic sturucture.

Proof

For any $U \in TM$, we have

$$h(U,TU) = g(U,TU)H = 0.$$

From equation (2.4)(i), (2.5), (2.6) and (2.8), the above equation reduced to

$$-\eta(U)\phi U = \phi(\nabla_U U + h(U,U)) + A_{NU}U - \nabla_U^{\perp}NU - \nabla_U TU$$

Which on making use of (2.8) and comparing tangential components simplifies as

$$-\eta(U)TU = T\nabla_U U - \nabla_U TU + th(U,U) + A_{NU}U.$$

Using (2.15) and (2.19), we obtain

$$\eta(U)TU = (\overline{\nabla}_U T)U - g(U,U)tH - A_{NU}U.$$

Taking the inner product with ξ , we get

$$g\big((\overline{\nabla}_U T)U,\xi\big) = g(g(H,NU)U,\xi)$$

$$or, \quad (\overline{\nabla}_U T)U = g(H,NU)U. \qquad (3.5)$$

Thus, the result follows from (3.5).

Theorem 3.3

A totally umbilical proper slant submanifold $\mathbf{M}$ of a nearly Kenmotsu manifold $\overline{\mathbf{M}}$ is totally geodesic if $\mathbf{H}$ **and** $\nabla_{\mathbf{U}}^{\perp}\mathbf{H}$ **lie** $\boldsymbol{\mu}$., for all $\mathbf{U} \in \mathbf{TM}$.

Proof

For any $U \in TM$, we have

$$\overline{\nabla}_U \phi U = (\overline{\nabla}_U \phi)U + \phi\overline{\nabla}_U U.$$

Using (2.4)(i), (2.5), (2.6) and (2.8), the above equation takes the form

$$\nabla_U TU + g(U,TU)H - A_{NU}U + \nabla_U^{\perp}NU = -\eta(U)\phi U + T\nabla_U U + N\nabla_U U + g(U,U)\phi H.$$

Taking the product with ϕH and using (2.2), we obtain

$$g(\nabla_U^{\perp}NU,\phi H) = -\eta(U)g(\phi U,\phi H) + g(N\nabla_U U,\phi H) + g(U,U)g(H,H).$$

Using the fact that $H \in \mu$, we derive

$$g(\nabla_U^{\perp}NU,\phi H) = g(U,U) \parallel H \parallel^2.$$

Then from equation (2.6), we get

$$g(\overline{\nabla}_U NU,\phi H) = g(U,U) \parallel H \parallel^2. \qquad (3.6)$$

Now for any $U \in TM$, we have

$$\overline{\nabla}_U \phi H = (\overline{\nabla}_U \phi)H + \phi\overline{\nabla}_U H$$

Using (2.6), (2.8) and (2.10), we obtain

$$-A_{\phi H}U + \nabla_U^{\perp}\phi H = P_U H + Q_U H - TA_H U - NA_H U + t\nabla_U^{\perp}H + n\nabla_U^{\perp}H.$$

Taking the product with NU and in view of fact $n\nabla_U^{\perp}H \in \mu$ the above equation becomes

$$g(\nabla_U^{\perp}\phi H, NU) = -g(NA_H U, NU) + g(Q_U H, NU).$$

In view of (3.4) and (3.7), we get

$$g(\nabla_U^{\perp}\phi H, NU) = -\sin^2\theta\{g(U,U)g(H,H) - \eta(A_H U)\eta(U)\} + g(Q_U H, NU) \qquad (3.7)$$

Since $\overline{\nabla}$ is a metric connection and NU and ϕH are orthogonal, thus on using (2.6), we derive

$$g(\overline{\nabla}_U NU,\phi H) = \sin^2\theta g(U,U) \parallel H \parallel^2 - g(Q_U H, NU) \qquad (3.8)$$

Then from (3.6) and (3.8), we obtain

$$0 = \cos^2\theta g(U,U) \parallel H \parallel^2 + g(Q_U H, NU). \qquad (3.9)$$

Now from (2.14), we have

$$g(Q_U H, NU) = g((\overline{\nabla}_U n)H, NU) + g(NA_H U, NU).$$

Using (3.4), we obtain

$$g(Q_U H, NU) = g((\overline{\nabla}_U n)H, NU) + \sin^2\theta g(U,U) \parallel H \parallel^2 \qquad (3.10)$$

Thus from (3.9), (3.10) and (2.18) and the fact that

$H \in \mu$, we obtain

$$g(U,U) \parallel H \parallel^2 = 0. \qquad (3.11)$$

Since M is proper slant then it follows from (3.11) that $H = 0$, that is, M is totally geodesic in $\overline{M}$. Hence, the theorem is proved.

HEMI-SLANT SUBMANIFOLDS OF A NEARLY KENMOTSU MANIFOLD

In this section we will obtain the integrability conditions of the distributions of hemi-slant submanifold of a nearly Kenmotsu manifold. Also, we obtain a classification result on a totally umbilical hemi-slant submanifold in a nearly Kenmotsu manifold.

Definition 4.1

Let $\overline{\mathbf{M}}$ be an almost contact Riemannian manifold and $\mathbf{M}$ a real submanifold of $\overline{\mathbf{M}}$. Then $\mathbf{M}$ is said to be hemi-slant submanifold of $\overline{\mathbf{M}}$ if there exist two orthogonal distributions $\mathbf{D}_\theta$ and $\mathbf{D}_\perp$ on $\mathbf{M}$ such that

1. $\mathbf{TM}$ admits the orthogonal direct decomposition $\mathbf{TM} = \mathbf{D}_\perp \oplus \mathbf{D}_\theta \oplus \langle \xi \rangle$.
2. $\mathbf{D}_\perp$ is anti-invariant distribution i.e., $\phi \mathbf{D}_\perp \subseteq \mathbf{T}^\perp \mathbf{M}$.
3. $\mathbf{D}_\theta$ is a non-zero slant distribution with slant angle $\theta \neq \frac{\pi}{2}$.

In this case, angle θ is a slant angle of $\mathbf{M}$. The anti-invariant distribution $\mathbf{D}_\perp$ of a hemi-slant submanifold is a slant distribution with angle $\theta = \frac{\pi}{2}$. It is clear that hemi-slant submanifolds are particular cases of bi-slant submanifolds. Moreover, it is also clear that if $\theta = 0$, then a hemi-slant submanifold is a CR-submanifold.

Lemma 4.1.

Let $\mathbf{M}$ be a hemi-slant submanifold of a nearly Kenmotsu manifold $\overline{\mathbf{M}}$, then the following holds

$$\phi[Z,W] = 3(A_{\phi W} Z - A_{\phi Z} W)$$

for all $Z, W \in D_\perp$.

Proof

For any $Z, W \in D_\perp$ and $U \in TM$, using (2.2), (2.5) and (2.7), we obtain

$$2g(A_{\phi W} Z, U) = -g(\phi \overline{\nabla}_Z U, W) - g(\phi \overline{\nabla}_U Z, W)$$

The above equation together with (2.4) (ii), implies

$$2g(A_{\phi W} Z, U) = -g(\overline{\nabla}_Z \phi U, W) - g(\overline{\nabla}_U \phi Z, W)$$
$$+\eta(Z) g(\phi U, W) + \eta(U) g(\phi Z, W)$$
$$= -g(\overline{\nabla}_Z \phi U, W) - g(\overline{\nabla}_U \phi Z, W).$$

Using (2.2) and (2.6), we get

$$2g(A_{\phi W} Z, U) = -g(\phi \overline{\nabla}_Z W, U) + g(A_{\phi Z} W, U).$$

Transvecting U both sides, we get

$$2A_{\phi W} Z = A_{\phi Z} W - \phi \overline{\nabla}_Z W. \qquad (4.1)$$

Similarly,

$$2A_{\phi Z} W = A_{\phi W} Z - \phi \overline{\nabla}_W Z. \qquad (4.2)$$

Thus, the result follows from (4.1) and (4.2) on their subtraction. In view of $Ker(T) = D_\perp$, this Lemma leads to the following proposition.

Proposition 4.1

Let $\mathbf{M}$ be a hemi-slant submanifold of a nearly Kenmotsu manifold $\overline{\mathbf{M}}$, then the anti-invariant distribution $\mathbf{D}_\perp$ is integrable if and only if

$$A_{\phi W} Z = A_{\phi Z} W,$$ for any $\mathbf{Z}, \mathbf{W} \in \mathbf{D}_\perp$.

Proof

The proof follows from the Lemma 4.1. Now, for the integrability of slant distribution $\mathbf{D}_\theta$, we have the following proposition.

Proposition 4.2

Let $\mathbf{M}$ be a hemi-slant submanifold of a nearly

Kenmotsu manifold $\bar{\mathbf{M}}$, then the slant distribution $\mathbf{D}_\theta$ is integrable if and only if

$$2(\bar{\nabla}_Y\phi)X + h(X,TY) - h(Y,TX) - \nabla_Y^{\perp}NX + \nabla_X^{\perp}NY \in ND_\theta$$

for any $X, Y \in D_\theta$.

Proof

For any $X, Y \in D_\theta$, we have

$$\phi[X,Y] = \bar{\nabla}_X\phi Y - (\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X - \bar{\nabla}_Y\phi X$$

From equation (2.4) (ii) and (2.8), we obtain

$$\phi[X,Y] = 2(\bar{\nabla}_Y\phi)X + \bar{\nabla}_X TY + \bar{\nabla}_X NY$$
$$-\bar{\nabla}_Y TX - \bar{\nabla}_Y NX + \eta(X)\phi Y + \eta(Y)\phi X.$$

Using (2.5) and(2.6), we get

$$\phi[X,Y] = 2(\bar{\nabla}_Y\phi)X + \nabla_X TY + h(X,TY) - A_{NY}X + \nabla_X^{\perp}NY$$
$$-\nabla_Y TX - h(Y,TX) + A_{NX}Y - \nabla_Y^{\perp}NX. \quad (4.3)$$

Taking inner product with FW, for any $W \in D_\perp$, we get

$$g(\phi[X,Y],FW) = g(2(\bar{\nabla}_Y\phi)X,FW)$$
$$+g(h(X,TY) - h(Y,TX) - \nabla_Y^{\perp}NX + \nabla_X^{\perp}NY, FW).$$

Thus on using (2.2), we obtain

$$g([X,Y],W) = g(2(\bar{\nabla}_Y\phi)X,\phi W) + \eta([X,Y])\eta(W)$$
$$+g(h(X,TY) - h(Y,TX) - \nabla_Y^{\perp}NX + \nabla_X^{\perp}NY, \phi W). \quad (4.4)$$

Hence we get our assertion from (4.4).

Theorem 4.1.

Let M be a totally umbilical hemi-slant submanifold of a nearly Kenmotsu manifold $\bar{\mathbf{M}}$, then atleast one of the following statements is true

1. $dimD_\perp = 1$
2. $H \in \mu$
3. $\mathbf{M}$ is proper slant that is, $\mathbf{D}_\perp = \{0\}$.

Proof

As we know that for a nearly Kenmotsu manifold $(\bar{\nabla}_U\phi)U = -\eta(U)\phi U$, using this fact we obtain

$$(\bar{\nabla}_Z\phi)Z = 0. \quad (4.5)$$

For any $Z \in D_\perp$. The tangential and normal components of (4.5) are $P_Z Z = 0$ and $Q_Z Z = 0$, respectively. From (2.10) and tangential component of (4.5), we obtain

$$(\bar{\nabla}_Z T)Z = A_{NZ}Z + th(Z,Z)$$

or,

$$T\nabla_Z Z = -Zg(H,NZ) - \| Z \|^2\, tH.$$

Taking the inner product with $W \in D_\perp$, we obtain

$$g(H,NZ)g(Z,W) + \| Z \|^2\, g(tH,W) = 0.$$

Thus, the equation (4.6) has a solution if either $dimD_\perp = 1$ or $H \in \mu$ or $D_\perp = \{0\}$. This completes the proof.
Now, we are in position to prove our main theorem.

Theorem 4.2

Let $\mathbf{M}$ be a totally umbilical hemi-slant submanifold of a nearly Kenmotsu manifold $\bar{\mathbf{M}}$, then atleast one of the following statements is true:

1. M is totally real,
2. M has a nearly cosymplectic structure
3. M is totally geodesic in $\bar{M}$, if $\nabla_U^{\perp}H \in \mu$, *for all* $U \in TM$,
4. $dimD_\perp = 1$.

Proof

Suppose $H \neq 0$ then by equation (3.11), we obtain $D_\theta = \{0\}$, thus from definition M is totally real which is case (i). If $D_\theta \neq \{0\}$ and $H \in \mu$, then by Theorem 3.2, M has a nearly cosymplectic structure. Moreover, if $\nabla_U^{\perp}H \in \mu$, for any $U \in TM$ and M has nearly cosymplectic structure, then by Theorem 3.3, M is totally geodesic in $\bar{M}$. Finally, if $H \boxed{\in} \mu$, then equation (4.6) has a solution if either $dimD_\perp = 1$ which is the case (iv). Thus, the theorem is proved completely.

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