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Full Length Research Paper

The hypernucleus ${}^{9}_{\Lambda}Be$ as a three body system

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The different properties of ${}^{9}_{\Lambda}Be$ hyper-nucleus are studied using hyper-spherical harmonics method. The ${}^{9}_{\Lambda}Be$ hyper-nucleus is considered as a three body system consisting of Λ particle and two α particles. The effect of different types of the α–α interactions on the calculated binding energy of Λ in ${}^{9}_{\Lambda}Be$ is studied. Different sets of Λ–α interactions are also considered. The Fabre optimal subset is adopted to obtain a good convergence for the calculated binding energy using the renormalized Numerove method. The present calculations are in good agreement with the available experimental data. The more realistic α–α interaction the more accurate the ${}^{9}_{\Lambda}Be$ binding energy obtained.

Key words: ${}^{9}_{\Lambda}Be$ hyper-nucleus, Λ particle and α particles, Λ–α interactions.

INTRODUCTION

The ${}^{9}_{\Lambda}Be$ hyper-nucleus serves as a very nice testing ground for the hyper nuclear cluster model for many different reasons. In fact, the ${}^{9}_{\Lambda}Be$ has a simple α–α–Λ structure at low excitation energies (Motoba et al., 1985). On the other hand, some complicated problems arise at high energies (Ymada et al., 1988) due to the α–α–Λ structure of the ${}^{9}_{\Lambda}Be$. Therefore, the theoretical study of this simple and complex structure is very interesting and challenging .Also we can give a prediction to the existence and realism of many interesting features such as super symmetric states (Dalitz and Gal, 1976) (hyper nuclear states of new symmetry), the drastic change of the electromagnetic transition states.

It is well known that the ${}^{9}_{\Lambda}Be$ hyper-nucleus is composed of two α particles and Λ hyperons. The structure of α particles in ${}^{9}_{\Lambda}Be$ hyper-nucleus is assumed to be not disturbed by the existence of other particles. So the distribution of the nucleons in the α particles in the ${}^{9}_{\Lambda}Be$ hyper-nucleus still keeps the same form as that obtained from the high energy electron scattering experiments with α particles (Daskalayannis et al., 1982) problem. It is also well known that hyper-nucleus ${}^{8}_{\Lambda}Be$ nucleus has the structure of two loosely bound particles (6.84±0.05). It might be expected that the hyper-nucleus ${}^{9}_{\Lambda}Be$ still keeps the structure of two α particles with Λ in it. Accordingly, the hyper-nucleus ${}^{9}_{\Lambda}Be$ can be strongly considered as a three body system. Therefore in calculating the binding energy, we have assumed the hyper-nucleus ${}^{9}_{\Lambda}Be$ as a three body system consisting of two α particles and a Λ particle.

The ground state energy of ${}^{9}_{\Lambda}Be$ hyper-nucleus has been calculated with different methods using different types of the α–α interactions. A Hartree Fock calculation was made by Chen et al. (Huazhong, 1984). Also cluster model calculation was made by Motoba and Banda, (1984) by solving a set of coupled integral equations with value of 6.39 MeV for the ${}^{9}_{\Lambda}Be$ binding energy. All these calculations obtained binding energies for the hyper-

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nucleus ${}^{9}_{\Lambda}Be$ ground state that are still far from the experimental value of 6.71 MeV. Microscopic variational calculations were made by Emiko Hiyama et al. (1997) in the framework of the Jacobian coordinate Gaussian basis method (Hiyama et al., 1996) using cluster model with binding energies around the experimental value. Faddeev calculations (Vladimir et al., 2004) in configuration space were performed using various $\alpha-\Lambda$ interactions and $\alpha-\alpha$ interactions. In these calculations, only the nuclear component of the $\alpha-\alpha$ interactions was taken into account giving binding energy values in disagreement with the experimental one due to the influence of the repulsive core of the $\alpha-\alpha$ interaction. The different properties of the ${}^{9}_{\Lambda}Be$ were studied applying different methods of calculations using relativistic mean filed theory (Yoshikazu et al., 2004) and unified quark-model baryon- baryon interactions (Hong et al., 2006) . The hyper-spherical harmonics method (HH) was also applied to determine the ground state energy of ${}^{9}_{\Lambda}Be$ using different forms of $\alpha-\Lambda$ and $\alpha-\alpha$ interactions (Verma and Sural, 1979; Verma and Sural, 1980; Sing-nang and Shuang-he, 1986; Sing-nan et al., 1988)

The aim of the present work is to calculate the binding energy of Λ hyperons in the ${}^{9}_{\Lambda}Be$ hyper-nucleus using different types of $\alpha-\Lambda$ and $\alpha-\alpha$ interactions. Our calculated results are then compared with the available experimental data as well as with those obtained applying other methods. The hyper-spherical harmonics method (HH) is applied where the Fabre optimal subset (Ali and Bomer, 1966; Ballot et al., 1980; Osman et al., 1991) is used to obtain a set of coupled differential equations to be numerically solved in order to obtain the energy eigen values and eigenfunctions of the ${}^{9}_{\Lambda}Be$ hyper-nucleus. The more solved equation, the more accurate the binding energy obtained.

In the (HH) method, the wave function describing a system of N particles (in the center of mass system) is expanded in terms of a complete set of orthonormal functions of 3N-4 variables. The expansion coefficients are functions of a single variable that represents the length of 3N-3 dimensional vector. By substituting the wave function expansion into the Schroedinger equation describing the system, one obtains an infinite set of coupled differential equations for the expansion coefficients. A multipole potential is also obtained by expanding the two body interaction on a complete set of hyper-spherical harmonics. This multi-pole potential is very helpful and useful when used in the Schroedinger equation.

As for the three-body system, the angular harmonics are functions of five angular variables. In order to determine the potential matrix of the three-body Schroedinger equation, the matrix elements of the multi-pole potentials between a pair of such hyperspherical harmonics were calculated. The symmetry of the system under study rules out some harmonics from appearing in the set of coupled equations. Further, the centrifugal barrier terms occurring in the set of coupled equations grow considerably with higher harmonics. One can therefore, truncate this infinite set (Ballot et al., 1980; Osman et al., 1991; Osman and Allam, 1995) and work with a finite set of coupled differential equations or a corresponding one dimensional integral equation. In the work of Verma and Sural (Verma and Sural, 1979; Verma and Sural, 1980) a set of coupled differential equations was converted into a set of coupled integral ones which were then solved numerically to give binding energy of the ${}^{9}_{\Lambda}Be$ nucleus. The numbers of equations that have to be retained in any calculation using the (HH) method will, of course, depend on the nature of the potential used. In the present work, the Fabre optimal subset was used to obtain a converged set of coupled differential equations in a single variable, namely, the hyper-radius. By numerically solving these equations, the eigenvalues and eigenfunctions of the hyper-nucleus ${}^{9}_{\Lambda}Be$ wave function were determined.

THEORETICAL WORK

Let the position vectors of the two α particles be denoted by r_1 and r_2, respectively, and their masses by $m_{\square}$. The position vector of the Λ particle is denoted by r_3 and its mass by m_{Λ}. The total mass of the hyper-nucleus ${}^{9}_{\Lambda}Be$ is $M = 2m_{\square} + m_{\square}$ and the mass of α particle is taken to be $m_{\square} = 4m$ and that for the lambda Λ particle is taken to be $m_{\square} = 6/5\ m$, where m is the nucleon (proton or neutron) mass. The Jacobi coordinates set used by Clare and Levinger (1985) was chosen here:

$$\eta = \alpha(r_1 - r_2) \qquad (1)$$

$$\xi = \beta(r_1 + r_2 - 2r_3) \qquad (2)$$

where

$$\alpha = \sqrt{\frac{2}{3}} \qquad and \qquad \beta = \frac{\alpha}{2}\sqrt{\frac{m_{\Lambda}}{m_{\Lambda} + 2m}}$$

The inter-particle separation is expressed as:

$$r_{12} = \sqrt{\frac{3}{2}}\eta\,, \quad r_{13} = \frac{1}{2\beta}\xi + \frac{1}{2\alpha}\eta\,,$$

and

$$r_{23} = \frac{1}{2\beta}\xi - \frac{1}{2\alpha}\eta \qquad (3)$$

Now, the hyper-spherical coordinates ρ and θ are introduced, where ρ is the hyper-radius and θ is the hyper-spherical angle. As a result, the following relations are obtained

$$\eta = \rho\sin\theta \quad , \quad \xi = \rho\cos\theta \quad , \quad \rho^2 = \eta^2 + \xi^2 \qquad (4)$$

with

$$\tan\theta = \frac{\eta}{\xi} \qquad (0 \le \theta \le \frac{\pi}{2}) \qquad (5)$$

The non relativistic Schroedinger equation for the hyper-nucleus ${}^{9}_{\Lambda}Be$ nucleus, after separating out the motion of the center of mass, can be written as:

$$\left\{\frac{-\hbar^2}{2\mu}(\nabla^2_{\eta} + \nabla^2_{\xi)} + \sum_{i>j} V(r_{ij}) - E\right\}\Psi(\eta,\xi) = 0 \qquad (6)$$

where μ is chosen to be μ = M/3 and $V(r_{ij})$ is the two-body central potential. Therefore, the Schroedinger equation for the hyper-nucleus ${}^{9}_{\Lambda}Be$ system expressed in terms of the hyper-spherical coordinates becomes:

$$\left\{\frac{-\hbar^2}{2\mu}\nabla^2 + V(\rho) - E\right\}\Psi(\rho) = 0 \qquad (7)$$

Expanding the wave function Ψ(ρ) on the HH basis, $Y_{(L)}(\Omega)$, gives

$$\Psi(\rho) = \rho^{\frac{-5}{2}} \sum_{[L]} Y_{[L]}(\Omega) U_{[L]}(\rho) \qquad (8)$$

where Ω is a set of 5 angular coordinates describing the position of a point at the surface of the unit hyper-sphere. (L) is a set of 5 numbers labeling the polynomial, and $U_{(L)}(\rho)$ are the radial wave functions. Substituting the expansion (8) into Equation (7), yields an infinite set of second order coupled differential equations written as:

$$\{\frac{-\hbar^2}{\mu}[\frac{d^2}{d\rho^2} - \frac{(L+2)^2 - \frac{1}{4}}{\rho^2}] - E\}U_{[L]}(\rho) + \sum_{[L]}\langle Y_{[L]}(\Omega)V(\rho)Y_{[L']}\rangle U_{[L']}(\rho) = 0 \qquad (9)$$

which is subsequently truncated in order to be treated numerically. This can be done by using the Fabre optimal subset (Ballot et al., 1980). As a result, the infinite set of coupled equations is transformed into a finite set of coupled ones to be solved numerically. For the case of central potential, the ground state of the hyper-nucleus ${}^{9}_{\Lambda}Be$ nucleus is described by even number of the grand orbital momentum L = 2K + l, due to the parity conservation. Then, the finite set of coupled equations, for orbital momentum l = 0, becomes:

$$[T_{2K} - E]U_{2K}(\rho) + \sum_{K'}(-)^{K} V^{2K'}_{2K''}(\rho)U_{2K'}(\rho) = 0 \qquad (10)$$

with the kinetic energy operator and the potential matrix elements being expressed as:

$$T_{2K} = \frac{-\hbar^2}{\mu}[\frac{d^2}{d\rho^2} - \frac{(2K+2)^2 - \frac{1}{4}}{\rho^2}] \qquad (11)$$

and

$$V^{2K'}_{2K''}(\rho) = \sum_{K''} C^{2K'}_{2K''}(K'',0,C)V_{2K''}(\rho) \qquad (12)$$

respectively. The geometrical coefficients, $C^{2K'}_{2K''}(K'',0,C)$, appearing in Equation (12) couple the set of equations with the main equation for which K = 0 for each component of the central components of the two-body potentials. Explicit expressions for these coefficients are given in Ballot et al. (1980). The multi-pole potentials, $V_{2K''}(\rho)$, given in Equation (12) introduces the multi-poles of the central parts of the two-body potential and are expressed as :

$$V_{2K''}(\rho) = \sum_{i>j} V^{ij}_{2K''}(\rho) \qquad (13)$$

with

$$V^{ij}_{2K''}(\rho) = \frac{16}{\pi}\int_0^{\infty} dx \sqrt{1-(\frac{x}{\rho})^2}\, x^2 V(x^2)\,{}_2F_1(-K, K+2, \frac{3}{2}, (\frac{x}{\rho})^2) \qquad (14)$$

where V(x) = $V(r_{ij})$ is the two-body potential. In the hyper-nucleus ${}^{9}_{\Lambda}Be$, the two body potentials refers to the α–α and the Λ-α interactions. Equation (14) may be written as:

$$V^{ij}_{2K}(\rho) = \frac{16}{\pi}\int_0^1 du(1-u^2)^{\frac{1}{2}} u^2 V(\rho u)\,{}_2F_1(-K, K+2, \frac{3}{2}, u^2) \qquad (15)$$

with V(ρu) = $V(r_{ij})$, which is more useful in our numerical calculations.

NUMERICAL CALCULATIONS AND RESULTS

In order to carry out the numerical calculations, the set of coupled differential equations represented by Equation (10) were written in matrix form as:

$$\left\{[I]\frac{d^2}{d\rho^2} + [Q]\right\}U(\rho) = 0 \qquad (16)$$

where (I) is the unit matrix and the column vector U(ρ) contains the partial waves $U_{2K}(\rho)$ as its components. Also the matrix element (Q) is given by:

$$[Q(\rho)] = \frac{\mu}{\hbar^2}\left\{E[I] - V_{eff}(\rho)\right\} \qquad (17)$$

The components of the effective potential matrix is given by:

$$V_{eff}(\rho) = V^{2K'}_{2K}(\rho) + \frac{\hbar^2}{\mu}[\frac{2K(K+1) + \frac{15}{4}}{\rho^2}] \qquad (18)$$

where $V^{2K'}_{2K}(\rho)$ is given by Equation (12).

The renormalized Numerov method (Jonson, 1978; Gordon 1969) was then used to solve (Ballot et al., 1980;

Osman et al., 1991; Osman and Allam, 1995) the set of coupled equations (16). In order to study the convergence of the hyper-nucleus $^{9}_{\Lambda}Be$ energy eigenvalue, the Fabre optimal subset was used, using a different Gaussian forms of the $\alpha-\Lambda$ and $\alpha-\alpha$ interactions. In order to attain the convergence of $^{9}_{\Lambda}Be$ eigenvalue, we have included terms up to K = 16. Therefore in studying the convergence of the hyper-nucleus $^{9}_{\Lambda}Be$ binding energy, we have solved K coupled equations for K = 1,2,.....,K_{max}. Also, the decrease in hyper-nucleus $^{9}_{\Lambda}Be$ binding energy was noted by increasing the number of terms in the HH expansion of the wave function. We have solved K = 16 coupled equations in order to obtain for the hyper-nucleus $^{9}_{\Lambda}Be$ binding energy value of -6.584 MeV, which is in good agreement with the experimental value of -6.71 MeV and more accurate than that obtained by other methods.

In the present calculations, a two body potential of Gaussian form is used for both of the α -α and Λ-α pairs. In order to study the effect of types of interactions on the $^{9}_{\Lambda}Be$ binding energy, we have used three types of α-α interactions. The first interaction is a simple one which is suggested by Buck et al. (1977) and used by Verma and Sural (1979). The second is a realistic one which is suggested by Ali and Bomder (1966) and used by Singnan et al. (1988). The third one is a more realistic one which is obtained by double folding the nuclear density of the two α particles into the Volkov2 (Volkov, 1965) nucleon–nucleon interaction and is used by Shuang-he et al. (1988). All the considered types of α–α interactions are of Gaussian shape and given by the following relation:

$$V^{\alpha\alpha}(r) = \sum_{i=1}^{6} V_i \exp\left(-\frac{r^2}{d_i^2}\right)$$

where the parameters V_i and d_i are given in Table 1.

We have considered for the coulomb part of the above α-α interactions, the following coulomb potential **$V_C(r)$** which is given by (Verma and Sural, 1979):

$$V_C(r) = 4e^2 erf(\lambda r)/r$$

with $\lambda = 0.75$ fm^{-1}.

For the Λ-α interaction, we have considered one that is used to obtain the correct binding energy of Λ hyperons in $^{5}_{\Lambda}He$.We have used the so called Verma and Sural (VS3) ρ_{DG} Λ-α interaction given by Daskalayannis et al. (1982) in which a double Gaussian form nucleon density distribution of α particle is considered and named as ρ_{DG}. In the present calculations, we have studied the effect of different sets of $\alpha-\alpha$ interactions on the $^{9}_{\Lambda}Be$ binding energy as well as convergence of the $^{9}_{\Lambda}Be$ binding energy. We have considered that the hyper-nucleus $^{9}_{\Lambda}Be$ has spin 1/2 and for the α particle, the spin is zero and that for Λ particle is 1/2. Therefore, we have used the important property that the total orbital momentum of the $^{9}_{\Lambda}Be$ ground state is zero with zero magnetic momentum.

DISCUSSION AND CONCLUSIONS

In the present work, the $^{9}_{\Lambda}Be$ hyper-nucleus was investigated by using a Gaussian shape α-α and Λ-α two body potentials. The effect of three sets of α-α interaction on the $^{9}_{\Lambda}Be$ binding energy was studied. The first interaction is a simple one suggested by Buck et al. (1977) which is used by Verma and Sural (1979). The second is a realistic one which is suggested by Ali and Bodmer (1966) and used by Singnan et al. (1988) .The third one is a more realistic one which is obtained by double folding the nuclear density of the two α particles into the Volkov2 (Volkov, 1965) nucleon–nucleon interaction and is used by Shuang-he et al. (1988). All the considered sets of α–α interactions are of Gaussian shape. For the Λ-α interaction, we have used VS3 ρ_{DG} interaction (Daskalayannis et al., 1982) in which the nucleon distribution of the α particles is considered to be of double Gaussian form. In applying the HH method, the Fabre optimal subset was used so as to obtain a fast and good convergence for the hyper-nucleus $^{9}_{\Lambda}Be$ binding energy. In applying the Buck et al. (1977) α–α potential (set1), up to K = 18 coupled equations were solved to obtain for $^{9}_{\Lambda}Be$ hyper-nucleus binding energy a value of (-6.521) MeV, which is close to the experimental value (-6.71MeV) and is more accurate than that obtained by Verma and Sural (Hiyama et al., 1997) (-6.39 MeV) when solving a coupled set of integral equations in the K-Harmonics method. The first four partial waves corresponding to the Buck et al. (1977) α–α potential are given in Figure 1. Using the Ali and Bodmer (1966) α–α potential (set2), up to 16 coupled equations were solved to obtain for the $^{9}_{\Lambda}Be$ binding energy a value of (-6.543 MeV) which is close to the experimental value and more accurate than that obtained by Singnang King (Sing-nang and Shuang-he, 1986) (-7.275 MeV). The first four partial waves corresponding to the Ali and Bodmer (Sing-nan et al., 1988) α–α potential are given in Figure 2. In using the double folded α–α potential (Verma and Sural, 1980) (set3) up to K = 14 coupled equations were solved to obtain for the $^{9}_{\Lambda}Be$ hyper nucleus a binding energy a value of (-6.814 MeV), which is in good agreement with the experimental value (-6.71MeV) and more accurate than that obtained by Shuang-he Shi et al. (1988) (-

Table 1. Parameters for the different sets of Gaussian shape α–α interactions.

Pot	i	1	2	3	4	5	6
Ref(12)	V_i (MeV)	-122.623	-	-	-	-	-
	d_i (Fm)	2.132	-	-	-	-	-
Ref(14)	V_i (MeV)	-160	400	-	-	-	-
	d_i (Fm)	2.105	1.275	-	-	-	-
Ref(15)	V_i (MeV)	-296.3	869	-73.82	506.36	-55.03	405.89
	d_i (Fm)	1.4925	2.1274	1.8728	2.029	1.2728	1.6838

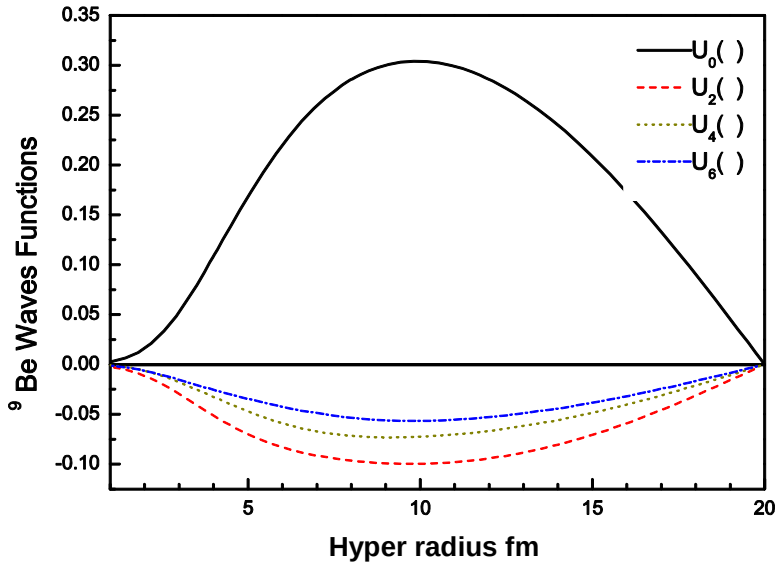

Figure 1. First four hyperradial partial waves $U_{(2K)}$ for the hypernucleus ${}^{9}_{\Lambda}Be$ generated by Buck et al (1986) α–α interaction.

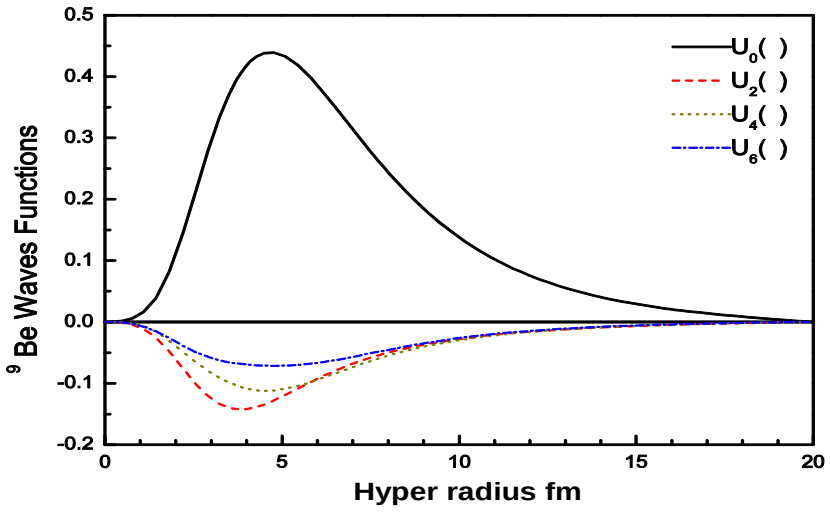

Figure 2. First four hyperradial partial waves $U_{(2K)}$ for the hypernucleus ${}^{9}_{\Lambda}Be$ generated by Ali and Bodmer (1988) α–α interaction.

Table 2. Energy eigenvalues E_K of ${}^{9}_{\Lambda}Be$ as a function of the number K of coupled equations.

K	0	2	3	4	5	6	7	
E_K(MeV)	-1.024	-1.928	-2.713	-3.364	-3.806	-4.219	-4.714	
K	8	9	10	11	12	13	14	Exp.
E_K(MeV)	-5.148	-5491	-5.824	-6.198	-6.279	-6.481	-6.584	-6.71

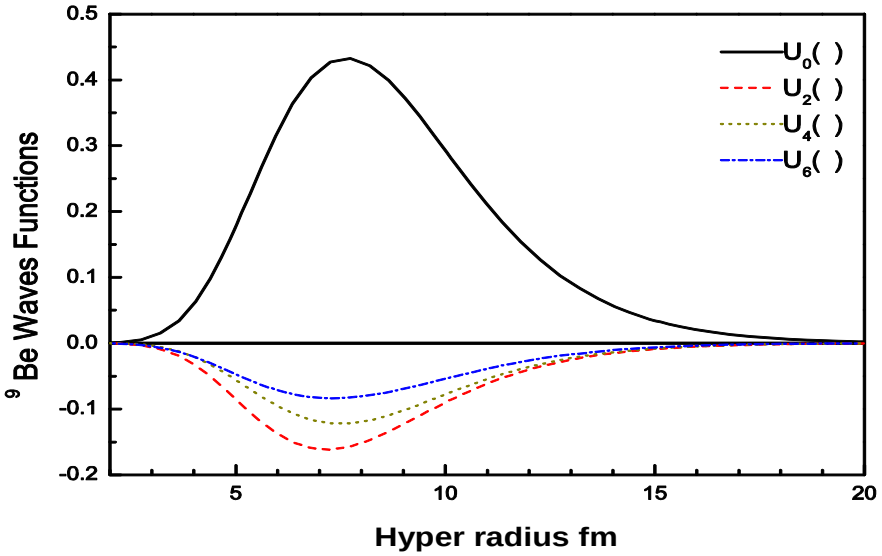

Figure 3. First four hyperradial partial waves $U_{(2K)}$ for the hypernucleus ${}^{9}_{\Lambda}Be$ generated by double folded (Verma and Sural, 1979) α–α interaction.

6.9025 MeV). The decrease in the calculated binding energy value and the corresponding increase in the number K of the coupled equations for the VS3 ρ_{DG} Λ–α interactions (Daskalayannis et al., 1982) together with the double folded α–α potential (Verma and Sural, 1980) (set3) are presented in Table 2. This indicates the fast convergence for the ${}^{9}_{\Lambda}Be$ binding energy values which is obtained when using the Fabre optimal subset.

The first four partial waves generated by these potentials are shown in Figures 1 to 3. It is shown from the figures that, the obtained hyper radial partial waves are nodeless, a general feature of the lowest partial waves of the ground state of an infinite set of coupled equations corresponding to local but non-central potentials (Ballot et al., 1980; Osman et al., 1991). The sudden drop of the first four partial waves of Figure 1 indicates the importance of using more realistic α–α potential than the simple one (Buck et al., 1977) used by Verma and Sural. (1979). Thus these three figures provide a useful study for the effect of different sets of α–α interactions on the ${}^{9}_{\Lambda}Be$ properties as well as a good check on the accuracy of the present numerical work. As a future work, electromagnetic properties for the ${}^{9}_{\Lambda}Be$ nucleus, such as the magnetic and charge form factors, could be studied making use of the partial waves generated by different α-α and Λ-α two body potentials.

Finally, it is concluded that, the ${}^{9}_{\Lambda}Be$ hyper-nucleus binding energy is affected by using different sets of the α–α interactions in which both the nuclear and coulomb components should be taken into account. The more realistic the α–α interactions, the more accurate the values for the ${}^{9}_{\Lambda}Be$ binding energy. We also conclude that our calculated ${}^{9}_{\Lambda}Be$ binding energy (-6.584 MeV), which is in good agreement with the experimental value of (-6.71 MeV), is more accurate than that obtained by other methods (Verma and Sural, 1979; Verma and Sural, 1980; Sing-nang and Shuang-he, 1986; Sing-nan et al., 1988; Volkov, 1965). Furthermore, using the Fabre optimal subset gives a good and faster convergence in the calculated binding energy.

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