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Full Length Research Paper

Optimal sensor placement for fault detection and isolation by the structural adjacency matrix

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In this paper, we consider the optimal sensors placement problem for faults detection and isolation. When the detection and isolation of faults of an existing system's sensors is impossible or uncertain, a reconfiguration sensor placement of this system should be reconsidered. The propagation of faults (Observability/Detectability) was studied in a structured linear system on a directed graph (digraph). An optimisation of sensor placement is necessary to isolate detectable faults and obtain an optimal structural isolability. To have a maximal structural isolability, we must find all the vertex-disjoint paths knowing that this property depends upon additional sensors. This novel approach is illustrated over a two tanks application applying on a directed graph to obtain the maximal and optimal structural detectability and isolability of the system.

Key words: Optimal sensor placement, fault detection and Isolation, linear structured systems, digraphs, structural adjacency matrix.

INTRODUCTION

This paper deals with the fault detection and isolation (FDI) problem of linear systems. This research work has received considerable attention in the past years (Isermann, 1997; Chen et al., 1996; Frank, 1996; Patton, 1994). Generally in the FDI problems, intrinsic solvability conditions were considered depending upon the internal structure of the system but not on the specific parameter values. We look for internal structures which are well suited for diagnosis.

A structural controllability was introduced by Lin (1974). The FDI problem solvability conditions were given by Commault et al. (2002) in terms of graph that can be associated in a natural way to a structured system. In general, the FDI solvability conditions are still not perfect. A specific attention in system observation, supervision and abrupt change is given to sensor placement problems (Bassville et al., 1987; Krysander and Frisk, 2008; Commault and Dion, 2007; Commault and Dion, 2003; Frisk and Krysander, 2007; Fan et al., 2009; Chamesedine et al., 2007).

The structural and graphical approaches have also been used in fault detection problems in various contexts (Dion et al., 2003; Commault and Dion, 2003; Frisk and Krysander, 2007; Fan et al., 2009; Izadi-Zamanabadi and Staroswiecki, 2000; Mellal et al., 2011; Chamesedine et al., 2007).

This study contributes on the sensor placement optimisation by generating new structural properties allowing the easiest and the maximum faults detectability and isolability in the system.

Moreover, the proposed matrix (the structural adjacency matrix) inspired from the graph theory, allow us to associate a digraph with a linear structured system without going through an analytical approach. This approach aid to present system's structure and it is useful in diagnostic domain. The obtained matrix allows us to optimise the sensors placement. However, in the existing literature structural representation of linear systems considers only the observability (the structural observability problem) (Commault and Dion, 2007; Commault and Dion, 2003; Lin, 1974; Boukhobza et al., 2007; Chamesedine et al., 2007). In addition, this developed matrix has never been used in this context

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and makes it easy to obtain the structural digraph without need to a prior complete system state or analytical model.

LINEAR STRUCTURED SYSTEM

Let us consider the following linear time-invariant system

$$\sum\nolimits_{\Lambda}\begin{cases}\dot{x}(t) = Ax(t) + Lf(t)\\ y(t) = Cx(t) + Mf(t)\end{cases} \quad (1)$$

Where $x(t) \in R^n$, is the state vector, $f(t) \in R^r$, is the fault vector and $y(t) \in R^p$, is the measured output vector. A, C, L and M are matrices with their corresponding dimensions. In this part we recall some definitions and results on linear structured systems (Commault and Dion, 2003). More details can be found in the work of Commault et al. (2010), Dion and Commault (1993), and Dion et al. (2003). The system given by Equation 1 is called a linear structured system if the entries of the composite matrix J fixed zeros or independent parameters (not related by algebraic equations). Where $\Lambda = \{\lambda_1, \lambda_2 ..., \lambda_k\}$ denotes the set of independent parameters of the composite matrix J = (A C; L M).

For the sake of simplicity the dependence of the system matrices on Λ will not be made explicit in the notation. A structured system represents a large class of parameter dependent linear systems. The structure is given by the location of the fixed zero entries of J. This structure often comes from physical particularities of the system (as a subsystem interconnection); thus, the only exact information of the system is its structure (that is, the absence of direct relations between variables and state variables for example).

For such systems one can study generic properties that is, properties which are true for almost all values of the parameters collected in Λ (Murota, 2000; Wonham, 1985).

Moreover, a property is said to be generic (or structural) if it is true for all values of the parameters (that is, any $\Lambda \in R^k$) outside a proper algebraic variety of the parameter space, that is, the zero set of a finite number of nontrivial polynomials in the parameters.

A directed graph G(ΣΛ) = (Z, W) can be associated to the structured system ΣΛ of Equation 1 where the matrix is $J = \begin{pmatrix} A & L \\ C & M \end{pmatrix}$ structured.

The vertex set is Z = F ∪ X ∪ Y where F, X and Y are the faults, the states and the output sets are given by {f1, f2..., fr}, {x1, x2..., xn} and {y1, y2..., yp}, respectively. The arc set is W = {(fi, xj) | Lji≠0} ∪ {(xi, xj) | Aji≠0} ∪ {(xi, yj) | Cji≠0} ∪ {(fi, yj) | Mji≠0}, where Aji(resp. Cji ,Lji ,Mji) denotes the entry (j, i) of the matrix A(resp. C ,L,M).
Moreover, the directed path in G(ΣΛ) from the vertex iμ0 to the vertex iμq is the arcs sequence (iμ0, iμ1), (iμ1, iμ2)..., (iμq−2, iμq−1), (iμq−1, iμq) such that iμt∈ Z for t = 0, 1..., q and (iμt−1, iμt) ∈ W for t=1, 2...,q.

The path length is the number of its arcs, each arc being counted the number of times it appears in the sequence. For the last sequence, the path has length q. Sometimes, we denote the *path* P by the sequence of vertices such that P = (iμ0, iμ1..., iμq−1, iμq).

Moreover, if iμ0 ∈ F and iμq ∈ Y, thus, P is called a fault-output path. A path where iμ0 = iμq is called a circuit. A path set with no common vertex is said to be a disjoint vertex.

A *k*-linking is a set of k vertex disjoint fault-output paths; it is also called a linking of size *k*. A linking is maximal when k is maximal.

Using the structured systems with theirs associated graphs many important results have been obtained for these systems on structural controllability, decoupling, disturbance rejection (Commault et al., 2010; Dion and Commault, 1993; Dion et al., 2003; Lin, 1974; Boukhobza et al., 2007).

As first example of these definitions the graph characterization of the structural observability were given by Lin (1974) and Murota (2000).

Let us consider ΣΛ to be the linear structured system given by Equation 1 with its associated graph G(ΣΛ). The system (in fact the pair(C, A)) is structurally observable if and only if:

1) There exists a state-output path starting from any state vertex in X,
2) There exists a set of vertex disjoint circuits and state-output paths which cover all state vertices.

However, in the existing literature of structural representation, the linear systems consider only the structural observability problem. In this paper, we look to define new structural proprieties for the FDI problem.

FDI PROBLEM SOLVABILITY WITH A SENSOR PLACEMENT OPTIMIZATION

It has been seen that the solvability of the FDI problem (observability condition) is based on the maximal size of fault-output linking. Here, consider some properties of these maximal linking and derive some useful consequences for fault detection problem and then for isolation of detectable faults.

Solvability of the fault detection problem with additional sensors

Here, we propose a method on the solvability of the fault detection problem with additional sensors. In graph terms, we try to get a size r linking based on the addition new output vertices and new edges which link the new states or fault vertices to them.

We define F1 as the set of fault vertices, Y1 the set of output vertices and X1 the set of state vertices in any fault-output paths from F1 to Y1.

Now, we think of a new structured system defined by its graph with input set F1 and output set Y1, state set X1 is the set of edges which corresponds to the edges in any path from F1 to Y1.

A variable w_i is said to be measurable with additional sensors if we can add a new sensor with its corresponding output z_j such that in the graph of the composite system G(ΣΛ), there is an edge (w_i, z_j) between this variable and the new output. Let us denote by Wm the set of measurable faults and state variables.

Lemma 1

Consider a linear structured system ΣΛ with graph G(ΣΛ). If we suppose that the FDI problem without additional sensors has no solution thus, the FDI problem with additional sensors has a solution if and only if:

$$(F1 \cup X1) \cap Wm \neq \Phi \quad (2)$$

Proof: Since the FDI problem without additional sensors has no solution, therefore F1 ≠ Φ. If the condition of Equation 2 is not satisfied, there is no edge connecting F1 ∪ X1 to the additional vertices of Z, therefore the FDI problem with additional sensors has no solution. To check this, if we do additional measurements which allow to solve the problem by simply adding new output vertices and new edges and verify on the modified graph that the condition is satisfied.

New structural propositions

Up to this point, we have defined the FDI problem as a classical problem based on the satisfaction of the observability condition (Lin, 1974). Now, we define new structural proprieties to represent a complete FDI problem (Krysander and Frisk, 2008; Frisk and Krysander, 2007), such as structural detectability and structural isolability.

As defined previously, the detectability condition in terms of linear structured systems is the same as the observability condition.

The additional sensor set lets the observability condition be maximal and full, we speak now about the maximal FDI.

Proposition 1 (Maximal structural detectability)

In the oriented graph associated to a linear structured system we can detect a fault if there is a path from the fault vertex to the output vertex and only one state vertex is on the path. We get maximal fault detection, if all faults vertices are connected to the output by adding new sensors (output vertices).

Proof: Since the fault detection problem without additional sensors has no solution, because the condition of Equation 2 is not satisfied as a result is no edge connecting F1∪ X1 to the additional vertices of Z (we suppose that the studied system has some fault-states links not connected to the output). Note that a k-linking is a set of k vertex disjoint fault-output paths. A linking is maximal when k is maximal. By adding the corresponding additional vertices of Z to the fault-state no output-connected will take place, while in a full output connection, we will have maximal fault detection and an optimal sensor placement.

Now we have to test if the new sensors placement with the maximal fault detection allows us to isolate all faults.

As given in the previous paragraphs the system ΣΛ without additional sensors represented by the graph G(ΣΛ) is not satisfied over the condition given by Equation 2, therefore, it has no maximal k-linking. So, it has at most one non-detected fault, which means, not all the faults are detectable and so we cannot isolate the faults.

Proposition 2 (Maximal structural isolability)

In the oriented graph associate to a linear structured system we can isolate a detectable fault if we have maximal faults detection, and if there is a path from the fault vertex to the output vertex and only one state vertex is on the path.

Proof: In the system ΣΛ without additional sensors represented by the graph G(ΣΛ) did not have a maximal linking, noting that at least one fault does not belong to any linking, thus, this fault is not detectable. In this case, there exist some faults that cannot be isolated from others because they have common outputs. By adding a new sensor to the system ΣΛ represented by a new output-edge in the graph G(ΣΛ) it will make a linking to fault-output path. Thus, each fault vertex is detected with just one sensor (output vertex) connected to a shared state vertex.

We consider this as a maximal isolation of all the detectable faults, but how about an optimal sensor placement that allows to isolate the maximal faults (without additional sensors). This will be solved by Proposition 3.

Proposition 3 (Optimal structural isolability)

In the oriented graph associated to a linear structured system we can isolate a maximum of detectable faults without any additional sensor; this is called an optimal isolability. The optimum isolation of all the detectable faults using the proprieties in the digraph is obtained by changing the direction of one arc connected to the state vertex between two adjacent faults vertices, which share

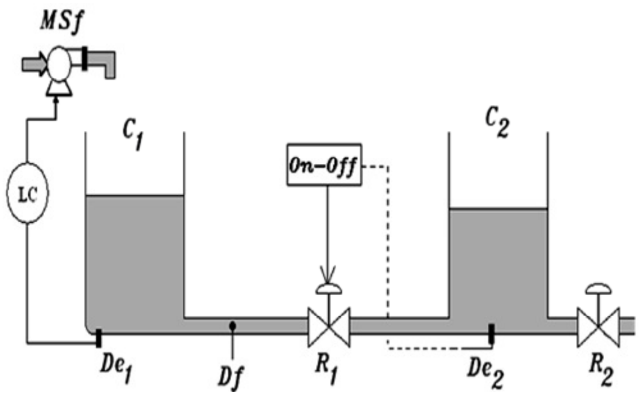

Figure 1. Two tanks system.

the same non-connected output state vertex (without losing the structural Observability condition).

Proof: In a system ΣΛ without additional sensors represented by the graph G(ΣΛ), some faults share the same state vertex and they are connected to a common output vertex that make them observable, but not detectable and not isolable. So in each fault-output path we can make a linking but it is not allowed at the same time (except if we add a new output). However, by changing the arc connected between the two states (from the shared state to the non-connected output state) we get two linking (two detected/isolated faults). This isolation of the faults is practical in example for visual faults and remarkable changes.

STUDY CASE

This study case was chosen to test our proposed approach with the existing ones such as bond graph and FDI problems (Benazzouz et al., 2009).

The chosen application consists of two tanks system given in Figure 1. C1 and C2 are the two tanks, R1 and R2 are two valves, Msf is the pump, u1 and u2 are commands, De1, De2 and Df1 are sensors used for detecting faults and monitoring the system. r1, r2 …, r6 are the residues or the faults indicators (Mellal et al., 2011). Mb is monitorability/detectability of the fault if Mb = 1, the fault is detectable on the element of the system, in this application all faults are detectable with a maximal faults detection. Ib is the isolability of the detectable fault if Ib = 1, the fault is isolable from the other faults. In this application the elements C2 and R2 have the same fault signatures, therefore, they give the same information on the fault and we cannot locate the faulty element (Mellal et al., 2011).

The fault signature matrix of two tanks system (Benazzouz et al., 2009) is given in Table 1. To isolate all the faults we must add new sensors in the system. Based on the proposed analysis, the digraphs were used to solve the FDI problem and isolate all the detectable faults of the two tanks system, because the digraph illustrates the fault propagation through a system.

To draw the directed graph of the two tanks system the structural adjacency matrix was defined. This matrix is a squared matrix composed by elements of the system in lines and columns (Alem and Benazzouz, 2011).

Definition 1

Assume G = (V, A) a digraph, where V is the vertices and A is the arcs of the graph G. The structural adjacency matrix of G is a squared matrix, M = (mij), of size n × n, defined as:

$$\begin{cases} \text{mij} = 1 & \text{if } (i, j) \in A \\ \quad 0 & \text{else} \end{cases} \tag{3}$$

To obtain the adjacency matrix and the digraph of the two tank system, Figure 1 was referred to and Table 1 to select the concerned elements and to verify the structural connections between them. Table 2 represents the structural adjacency matrix of the two tanks system. This matrix illustrates flows and information propagations through the system.

Note that, the structural adjacency matrix is zero-diagonal because there are no circuits (between the elements themselves) and is symmetric. Considering sensors De1, De2 and Df1, which are used to detect faults on elements Msf, C1, R1, C2 and R2 of the two tanks system. Note that f1, f2, f3, f4 and f5 designates faults of elements Msf, C1, R1, C2 and R2, respectively. Figure 2 represents the digraph of the two tanks system.

To apply our proposed approach to solve the FDI problem for the two tank system, new sensors to detect the maximum of faults were first added because it can be seen from the graph given in Figure 2 that R2 has no direct connection with the output. This remark is also confirmed in Table 2. Therefore, we propose to add Df2 as a sensor, this will allow that the fault f5 affecting R2 is detectable and isolated from the fault f4 affecting C2. This problem was mentioned Table 1, where Ib = 0 for C2 and R2.

With the additional sensor set all the faults on the elements can be detected because each of these is connected directly to the output (first proposition).

However, Msf is a pump and the faults on the upstream sources are remarkable and known (that is, faults on the commands). By eliminating the commands u1 and u2 we will simplify the system. Based on the graph, if we inverse the effect of the fault f1 on the Msf this will isolated directly from the others faults, since it is easy to isolate a fault on sources and inputs (Commault et al., 2002; Frisk and Krysander, 2007) (second and third proposition).

The obtained new directed graph is given in Figure 3. Based on this new graph, we can isolate all the detectable faults of the two tanks system.

Table 1. Fault signature matrix of two tanks system.

Variable	r_1	r_2	r_3	r_4	r_5	r_6	M_b	I_b
CI	1	0	0	0	0	0	1	1
RI	0	1	0	0	0	0	1	1
C2	0	0	1	0	0	0	1	0
R2	0	0	1	0	0	0	1	0
Msf	1	0	0	1	0	0	1	1
u_1	0	0	0	1	0	1	1	1
u_2	0	0	0	0	1	0	1	1
De_1	1	1	0	0	0	1	1	1
De_2	0	1	1	0	1	0	1	1
Df_1	1	0	1	0	0	0	1	1

Table 2. Structural adjacency matrix of the two tanks system.

Two tank	Msf	C1	RI	C2	R2	De1	De2	v
Msf	0	1	0	0	0	1	0	0
C1	1	0	1	0	0	1	0	1
R1	0	1	0	1	0	0	1	1
C2	0	0	1	0	1	0	0	0
R2	0	0	0	1	0	0	0	0
De1	1	1	0	0	0	0	0	0
De2	0	0	1	1	0	0	0	0
Df1	0	1	1	0	0	0	0	0

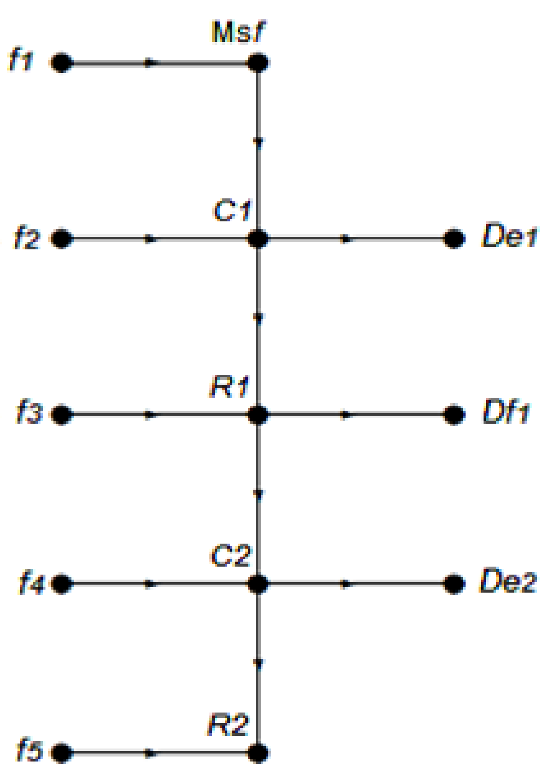

Figure 2. Digraph of two tanks system.

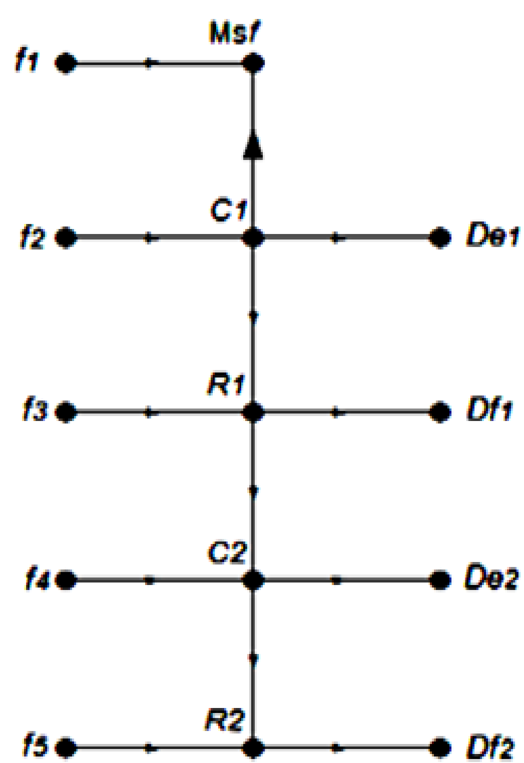

Figure 3. New digraph of two tanks system.

Conclusion

In this paper, we have considered a particular FDI problem and interested in a case when this problem has no solution using the measurements available on the system. We focused our goal of faults detection problem with additional sensors, then on the maximal and the optimal (structural) isolation of the detectable faults on linear structured systems by using oriented graphs (Digraphs).

We have presented the graph G(ΣΛ) which can be naturally associated with the structured system ΣΛ . This graph gives a visual representation of the internal structure and the solvability of several structural problems which can be extended to other applications.

The structural adjacency matrix has been defined as a simple method to draw the digraph. The obtained matrix is a qualitative approach to solve the FDI problems and the optimal sensors placement on system or components level which have been never used in this context. This matrix illustrates the propagation of information and the flows through the system. So it can be used in other contexts such as in system and control theory.

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