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Full Length Research Paper

The electrostatic potential at the center of associative magic squares

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Magic squares are one of the most wonderful math discussions which make a lot of scientific questions and problems in math and other sciences, very difficult to solve. With such a great world and with such history and long antiquity, their usages in physics are not so much. This work studies magic squares together with some physical concepts and in fact it analyzes the electrostatic potential at the center of natural and associative magic squares and derives interesting relations and results.

Key words: Associative magic square, electrostatic potential, contour plot.

INTRODUCTION

Authoritative writers on the history of mathematics still frequently assert that magic squares are of great antiquity, and that they have been known from very ancient times in China and India (Cammann, 1960).

Generally a natural magic square is a $n \times n$ ($n > 2$) matrix of the non-repetitive integers from 1 to n^2 such that the sum of each row, column, and main diagonals is equal to:

$$M_n = \frac{n(n^2+1)}{2} \qquad (1)$$

In which n is the order of the square and M_n is the magic constant (Mathworld, 2011).

In a natural magic square, if all pairs of elements which are antipodal to each other have the same pair sum:

$$a_{ij} + a_{n-i+1,n-j+1} = (n^2+1); \quad i,j = 1,\dots,n \qquad (2)$$

the square will be named associative as shown in Figure 1.

For odd n, the center element, which can be seen as pairing with itself, must be half of this constant (Loly et al., 2009).

As we know, integers $1, 2, 3, \dots, n^2$ can be located in $n^2!$ different ways in a $n \times n$ square but only some of these ways can create a magic square. Generally, without considering the magic squares resulting from reflection or rotation, number of these squares for $n = 3, 4, 5$ is exactly calculated, but it is good to know that for squares with higher order, scientists could only estimate the number of them with the Monte Carlo method. For example number of 8×8 associative magic squares is about $(2.5228 \pm 0.0014) \times 10^{27}$ (Pinn and Wieczerkowski, 1998; Trump, 2001; Gaspalou, 2005). Note that there are no singly-even associative magic squares (Benson and Jacoby, 1976).

Here, some examples are given as a pre-requisite for magic squares applications before going further. Suppose, point masses, electric charges and etc are mentioned instead of numbers. We review a summary of some papers about center of mass, moment of inertia and electric dipole moment.

When a rigid body has vibration or rotational motion in addition to translational motion, there is a point named center of mass which its motion is like one element motion that is affected by external forces. In fact, center of mass is a point that represents the whole body or parts of a system motion. If point masses are put in every

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square instead of numbers, the coordinates of the center of mass of all these discrete masses in relation to the center of the square will be point (0,0) and in its geometrical center. This is apparently obvious. Asker Ali Abiyev with defining a new kind of magic square, proved that center of mass of all natural magic squares of any order is their geometrical center. In fact, a novel method for the determination of mass center of Abiyev's natural magic squares is presented (Abiyeva et al., 2004; Abiyeva, 2011).

There is a tendency that a rigid body likes to have, in relation to a certain axis and to its first rotational situation. We simplify this expression by defining the quantity as the moment of inertia I (Halliday et al., 2004):

$$\mathrm{I} = \sum_i m_i r_i^2 \qquad (3)$$

Peter Loly shows that moment of inertia of natural magic squares of order n about an axis perpendicular to its centre only depends on the order of square and the row and column properties, and not on the diagonals of magic squares, so that it actually applies to the larger class of semi-magic squares which lack one or both diagonal magic sums of magic squares. He uses the feature of equality of columns and rows and also the perpendicular axis theorem, and he shows that the moment of inertia of a natural magic square of order n is equal to (Loly, 2004):

$$I_n = \frac{1}{12} n^2 (n^4 - 1) \qquad (4)$$

Also in the other paper, by deriving inertial tensor for magic cubes, shows that moment of inertia of order n of them are equal to (Rogers and Loly, 2004):

$$I_n = \frac{1}{12} n^3 (n^3 + 1)(n^2 - 1) \qquad (5)$$

Electric dipole moment is a molecules feature and is used in physics and chemistry in the subject of polarity of molecules. It is determined by the geometry (size, shape, and density) of the charge distribution. Thus, the dipole moment of a collection of point charges is:

$$P = \sum_{i=1}^{n} q_i r_i \qquad (6)$$

in which the r_i gives the displacement of charge q_i from the origin (Griffiths and College, 1999; Yadav et al., 2010).

If we put point electric charges instead of numbers in natural magic squares, like the proof of mass center problem, we can show that dipole moment of each square with any order in relation to center of the square is zero: $\sum_{i=1}^{n^2} q_i r_i = 0$; and if each magic square changes to its bone (Subtracting the average of the numbers in the square, $\frac{(n^2+1)}{2}$, from each number, yields a skeleton square of normalized numbers, "the bones", consisting of positive and negative numbers, and, for odd order squares, 0. This is a convenient way of seeing symmetry in the square), then dipole moment in relation to each arbitrary origin will be equal to zero (White, 2011). And in the other paper, with the electric multipoles expansion for magic cubes, shows that all of the components of the quadrupole moment vanish and in addition quadrupole of magic squares is non-zero (Rogers and Loly, 2005).

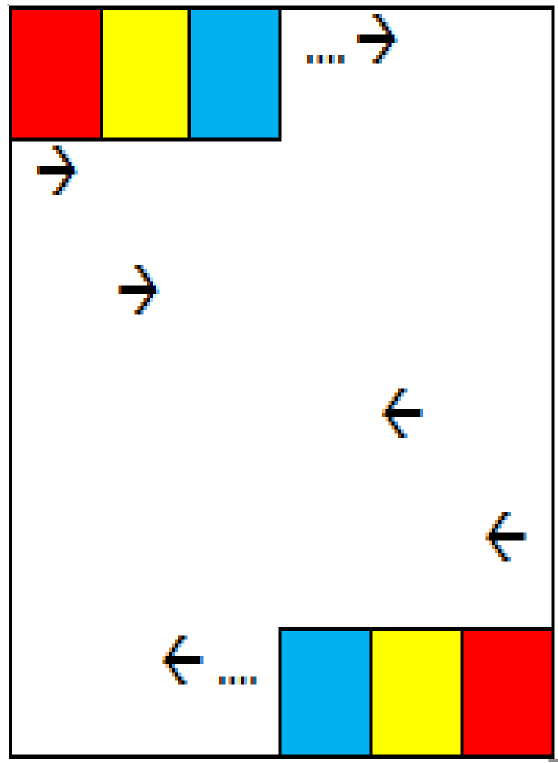

Figure 1. A schematic of associative magic squares.

In this work, at first we show some symmetry in contour plots of electrostatic potential of 3×3 and 4×4 natural magic squares with Mathematica software, then discuss about electrostatic potential at the center of 4×4 and other order of natural magic squares. After that, we analyze the electrostatic potential at the center of associative magic squares and demonstrate that value of "total electrostatic potential" at the center of associative magic squares is exactly equal to a constant amount which depends only on the order of the square and size of the distance between a cell and center of the square which is interesting because of the number of these magic squares for any order. At last, we will show that the electrostatic potential of associative magic squares is a mean state of the electrostatic potential at the center of n×n normal squares. We suppose that a n×n normal square is a matrix of the non-repetitive integers from 1 to n^2 with random substitution of numbers which is not magic. On the other hand, by considering some special conditions, we study that the electric dipole potential at these squares center is constant.

Contour plots of 3×3 and 4×4 natural magic squares

Electrostatic is a branch of physics which studies the static electric charges. We know that the curl of a radial vector $A = f(r)\hat{r}$ is zero, and that is expressible as the gradient of a scalar, and hence it is a conservative vector, so we can show that any electrostatic electric field can be written as a gradient of a scalar function, since

$$\vec{E}(r) = -\vec{\nabla}\varphi(r) \qquad (7)$$

and then

$$\vec{\nabla} \times \vec{E} = 0 \qquad (8)$$

Therefore E is representable as the gradient of a scalar function φ, as follows:

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{q}{|r-\hat{r}|} \qquad (9)$$

in which the function φ is called the electrostatic potential of the charge q and the SI unit of it, is joules per coulomb, which is defined as a volt (V) (Halliday et al., 2004; Nayfeh and Brussel, 1985; Arabshahi, 2011; Jaleh et al., 2011).

We drew the contour plot of electrostatic potential of single natural magic square of order 3 and all contour plots of order 4 (880 shapes). In general, we got different plots and only in some cases, saw symmetry in them as shown in Figure 2. There might be a relation in the sense of a study begun by Craig Knecht who found ponds and lakes by treating magic squares as an array of pillars. Also, thoughts are to see if there is any connection with the sets of singular values in the magic square spectra paper from Peter Loly (Knecht, 2007; Loly et al., 2009).

Electrostatic potential at the center of 4×4 natural magic squares

As we know, there is just one natural magic square of order 3 which the value of electrostatic potential at the center of it is about: $\frac{34.1}{4\pi\varepsilon_0}$. We calculated the total electrostatic potential at the center of 10 different samples of natural magic squares. It was interesting that all of the values were completely the same and equal to a constant amount. This feature is because of these squares special treatment and in fact regular and symmetrical sums at them. Now we easily demonstrate for all of the natural magic squares of order 4 that value of total electric potential in their center is fixed. Consider the following squares:

$$\begin{bmatrix} \alpha & \beta & \gamma & \delta \\ \varepsilon & \epsilon & \zeta & \eta \\ \theta & \vartheta & \iota & \kappa \\ \lambda & \mu & \nu & \xi \end{bmatrix}, \begin{bmatrix} A-a & C+a+c & B+b-c & D-b \\ D+a-d & B & C & A-a+d \\ C-b+d & A & D & B+b-d \\ B+b & D-a-c & A-b+c & C+a \end{bmatrix} \qquad (10)$$

In 1910 Bergholt found a parameterization for the fourth order natural magic square as shown in right square of Equation (10), which it shows (Bergholt, 1910; Loly et al., 2009):

→

$$\alpha+\delta+\lambda+\xi = \epsilon+\zeta+\vartheta+\iota = A+B+C+D = M_4\,(magic\ sum) \qquad (11)$$

So:

$$\varphi = \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n^2}\frac{q_i}{r_i} = \frac{\alpha+\delta+\lambda+\xi}{\sqrt{4.5}} + \frac{\epsilon+\zeta+\vartheta+\iota}{\sqrt{0.5}} + \frac{\beta+\gamma+\varepsilon+\eta+\theta+\kappa+\mu+\nu}{\sqrt{2.5}} = \frac{34}{\sqrt{4.5}} + \frac{34}{\sqrt{0.5}} + \frac{68}{\sqrt{2.5}} \approx 107.117\right]$$

By studying and checking the higher order of natural magic squares, we found out that we cannot give an overall formula about certain series of natural magic squares. Values of potential at the center of squares with odd order (without considering any charges on square center), $4k+2$ and other $4k$ orders were different.

Electrostatic potential at the center of associative magic squares

Here, we will show the more interesting symmetry in associative magic squares. By using the features of these kind of magic squares and after a few calculation, reasoning and proving a series, we demonstrate that value of "total electrostatic potential" at the center of associative magic squares is exactly equal to a constant amount which depends only on the order of the square and size of the distance between a cell and center of that square. It is so interesting to be able to arrange n^2

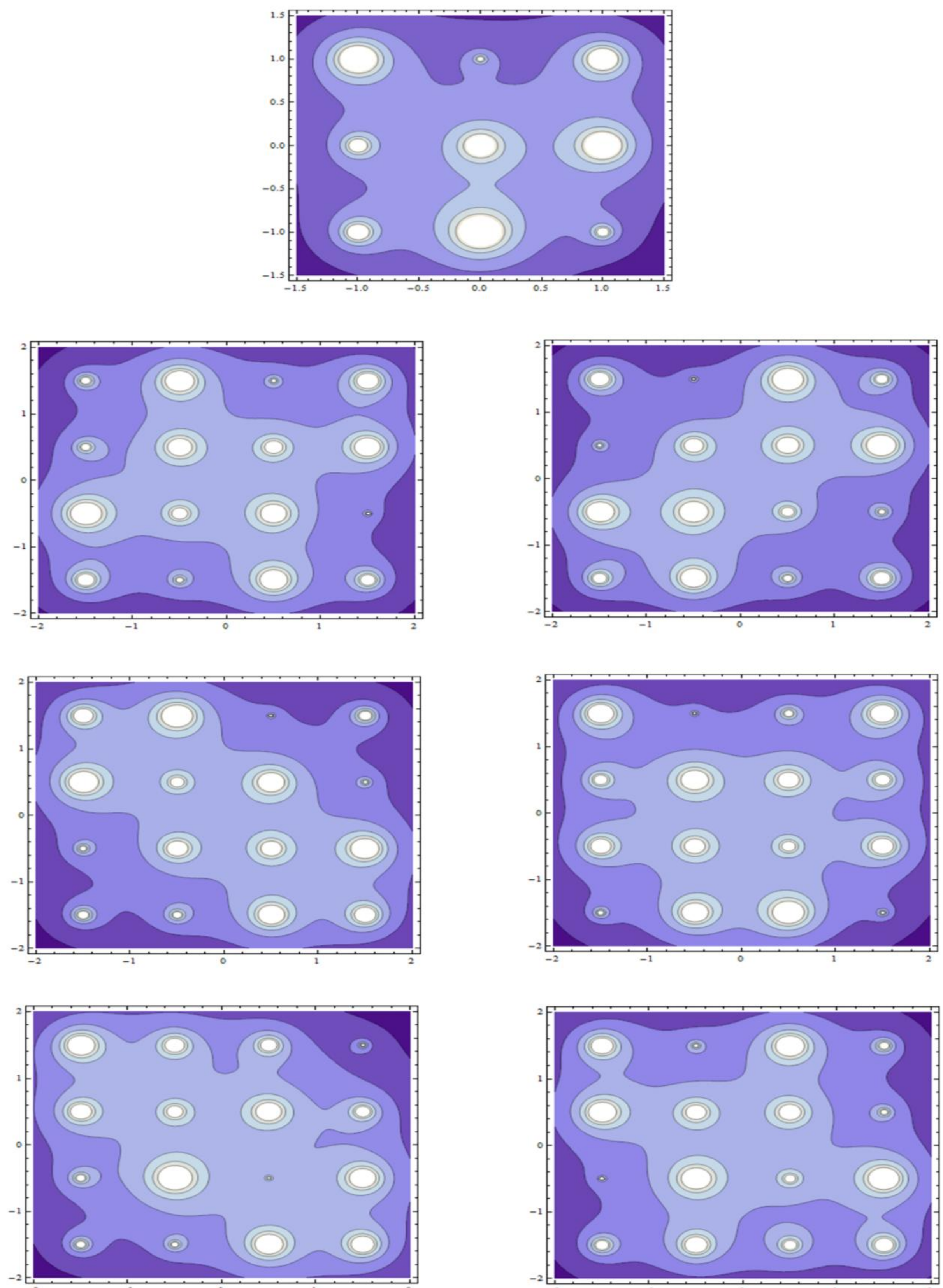

Figure 2. Contour plot of electrostatic potential of order 3 and some regular contour plots of order 4 include a, b, c, d, e, f and g.

different electric charges that are in period of 1 to n^2 in many ways as to have a fixed value for total potential at the center of the square which is only related to square order and the "r" in that certain order. In attention to Equation (2), we should prove that the amount of the following series for any value of n is constant.

$$\varphi = \frac{1}{2} \times \frac{1}{4\pi\varepsilon_0} \sum_{i,j=1}^{n} \frac{a_{ij} + a_{[n-i+1],[n-j+1]}}{r_{a_{ij}}} = constant; \quad \left(r_{a_{ij}} = r_{a_{[n-i+1],[n-j+1]}} \right) \quad (12)$$

For square with odd order ($2k+1$), we have:

$$\varphi = \frac{1}{4\pi\varepsilon_0} \left[\frac{a_{1,1} + a_{n,n}}{r_{a_{1,1}}} + \frac{a_{1,2} + a_{n,n-1}}{r_{a_{1,2}}} + \frac{a_{1,3} + a_{n,n-2}}{r_{a_{1,3}}} + \cdots + \frac{a_{\frac{n+1}{2},\frac{n-1}{2}} + a_{\frac{n+1}{2},\frac{n+3}{2}}}{r_{a_{\frac{n+1}{2},\frac{n-1}{2}}}} \right] \quad (13)$$

Pair numbers that defined in associative magic squares are equal to each other so we factor from one of them, then make common denominator from all terms. For each square with certain order, the $r_{a_{ij}}$ are fixed, so adding or multiplying them is also a constant value and sum of our both magic numbers will be the fixed value of n^2+1. In fact, the antipodal pair charges factor out of the potential factors leave a purely geometrical sum of inverse distances as:

$$\varphi = \frac{1}{4\pi\varepsilon_0} \left[(n^2+1) \times C_1 \right] = C \quad (14)$$

For squares with even orders, only the indexes of last phrases will be changed but the result will always be the same.

$$\varphi = \frac{1}{4\pi\varepsilon_0} \left[\frac{a_{1,1} + a_{n,n}}{r_{a_{1,1}}} + \cdots + \frac{a_{\frac{n}{2},\frac{n-2}{2}} + a_{\frac{n+2}{2},\frac{n+4}{2}}}{r_{a_{\frac{n}{2},\frac{n-2}{2}}}} + \frac{a_{\frac{n}{2},\frac{n}{2}} + a_{\frac{n+2}{2},\frac{n+2}{2}}}{r_{a_{\frac{n}{2},\frac{n}{2}}}} \right]$$

(15)

As a simple analysis, we can say that if associative magic square changes to its bone, the sum of the potentials of two antipodal pair charges will be always zero and thus the total potential at the center of these squares will be zero.

C_1 value is equal to sum of inverses of all distances ($\sum 1/r$) which their number for odd orders is $\frac{n^2-1}{2}$ and for even orders is $\frac{n^2}{2}$, but some of these distances are repetitive values.

Some examples for C_1:

n = 3 $C_1 \approx$ 3.4142
n = 4 $C_1 \approx$ 6.3010
n = 5 $C_1 \approx$ 6.9101
n = 7 $C_1 \approx$ 10.4226
n = 8 $C_1 \approx$ 13.3235

Note that, as it showed on page 6, the electric potential for n=4 was about $\frac{1}{4\pi\varepsilon_0}$(107.117) which is consistent with the aforementioned example:

$$\varphi_4 \approx \frac{1}{4\pi\varepsilon_0} \left[(4^2+1) \times 6.3010 \right] \approx \frac{1}{4\pi\varepsilon_0} (107.117)$$

Now we want to prove that the electrostatic potential at the center of associative magic square of order n is equal to average of maximum and minimum electrostatic potential at the center of a $n \times n$ normal square.

$$\varphi_{CAMS} = \frac{\varphi_{Max_{CNS}} + \varphi_{Min_{CNS}}}{2} \quad (16)$$

"CAMS: center of associative magic square; CNS: Center of normal square". Maximum potential will be achieved when the summation of large electric charges is divided on minimum distances and the summation of small electric charges is divided on maximum distances. Also the minimum potential will be achieved when the summation of small electric charges is divided on minimum distances and the summation of large electric charges is divided on maximum distances.

Example:

$$\varphi_3 = \frac{\varphi_{Max_{CNS}} + \varphi_{Min_{CNS}}}{2} \approx \frac{1}{4\pi\varepsilon_0}\left[\frac{\frac{9+8+7+6}{1} + \frac{4+3+2+1}{\sqrt{2}} + \cdots + \frac{9+8+7+6}{\sqrt{2}}}{2} = \frac{10\left(\frac{4}{1}+\frac{4}{\sqrt{2}}\right)}{2} \approx 34.14\right]$$

$$\rightarrow \quad (1+\mathrm{n}^2), \left(2+(\mathrm{n}^2-1)\right), \left(3+(\mathrm{n}^2-2)\right), \ldots = (\mathrm{n}^2+1)$$

$$\rightarrow \quad \frac{\varphi_{Max_{CNS}}+\varphi_{Min_{CNS}}}{2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{(\mathrm{n}^2+1)\times 2\sum 1/r}{2} = (n^2+1)\times C_1\right] \qquad (17)$$

In fact, if the normal square changes to its bone, you will get as much negative as positive potentials at the center and thus zero will be the average of the potentials.

Function of electric potential is proportional to $(1/r)$. We have demonstrated that value of $\sum q\,(1/r)$ at the center of associative magic squares is constant. The associative condition, $a_{ij} + a_{n-i+1,n-j+1} = const$, makes we able to replace any arbitrary function of distance (r) instead of $(1/r)$, such as $f(r)$. It means that value of $\sum q_i f(r_i)$ at the center of associative magic squares is constant.

$$\sum q_i f(r_i) = Constant \qquad (18)$$

A (physical) electric dipole (P) consists of two equal and opposite charges ($\pm q$) separated by a distance d, where the approximate dipole potential at large distances will be as follows (Griffiths and College, 1999):

$$\varphi_{\mathrm{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{P.\hat{r}}{r^2} \qquad (19)$$

If we put the electric dipoles instead of numbers in squares, the d be unit ($d \ll r$) and the orientation of electric dipoles in a lattice be radial, consequently the equation (19) will be changed to: $\varphi_{\mathrm{dip}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$; which is one of the subsets of equation (18), so the electric dipole potential of our associative magic squares is constant at the center with these special conditions.

Conclusion

Science is a kind of attention to objects. Physicists are often looking for regularity and symmetry and their descriptions and the magic squares are one of the interesting sources in mathematics for them. By plotting the contours of electrostatic potential of natural magic squares, we can bring regular and symmetrical contours which are the essence of an interesting and new idea, but analyzing "how such regular contours can be derived directly and why some contour plots are not regular", needs to be studied in detail in the future. As we said, there might be a connection with topographical model of Craig Knecht or sets of singular values of magic squares. Between natural magic square of different orders, electrostatic potential just at the center of order 4 is a constant value which depends on the order of the square and size of the distance of charges from square center and then this beautiful connection is completely correct for any arbitrary order of associative magic squares which is because of existence of Equation (2) in addition to Equation (1) for these squares.

Electrostatic potential at the center of associative magic squares is equal to average of minimum and maximum potential at the center of normal squares. It seems that we are fun to play with numbers and in fact potential of our squares are a special case and completely balanced of non-magic squares.

Total value of the Equation (18) at the center of natural magic squares of order 3 and 4 and associative magic squares of an arbitrary order n is constant and depends on n and $f(r)$, which one of the sub results of it in physics is electric dipoles potential with radial orientation in our squares.

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