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Full Length Research Paper

Bianchi type I magnetized cosmological model for barotropoic fluid distribution in Lyra geometry

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Bianchi type I cosmological model for barotropic fluid distribution with magnetic field in Lyra geometry is investigated. To get the deterministic solution in terms of comic time t, we have assumed that σ^1_1 (eigenvalue of shear tensor σ^j_i) is proportional to expansion (θ). This leads to A = $(BC)^n$, where A, B and C are metric potentials and n is a constant. We also assume that current is flowing along x-axis, therefore, the magnetic field is in yz-plane. The behaviour of the model in the presence and absence of the magnetic field is discussed. We find that the model starts with a big-bang and the expansion in the model decreases as time increases. The displacement vector decreases slowly with time. The model possesses point type and cigar type singularities under different conditions. It has been shown that particle horizon exists in the model. The present investigation is new and is different from other author's solution. The physical and geometrical aspects of the model in the presence and absence of magnetic field are also discussed.

Key words: Bianchi I, magnetized, cosmological, barotropic, Lyra geometry.

INTRODUCTION

In the early stage of evolution of universe, the radiation was dominant over matter. But present day observations show that matter is dominant over radiation. Friedmann (1924) obtained the solution for dust distribution considering Friedmann-Robertson-Walker (FRW) space-time in which there was no need to introduce cosmological constant. But FRW model failed to describe early universe being unstable near singularity (Lifshitz and Khalatnikov, 1963). Therefore, spatially homogeneous and anisotropic Bianchi type I models are considered to understand the universe in its early stage of evolution.

The present day magnitude of magnetic energy is very small as compared to estimated matter density. It might not have been negligible during early stage of evolution of universe. Therefore, it is interesting to study cosmological model in the presence of magnetic field. Asseo and Sol (1987) speculated a primordial magnetic field of cosmological origin. Bronnikov et al. (2004) have pointed out that magnetic field vector specifies a preferred spatial direction, therefore, a cosmological model having global magnetic field is necessarily anisotropic. Thorne (1967) investigated locally rotationally symmetric (LRS) Bianchi type I cosmological model in the presence of magnetic field. Collins (1972) gave a qualitative analysis of Bianchi type I models with magnetic field. Cosmological models with magnetic field are also studied (Roy and Prakash, 1978; Bali, 1986; Bali and Tyagi, 1988; Bali and Jain, 1998).

Einstein derived his field equations of general relativity by geometrizing gravitation. Inspired by geometrizing gravitation, Weyl (1918) developed a theory to geometrize both gravitation and electromagnetism. But Weyl's theory was discarded due to non-integrability of length of vector under parallel displacement. Lyra (1951) introduced a gauge function into the structureless manifold and modified Riemannian geometry. This step removed the main obstacle of Weyl's theory and made length of vector integrable under parallel displacement. By introducing a new scalar tensor theory of gravitation, Sen (1957) investigated an analogue of Einstein's field equation. Halford (1972) has shown that constant

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displacement vector ϕ_μ, plays the role of cosmological constant in general relativity. In the frame work of Lyra geometry, a number of authors have investigated cosmological models (Singh and Singh, 1992; Singh et al., 2004; Rahman and Bera, 2001; Rahman et al., 2005; Pradhan et al., 2001, 2003; Bali and Chandnani, 2008; Ram et al., 2008). Bali et al. (2007) have investigated Bianchi type I dust filled universe in the presence of magnetic field in the frame work of Lyra geometry. Recently, cosmological models in the frame work of Lyra's geometry in different contexts are investigated by (Pradhan and Kumhar, 2009; Pradhan and Mathur, 2009; Pradhan and Yadav, 2009; Pradhan, 2009; Pradhan et al., 2011; Agarwal et al., 2011; Pradhan and Singh, 2011).

In this paper, we have investigated Bianchi type I barotropic fluid distribution cosmological model in the presence of magnetic field in the frame work of Lyra geometry. The physical and geometrical aspects of the model in the presence and absence of magnetic with singularities in the model are also discussed. We find that the model represents point type and cigar type singularities under different conditions (MacCallum, 1971). The results obtained are new and are different from other author's work.

METRIC AND FIELD EQUATIONS

We consider Bianchi type I metric in the form:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{1}$$

where A, B and C are functions of t-alone.

Energy momentum tensor T_i^j for perfect fluid distribution in the presence of magnetic field is given by:

$$T_i^j = (\rho + p)\, v_i v^j + p\, g_i^j + E_i^j \tag{2}$$

Einstein's modified field equation in normal gauge for Lyra's manifold given by Sen (1957) as:

$$R_i^j - \frac{1}{2}R\, g_i^j + \frac{3}{2}\phi_i\, \phi^i - \frac{3}{4}\phi_k\, \phi^k\, g_i^j = -T_i^j \tag{3}$$

(in geometrized units where $8\pi G = 1$ and $c = 1$)

where

$$v_i = (0,0,0,-1);\, v^i\, v_i = -1;\, \phi_i = (0,0,0,\beta(t));\, v_4 = 1,$$

p is isotropic pressure, ρ is the matter density, v^i is the fluid flow vector and β is the gauge function.

E_i^j is the electromagnetic field tensor given by Lichnerowicz (1967) as:

$$E_i^j = \bar{\mu}\left[\, |h|^2 \left(v_i\, v^j + \frac{1}{2}g_i^j\right) - h_i\, h^j \right] \tag{4}$$

$\bar{\mu}$ being magnetic permeability and h_i the magnetic flux vector defined by:

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijk\ell} F^{k\ell}\, v^j \tag{5}$$

where $F^{k\ell}$ is the electro-magnetic field tensor and $\in_{ijk\ell}$ is the Levi-Civita tensor density. We assume that current is flowing along x-axis, so magnetic field is in the yz-plane. Thus, $h_1 \neq 0,\ h_2 = 0 = h_3 = 0 = h_4$ and F_{23} is the only non-vanishing component of F_{ij}. This leads to $F_{12} = 0 = F_{13}$ by virtue of Equation 5. We also find $F_{14} = 0 = F_{24} = F_{34}$ due to the assumption of infinite electrical conductivity of the fluid (Maartens, 2000). A cosmological model, which contains a global magnetic field, is necessarily anisotropic since the magnetic vector specifies a preferred spatial direction (Bronnikov et al., 2004). The Maxwell's equations of $F_{ij;k} + F_{jk;i} + F_{ki;j} = 0$ and $F^{ij}_{;j} = 0$ are satisfied by F_{23} = constant = H (say).

Equation 5 leads to the following equation:

$$h_1 = \frac{AH}{\bar{\mu}\, BC} \tag{6}$$

From Equation 4, we have:

$$E_1^1 = -\frac{H^2}{2\bar{\mu}\, B^2C^2} = -E_2^2 = -E_3^3 = E_4^4 \tag{7}$$

The modified Einstein's field Equation 3 for the metric of Equation 1 leads to the following equations:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \frac{3}{4}\beta^2 = -\left(p - \frac{H^2}{2\bar{\mu}\, B^2C^2}\right) \tag{8}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{3}{4}\beta^2=-\left(p+\frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (9)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{3}{4}\beta^2=-\left(p+\frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (10)$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}-\frac{3}{4}\beta^2=\rho+\frac{H^2}{2\bar{\mu}B^2C^2} \quad (11)$$

The energy conservation equation $T^{j}_{i;j}=0$ leads to the equation that follows:

$$\rho_4+(\rho+p)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)-\left[\frac{\partial}{\partial t}\left(\frac{H^2}{2\bar{\mu}B^2C^2}\right)+\frac{H^2}{\bar{\mu}B^2C^2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\right]=0 \quad (12)$$

And conservation of left hand side of Equation 3 leads to the equation that follows:

$$\left(R^j_i-\frac{1}{2}R\,g^j_i\right)_{;j}+\frac{3}{2}(\phi_i\,\phi^j)_{;j}-\frac{3}{4}(\phi_k\,\phi^k\,g^j_i)_{;j}=0 \quad (13)$$

This leads to this equation as follows:

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j}+\phi^\ell\Gamma^j_{\ell j}\right]+\frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial x^j}-\phi_\ell\,\Gamma^\ell_{ij}\right]$$
$$-\frac{3}{4}g^j_i\,\phi_k\left[\frac{\partial\phi^k}{\partial x^j}+\phi^\ell\Gamma^k_{\ell j}\right]-\frac{3}{4}g^j_i\,\phi^k\left[\frac{\partial\phi_k}{\partial x^j}-\phi_\ell\Gamma^\ell_{kj}\right]=0 \quad (14)$$

Equation 14 is automatically satisfied for i = 1,2,3. For i = 4, Equation 14 leads to:

$$\frac{3}{2}\beta\left[\frac{\partial}{\partial x^4}(g^{44}\phi_4)+\phi^4\,\Gamma^4_{44}\right]+\frac{3}{2}g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial x^4}-\phi_4\Gamma^4_{44}\right]$$
$$-\frac{3}{4}g^4_4\phi_4\left[\frac{\partial\phi_4}{\partial x^4}+\phi^4\Gamma^4_{44}\right]-\frac{3}{4}g^4_4\phi^4\left[\frac{\partial\phi_4}{\partial x^4}-\phi^4\Gamma^4_{44}\right]=0 \quad (15)$$

which leads to:

$$\frac{3}{2}\beta\beta_4+\frac{3}{2}\beta^2\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \quad (16)$$

SOLUTION OF FIELD EQUATIONS

Multiplying Equation 11 by γ and adding into Equation 8, we have:

$$\gamma\frac{A_4B_4}{AB}+(\gamma+1)\frac{B_4C_4}{BC}+\gamma\frac{A_4C_4}{AC}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+(1-\gamma)\frac{3}{4}\beta^2$$
$$=\gamma\rho-p+\frac{(\gamma+1)H^2}{2\bar{\mu}B^2C^2} \quad (17)$$

Applying the Barotropic fluid condition, that is, $p=\gamma\rho$, Equation 17 leads to:

$$\gamma\frac{A_4B_4}{AB}+(\gamma+1)\frac{B_4C_4}{BC}+\gamma\frac{A_4C_4}{AC}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+(1-\gamma)\frac{3}{4}\beta^2$$
$$=\frac{(\gamma+1)H^2}{2\bar{\mu}B^2C^2} \quad (18)$$

For the complete determination of the model of the universe, we assume that eigenvalue (σ^1_1) of shear tensor (σ^j_i) is proportional to the expansion (θ). This leads to:

$$A=(BC)^n \quad (19)$$

Using Equation 19, Equation 16 leads to:

$$\beta=\frac{N}{(BC)^{n+1}} \quad (20)$$

From Equation 8 and 9, we have:

$$\frac{B_{44}}{B}-\frac{A_{44}}{A}+\frac{C_4}{C}\left(\frac{B_4}{B}-\frac{A_4}{A}\right)=\frac{H^2}{\bar{\mu}B^2C^2} \quad (21)$$

From Equation 9 and 10, we have:

$$\frac{C_{44}}{C}-\frac{B_{44}}{B}=\frac{A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}\right) \quad (22)$$

which leads to

$$C^2\left(\frac{B}{C}\right)_4=\frac{L}{(BC)^n} \quad (23)$$

where L is a constant of integration.

Now we assume that:

$$BC = \mu \tag{24}$$

$$\frac{B}{C} = \nu \tag{25}$$

$$B^2 = \mu\nu \tag{26}$$

Thus

$$C^2 = \mu / \nu \tag{27}$$

Using Equation 24 and 25, Equation 23 leads to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^{n+1}} \tag{28}$$

Using Equation 19 and 20, Equation 18 leads to:

$$n\gamma\left\{\left(\frac{B_4}{B}\right)^2+\left(\frac{C_4}{C}\right)^2\right\}+(2n\gamma+\gamma+1)\frac{B_4C_4}{BC}+\frac{B_{44}}{B}+\frac{C_{44}}{C}+(1-\gamma)\frac{3}{4}\frac{N^2}{(BC)^{2n+2}} = \frac{H^2(1+\gamma)}{2\bar{\mu}(BC)^2} \tag{29}$$

Now using Equations 24 to 27, Equation 29 leads to:

$$\frac{\mu_{44}}{\mu}+\left(n\gamma+\frac{\gamma-1}{4}\right)\frac{\mu_4^2}{\mu^2}+\left(\frac{1-\gamma}{4}\right)\frac{\nu_4^2}{\nu^2}+\frac{3}{4}\frac{(1-\gamma)N^2}{\mu^{2n+2}}=\frac{(1+\gamma)H^2}{2\bar{\mu}\mu^2} \tag{30}$$

Again using Equation 28, Equation 30 leads to:

$$\frac{\mu_{44}}{\mu}+\left(n\gamma+\frac{\gamma-1}{4}\right)\frac{\mu_4^2}{\mu^2}+\left(\frac{1-\gamma}{4}\right)\frac{(L^2+3N^2)}{\mu^{2n+2}}=\frac{(1+\gamma)H^2}{2\bar{\mu}\mu^2} \tag{31}$$

which leads to:

$$2\mu_{44}+2\left(n\gamma+\frac{\gamma-1}{4}\right)\frac{\mu_4^2}{\mu}=\frac{(\gamma-1)}{2}\frac{(L^2+3N^2)}{\mu^{2n+1}}+\frac{(1+\gamma)H^2}{\bar{\mu}\mu} \tag{32}$$

Now we assume:

$$\mu_4 = f(\mu) \tag{33}$$

Therefore,

$$\mu_{44} = f f' \tag{34}$$

Where

$$f' = \frac{df}{d\mu}$$

Using Equations 33 and 34 in Equation 32, we have:

$$\frac{d}{d\mu}(f^2)+\alpha\frac{f^2}{\mu}=\frac{b}{\mu^{2n+1}}+\frac{(\gamma+1)K}{\mu} \tag{35}$$

Where

$$K = \frac{H^2}{\bar{\mu}} \tag{36}$$

$$\alpha = 2\left(n\gamma+\frac{\gamma+1}{4}\right) \tag{37}$$

$$b = \left(\frac{\gamma-1}{2}\right)(L^2+3N^2) \tag{38}$$

Now Equation 35 leads to:

$$f^2 = \frac{b}{(\alpha-2n)\mu^{2n}}+\frac{(\gamma+1)K}{\alpha}+\frac{M}{\mu^{\alpha}} \tag{39}$$

where M is a constant of integration.

To find the solution in terms of cosmic time t, we have assume M = 0 and n = 1, then Equation 39 becomes:

$$f^2 = \frac{b}{(\alpha-2)\mu^2}+\frac{(\gamma+1)K}{\alpha} \tag{40}$$

where

$$\alpha = \frac{5\gamma-1}{2} \tag{41}$$

Equation 40 leads to:

$$\frac{d\mu}{dt} = f = \frac{1}{\mu}\left[\frac{b}{(\alpha-2)}+\frac{(\gamma+1)K}{\alpha}\mu^2\right]^{1/2} \tag{42}$$

which on integration leads to:

$$\mu = \left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{1/2} \tag{43}$$

where ℓ is the constant of integration and

$$S = \frac{b\alpha}{(\alpha-2)(\gamma+1)} \tag{44}$$

Now Equation 28 leads to:

$$\frac{d\nu}{\nu} = \frac{L}{\mu^2}dt = \frac{L}{\mu^2}\left(\frac{dt}{d\mu}\right)d\mu$$

which leads to:

$$\log \nu = \int \frac{L}{\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]}dt \tag{45}$$

which again leads to:

$$\nu = \eta\left[\frac{K(t+\ell)\sqrt{\gamma+1}-\sqrt{S\alpha}}{K(t+\ell)\sqrt{\gamma+1}+\sqrt{S\alpha}}\right]^{\frac{L}{2}\sqrt{\frac{\alpha}{(\gamma+1)S}}} \tag{46}$$

where η is the constant of integration.
Now the metric potential are given by:

$$A^2 = \mu^2 = \left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right] \tag{47}$$

$$B^2 = \mu\nu = \eta\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{1/2}\left[\frac{K(t+\ell)\sqrt{\gamma+1}-\sqrt{S\alpha}}{K(t+\ell)\sqrt{\gamma+1}+\sqrt{S\alpha}}\right]^{\frac{L}{2}\sqrt{\frac{\alpha}{(\gamma+1)S}}} \tag{48}$$

$$C^2 = \frac{\mu}{\nu} = \eta\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{1/2}\left[\frac{K(t+\ell)\sqrt{\gamma+1}-\sqrt{S\alpha}}{K(t+\ell)\sqrt{\gamma+1}+\sqrt{S\alpha}}\right]^{\frac{L}{2}\sqrt{\frac{\alpha}{(\gamma+1)S}}} \tag{49}$$

Thus the metric (Equation 1) leads to:

$$\begin{aligned} ds^2 = -dt^2 &+ \left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]dx^2 + \eta\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{1/2} \\ &\left[\frac{K(t+\ell)\sqrt{\gamma+1}-\sqrt{S\alpha}}{K(t+\ell)\sqrt{\gamma+1}+\sqrt{S\alpha}}\right]^{\frac{L}{2}\sqrt{\frac{\alpha}{(\gamma+1)S}}}dy^2 + \eta\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{1/2} \\ &\left[\frac{K(t+\ell)\sqrt{\gamma+1}-\sqrt{S\alpha}}{K(t+\ell)\sqrt{\gamma+1}+\sqrt{S\alpha}}\right]^{\frac{-L}{2}\sqrt{\frac{\alpha}{(\gamma+1)S}}}dz^2 \end{aligned} \tag{50}$$

Let

$$t + \ell = \frac{\sqrt{\alpha s}\sec(\sqrt{K}\tau)}{K\sqrt{\gamma+1}} \tag{51}$$

Using Equation 51, Equation 43 leads to:

$$\mu = \sqrt{\frac{S}{K}}\tan(\sqrt{K}\tau) \tag{52}$$

and Equation 46 leads to:

$$\nu = \left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\left(\frac{\sqrt{K}}{2}\right)}\right]^d \tag{53}$$

where $\eta = \frac{1}{\left(\frac{\sqrt{K}}{2}\right)^d}$ and $d = L\sqrt{\frac{\alpha}{S(\gamma+1)}}$.

Using Equations 51 to 53, the metric (Equation 50) leads to:

$$\begin{aligned} ds^2 = &-\frac{\alpha S}{(\gamma+1)}\sec^2(\sqrt{K}\tau)\frac{\tan^2(\sqrt{K}\tau)}{K}d\tau^2 + S\frac{\tan^2(\sqrt{K}\tau)}{K}dx^2 \\ &+\sqrt{S}\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}}\right]\left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\frac{\sqrt{K}}{2}}\right]^d dy^2 + \sqrt{S}\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}}\right]\left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\frac{\sqrt{K}}{2}}\right]^{-d} dz^2 \end{aligned} \tag{54}$$

where the metric potentials are given by:

$$A^2 = \frac{S}{K}\tan^2(\sqrt{K}\tau) \tag{55}$$

$$B^2 = \sqrt{\frac{S}{K}}\tan(\sqrt{K}\tau)\left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\frac{\sqrt{K}}{2}}\right]^d \tag{56}$$

$$C^2 = \sqrt{\frac{S}{K}}\tan(\sqrt{K}\tau)\left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\frac{\sqrt{K}}{2}}\right]^{-d} \tag{57}$$

SOME PHYSICAL AND GEOMETRICAL PROPERTIES

Using Equations 24 and 43, Equation 20 gives the displacement vector:

$$\beta = \frac{N}{\mu^2} = \frac{N}{\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]} \quad (58)$$

which again leads to:

$$\beta = \frac{N\sqrt{K}}{\sqrt{S}\tan(\sqrt{K}\tau)} \quad (59)$$

The expansion θ is given by:

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = \frac{2K^2(\gamma+1)(t+\ell)}{[(\gamma+1)K^2(t+\ell)^2 - \alpha S]} \quad (60)$$

which leads to:

$$\theta = \frac{2\sqrt{\gamma+1}}{\sqrt{\alpha S}}\sec(\sqrt{K}\tau)\left[\frac{K}{\tan^2(\sqrt{K}\tau)}\right] \quad (61)$$

Using barotropic fluid condition, that is, $p = \gamma\rho$ and BC = μ, $A = BC$, $\frac{H^2}{\mu} = K$ in Equation 12, we have:

$$\rho_4 + 2(\gamma+1)\rho\frac{\mu_4}{\mu} - \left[\frac{\partial}{\partial t}\left(\frac{K}{2\mu^2}\right) + \frac{K}{\mu^2}\left(\frac{\mu_4}{\mu}\right)\right] = 0$$

which leads to:

$$\rho = \frac{Q}{[\mu^2]^{(\gamma+1)}} \quad (62)$$

where Q is the constant of integration.

Using Equation 43, Equation 62 leads to:

$$\rho = \frac{Q}{\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{(\gamma+1)}} \quad (63)$$

which again leads to:

$$\rho = \frac{Q\,K^{\gamma+1}}{S^{\gamma+1}[\tan(K\tau)]^{2(\gamma+1)}} \quad (64)$$

and

$$p = \gamma\rho = \frac{Q\gamma}{\left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right]^{\gamma+1}} = \frac{Q\gamma K^{\gamma+1}}{S^{\gamma+1}[\tan(\sqrt{K}\tau)]^{2(\gamma+1)}} \quad (65)$$

The shear (σ) is given by:

$$\sigma^2 = \frac{K^2}{4S\tan^4(\sqrt{K}\tau)}\left[\frac{L^2}{S} + \frac{(\gamma+1)\sec^2(\sqrt{K}\tau)}{3\alpha}\right] \quad (66)$$

Since $\frac{\sigma}{\theta} \neq 0,$ hence, anisotropy is maintained throughout.

The spatial volume (R^3) is given by:

$$R^3 = ABC = A^2 = \mu^2$$

which leads to:

$$R^3 = \left[\frac{(\gamma+1)K(t+\ell)^2}{\alpha} - \frac{S}{K}\right] = \frac{S}{K}\tan^2(\sqrt{K}\tau) \quad (67)$$

The deceleration parameter (q) is given by:

$$q = -\left[\frac{R_{44}/R}{R_4^2/R^2}\right] = \frac{1}{2} + \frac{3b\alpha^2}{2(\alpha-2)(\gamma+1)^2K^2(t+\ell)^2} \quad (68)$$

which leads to:

$$q = \frac{1}{2} + \frac{3}{2\sec^2(\sqrt{K}\tau)} \quad (69)$$

THE MODEL IN ABSENCE OF MAGNETIC FIELD

To find the solution in absence of magnetic field, we put $K \to 0$, from Equation 52:

$$\mu = \sqrt{S}\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]\tau$$

$$\mu = \sqrt{S}\,\tau \quad \text{when } K \to 0 \tag{70}$$

From Equation 53, we have:

$$\nu = \left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\left(\frac{\sqrt{K}\tau}{2}\right)}\right]^{d} \tau^{d}$$

$$\nu = \tau^{d} \quad \text{when } K \to 0 \tag{71}$$

The metric (54) leads to:

$$ds^2 = -\frac{\alpha S}{(\gamma+1)}\sec^2(\sqrt{K}\tau)\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]^2 \tau^2 d\tau^2 + S\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]^2 \tau^2 dx^2$$

$$+\sqrt{S}\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]\left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\left(\frac{\sqrt{K}\tau}{2}\right)}\right]^{d} \tau^{d+1} dy^2$$

$$+\sqrt{S}\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]\left[\frac{\tan\left(\frac{\sqrt{K}\tau}{2}\right)}{\left(\frac{\sqrt{K}\tau}{2}\right)}\right]^{-d} \tau^{1-d} dz^2$$

which leads to:

$$ds^2 = -\frac{\alpha S}{(\gamma+1)}\tau^2 d\tau^2 + S\tau^2 dx^2 + \sqrt{S}\tau^{1+d} dy^2 + \sqrt{S}\tau^{1-d} dz^2 \tag{72}$$

when $K \to 0$

where the metric potentials are given by:

$$A^2 = S\,\tau^2 \tag{73}$$

$$B^2 = \sqrt{S}\,\tau^{1+d} \tag{74}$$

$$C^2 = \sqrt{S}\,\tau^{1-d} \tag{75}$$

From Equation 59, the displacement vector:

$$\beta = \frac{N}{S\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]^2 \tau^2}$$

$$\beta = \frac{N}{S\,\tau^2} \qquad \text{when } K \to 0 \tag{76}$$

From Equation 61, the expansion θ is given by:

$$\theta = 2\sqrt{\frac{\gamma+1}{\alpha S}}\sec(\sqrt{K}\tau)\left[\frac{\sqrt{K}\tau}{\tan(\sqrt{K}\tau)}\right]^2 \frac{1}{\tau^2}$$

which leads to:

$$\theta = 2\sqrt{\frac{\gamma+1}{\alpha S}}\frac{1}{\tau^2} \quad \text{when } K \to 0 \tag{77}$$

From Equation 64:

$$\rho = \frac{Q}{S^{\gamma+1}\left[\frac{\tan(\sqrt{K}\tau)}{\sqrt{K}\tau}\right]^{2(\gamma+1)} \tau^{2(\gamma+1)}}$$

which leads to:

$$\rho = \frac{Q}{S^{\gamma+1}\,\tau^{2(\gamma+1)}} \quad \text{when } K \to 0 \tag{78}$$

Again from Equation 65:

$$p = \gamma\rho = \frac{Q\,\gamma\,K^{\gamma+1}}{S^{\gamma+1}\left[\frac{\tan(\sqrt{K}\tau)}{(\sqrt{K}\tau)}\right]^{2(\gamma+1)} (\sqrt{K}\tau)^{2(\gamma+1)}}$$

which leads to:

$$p = \gamma\rho = \frac{Q\gamma}{S^{\gamma+1}\,\tau^{2(\gamma+1)}} \quad \text{when } K \to 0 \tag{79}$$

The shear (σ) in absence of magnetic field is given by:

$$\sigma = \frac{1}{2\sqrt{S}}\left(\frac{L^2}{S} + \frac{(\gamma+1)}{3\alpha}\right)^{1/2} \frac{1}{\tau^2} \tag{80}$$

The spatial volume (R^3) and deceleration parameter (q) in absence of magnetic field are given by:

$$R^3 = S\ \tau^2 \qquad (81)$$

$$q = 2 \qquad (82)$$

DISCUSSION AND CONCLUSION

The model (Equation 54) in the presence of magnetic field starts with a big-bang at $\tau = 0$ and the expansion in the model decreases as τ increases where τ is defined in terms of cosmic t by Equation 51. The matter density $\rho \to \infty$ when $\tau \to 0$ and $\rho \to 0$ when $\tau \to \infty$. The spatial volume (R^3) increases as τ increases. The displacement vector (β) is initially large but decreases due to lapse of time. Since $\frac{\sigma}{\theta} \neq 0$, hence anisotropy is maintained throughout. Also $q > 0$, therefore, the model (Equation 54) represents decelerating model in the presence of magnetic field. The model (Equation 54) has point type singularity at $\tau = 0$ in the presence of magnetic field.

The model (Equation 72) in the absence of magnetic field, also starts with a big-bang at $\tau = 0$ and the expansion in the model decreases as τ increases. The matter density (ρ) $\to \infty$ when $\tau \to 0$ and $\rho \to 0$ when $\tau \to \infty$. The displacement vector (β) is initially large, but decreases due to lapse of time. Since $\frac{\sigma}{\theta} \neq 0$, hence anisotropy is maintained throughout. The spatial volume increases as τ increases. The deceleration parameter $q = 2 > 0$, hence model (Equation 72) represents decelerating model. The Hubble parameter (H) is initially large, but decreases due to lapse of time. The model (Equation 72) has point type singularity at $\tau = 0$ when $0 < d < 1$ and cigar type singularity at $\tau = 0$ when $1 < d$. (MacCallum, 1971). The deceleration parameter $q > 0$ in the presence and absence of magnetic field, because it gives the significance to study the early universe.

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