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Full Length Research Paper

Application of He's energy balance method for nonlinear vibration of thin circular sector cylinder

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This paper deals with application of a new kind of analytical technique for a non-linear problem called the He's Energy Balance Method (EBM). This methodology has been utilized to achieve approximate solutions for nonlinear free vibration of conservative thick circular sector slabs. In Energy Balance Method, contrary to the conventional methods, only one iteration leads to high accuracy of solutions. These solutions do not only have high degree of accuracy, but are also uniformly valid in the whole solution domain. EBM operates very well in the whole range of the parameters involved. Excellent agreement of the approximate frequencies and periodic solutions with the exact ones could be established. Some patterns are given to illustrate the effectiveness and convenience of the methodology. It has been indicated that the numerical results of other methods have same conclusion; while EBM is much easier, more convenient and more efficient than other approaches. The Energy Balance Method is a novel method which alleviates drawbacks of the traditional numerical techniques.

Key words: Thin circular sector cylinder, nonlinear vibration, energy balance method.

INTRODUCTION

Recently, nonlinear oscillator models have been widely considered in physics and engineering.

It is obvious that there are many nonlinear equations in the study of different branches of science which do not have analytical solutions. Therefore, these nonlinear equations must be solved using other methods. Many researchers have been working on various analytical methods for solving nonlinear oscillation systems in the last decades. Perturbation technique is one of the well-known methods (He, 1999); the traditional perturbation method contains many shortcomings. They are not useful for strongly nonlinear equations. So for overcoming the shortcomings, many new techniques have appeared in open literature. Some new perturbation methods include Homotopy perturbation (Bayat et al., 2010a), parameter–expansion (Kimiaeifar et al., 2010), parameterized perturbation (He,1999), energy balance (Bayat et al., 2011b; Bayat et al., 2011c; He, 2002), variational approach (Bayat et al., 2011d; Pakar et al., 2011; He, 2007) and the other analytical and numerical methods (Bayat et al., 2011e, 2011f, 2011g; Shahidi et al., 2011; Soleimani et al., 2011).

Among these methods, we have considered energy balance method (EBM) for solving the nonlinear vibration of thin circular sector slab in this paper. Recently, nonlinear analytical techniques for solving nonlinear problems have been dominated by different methods. By extending the Energy Balance Method proposed by He, approximate analytical formulas for the period and periodic solution have been established. Variational methods such as Raleigh-Ritz and Bubnov-Galerkin techniques have been, and will continue to be popular tools for nonlinear analysis. When contrasted with other approximate analytical methods, variational methods combine the following two merits:

1. They provide physical insight into the nature of the solution of the problem.
2. The obtained solutions are the best among all the possible trial-functions.

Comparison of the result which is obtained by this method with the obtained result by the other solution reveals that the He's Energy Balance Method is very effective and convenient.

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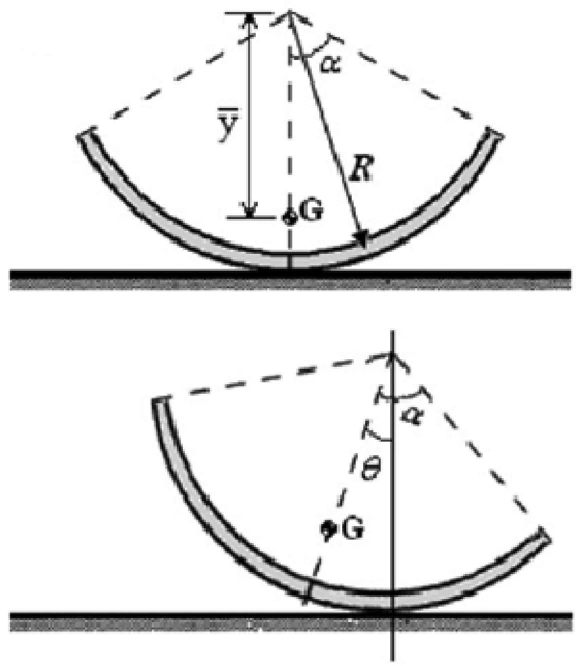

Figure 1. Geometric parameters of the homogeneous thin circular sector cylinder.

THIN CIRCULAR SECTOR CYLINDER FORMULATION

Swinging oscillation of thin circular sector cylinder in this condition a thin circular sector cylinder is considered as shown in Figure 1. As before thin circular sector cylinder rolls in an oscillatory motion back and forth on a flat stationary support, with no sliding effect. Governing equation of the oscillation is as follows:

$$\left(R^2 - R\bar{y}\cos(\theta)\right)\left(2\ddot{\theta}\right) + R\left(\bar{y}\sin(\theta)\right)\dot{\theta}^2 + \left(g\bar{y}\right)\sin(\theta) = 0 \quad (1)$$
$$\theta(0) = A,\quad \dot{\theta}(0) = 0,$$

where the geometrical parameters are shown in Figure 1. The height of mass center obtained as:

$$\bar{y} = \frac{R\sin(\alpha)}{\alpha} \quad (2)$$

Introducing the dimensionless time variable:

$$\bar{t} = \sqrt{\frac{1}{\bar{y}}}\,t = \left(\sqrt{\frac{R\sin(\alpha)}{\alpha}}\right)^{-1} t. \quad (3)$$

Equation (1) becomes:

$$\left(\frac{R}{\bar{y}} - \cos(\theta)\right)\left(2\ddot{\theta}\right) + \sin(\theta)\dot{\theta}^2 + \frac{g\bar{y}}{R}\sin(\theta) = 0 \quad (4)$$
$$\theta(0) = A,\quad \dot{\theta}(0) = 0.$$

And by introducing the dimensionless geometrical parameter

$$\lambda = \frac{\bar{y}}{R} = \frac{\sin(\alpha)}{\alpha} \quad (5)$$

Equation (4) becomes

$$\left(\frac{1}{\lambda} - \cos(\theta)\right)\left(2\ddot{\theta}\right) + \sin(\theta)\dot{\theta}^2 + g\lambda\sin(\theta) = 0 \quad (6)$$
$$\theta(0) = A,\quad \dot{\theta}(0) = 0.$$

BASIC IDEA OF EBM

In the present paper, we consider a general nonlinear oscillator in the form (He, 2002);

$$u'' + f(u(t)) = 0 \quad (7)$$

In which u and t are generalized dimensionless displacement and time variables, respectively. Its variational principle can be easily obtained:

$$J(u) = \int_0^t \left(-\frac{1}{2}u'^2 + F(u)\right) dt \quad (8)$$

where $T = \frac{2\pi}{\omega}$ is period of the nonlinear oscillator, $F(u) = \int f(u)\,du$.

Its Hamiltonian, therefore, can be written in the form:

$$H = \frac{1}{2}u'^2 + F(u) + F(A) \tag{9}$$

Or

$$R(t) = -\frac{1}{2}u'^2 + F(u) - F(A) = 0 \tag{10}$$

Oscillatory systems contain two important physical parameters, that is, the frequency ω and the amplitude of oscillation. A . So let us consider such initial conditions:

$$u(0) = A, \qquad u'(0) = 0 \tag{11}$$

We use the following trial function to determine the angular frequency ω:

$$u(t) = A\cos\omega t \tag{12}$$

Substituting (12) into u term of (10), yield:

$$R(t) = \frac{1}{2}\omega^2 A^2 \sin^2\omega t + F(A\cos\omega t) - F(A) = 0 \tag{13}$$

If, by chance, the exact solution had been chosen as the trial function, then it would be possible to make R zero for all values of t by appropriate choice of ω. Since Equation (12) is only an approximation to the exact solution, R cannot be made zero everywhere. Collocation at $\omega t = \frac{\pi}{4}$ gives:

$$\omega = \sqrt{\frac{2(F(A)) - F(A\cos\omega t)}{A^2\sin^2\omega t}} \tag{14}$$

Its period can be written in the form:

$$T = \frac{2\pi}{\sqrt{\dfrac{2(F(A)) - F(A\cos\omega t)}{A^2\sin^2\omega t}}} \tag{15}$$

APPLICATION

Its variational formulation can be readily obtained Equation (1) as follows:

$$J(\theta) = \int_0^t \left(\frac{1}{\lambda}\dot{\theta}^2 - \dot{\theta}^2\cos(\theta) - g\lambda\cos(\theta)\right)dt. \tag{16}$$

Its Hamiltonian, therefore, can be written in the form:

$$H = \left(\frac{1}{\lambda}\dot{\theta}^2 - \dot{\theta}^2\cos(\theta) - g\lambda\cos(\theta)\right) \tag{17}$$

And

$$H_{t=0} = -g\lambda\cos(A), \tag{18}$$

$$H_t - H_{t=0} = \left(\frac{1}{\lambda}\dot{\theta}^2 - \dot{\theta}^2\cos(\theta) - g\lambda\cos(\theta)\right) - (-g\lambda\cos(A)), \tag{19}$$

We will use the trial function to determine the angular frequency ω, that is:

$$\theta(t) = A\cos\omega t \tag{20}$$

If we substitute (20) into (19), it results in the following residual equation:

$$H_t - H_{t=0} = \begin{pmatrix} \frac{1}{\lambda}\left(A^2\omega^2\sin^2(\omega t)\right) \\ -\left(\left(A^2\omega^2\sin^2(\omega t)\right)\cos\left(A\cos(\omega t)\right)\right) \\ -g\lambda\cos\left(A\cos(\omega t)\right) \end{pmatrix} - \left(-g\lambda\cos(A)\right) \tag{21}$$

If we collocate at $\omega t = \frac{\pi}{4}$ we obtain:

$$\frac{1}{2}\frac{A^2\omega^2}{\lambda} - \frac{1}{2}\cos(\frac{\sqrt{2}}{2}A)A^2\omega^2 - g\lambda\cos(\frac{\sqrt{2}}{2}A) + g\lambda\cos(A) = 0 \tag{22}$$

This leads to the following result:

$$\omega = \frac{\sqrt{\left(2 - 2\cos(\frac{\sqrt{2}}{2}A)\lambda\right)g\left(\cos(\frac{\sqrt{2}}{2}A) - \cos(A)^2\right)\lambda}}{\left(1 - \cos(\frac{\sqrt{2}}{2}A)\lambda\right)A} \tag{23}$$

According to Equations (20) and (23), we can obtain the following approximate solution:

$$\theta(t) = A\cos\left(\frac{\sqrt{\left(2 - 2\cos(\frac{\sqrt{2}}{2}A)\lambda\right)g\left(\cos(\frac{\sqrt{2}}{2}A) - \cos(A)^2\right)\lambda}}{\left(1 - \cos(\frac{\sqrt{2}}{2}A)\lambda\right)A}t\right) \tag{24}$$

RESULTS AND DISCUSSION

In this section, to illustrate and verify the accuracy of this new approximate analytical approach, a comparison between Energy Balance Method and numerical ones are presented in Figures 2 to 4 for thin circular sector cylinder. Figures 2 and 3 show the motion of the system is a periodic motion and the amplitude of vibration is a function of the initial conditions.

The phase plane of the equation has been considered in Figure 4 for $A = \pi/12$, $g = 10$.

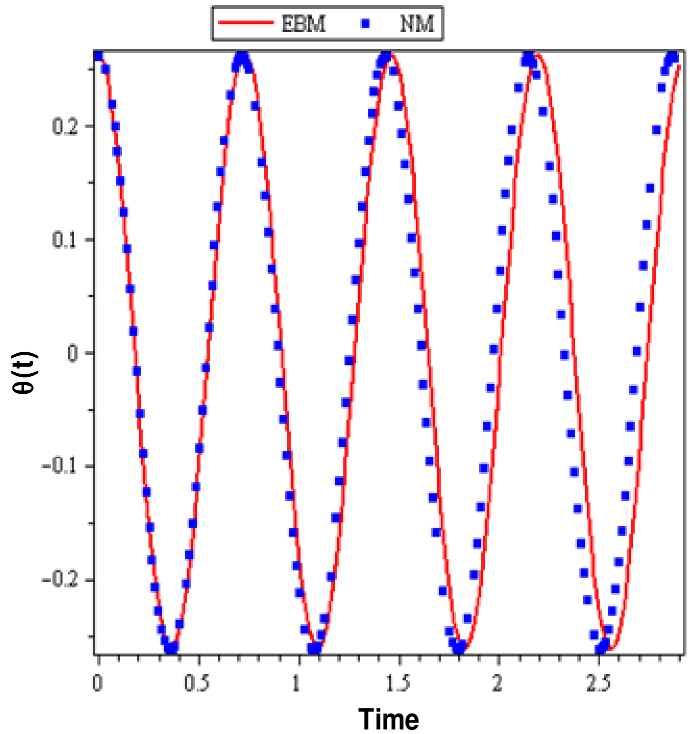

Figure 2. Comparison of analytical solution of $\theta(t)$ based on time with the numerical solution for $\alpha = \pi/6$, $A = \pi/12$, $g = 10$, $R = 5$.

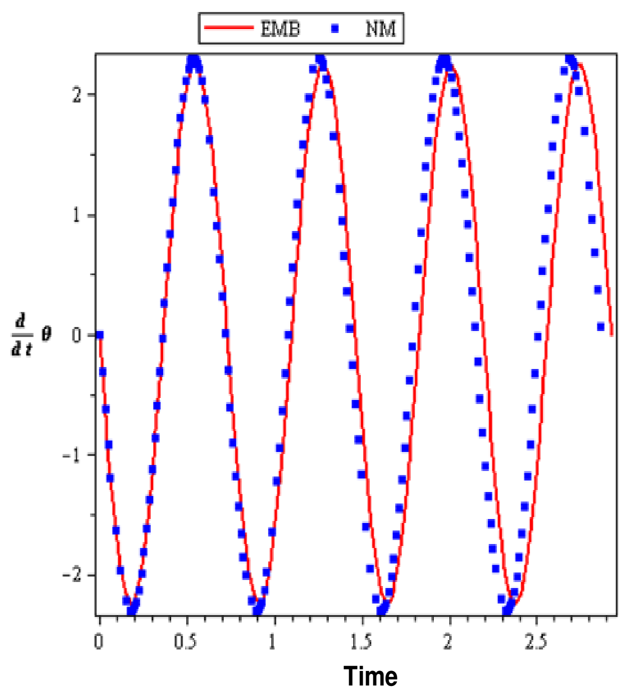

Figure 3. Comparison of analytical solution of $\dot{\theta}(t)$ based on time with the numerical solution for $\alpha = \pi/6$, $A = \pi/12$, $g = 10$, $R = 5$.

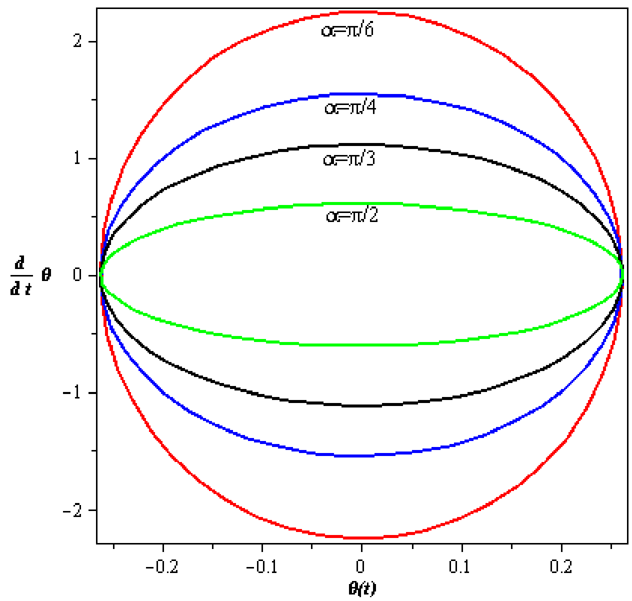

Figure 4. Phase plane, for $A = \pi/12, \quad g = 10$.

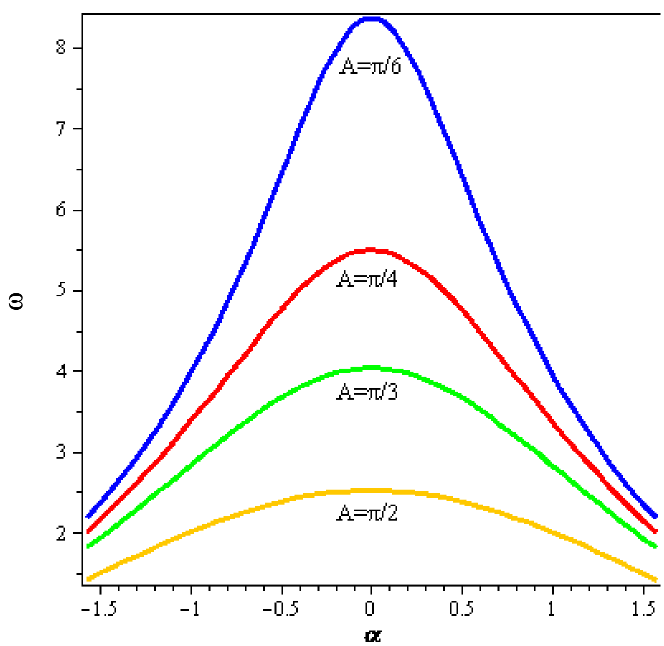

Figure 5. Comparison of frequency corresponding to various parameters of α for $A = \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$.

The effect of α on the frequency corresponding to $A = \pi/12$, $g = 10$ has been studied in Figure 5. The comparison of frequency corresponding to various parameters of α and amplitude (A) are shown in the Figures 6 and 7. It is evident that EBM shows an excellent agreement with the numerical solution and quickly convergent and valid for a wide range of vibration amplitudes and initial conditions. The accuracy of the results show that the EBM can be potentiality used for the analysis of strongly nonlinear oscillation problems accurately.

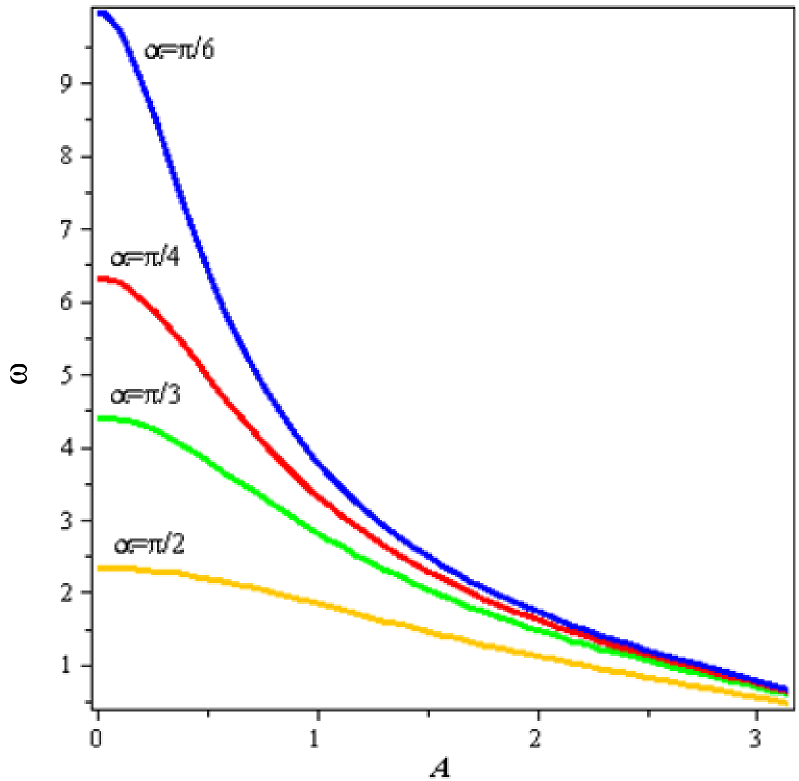

Figure 6. Comparison of frequency corresponding to various parameters of amplitude (A) for $A = \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$.

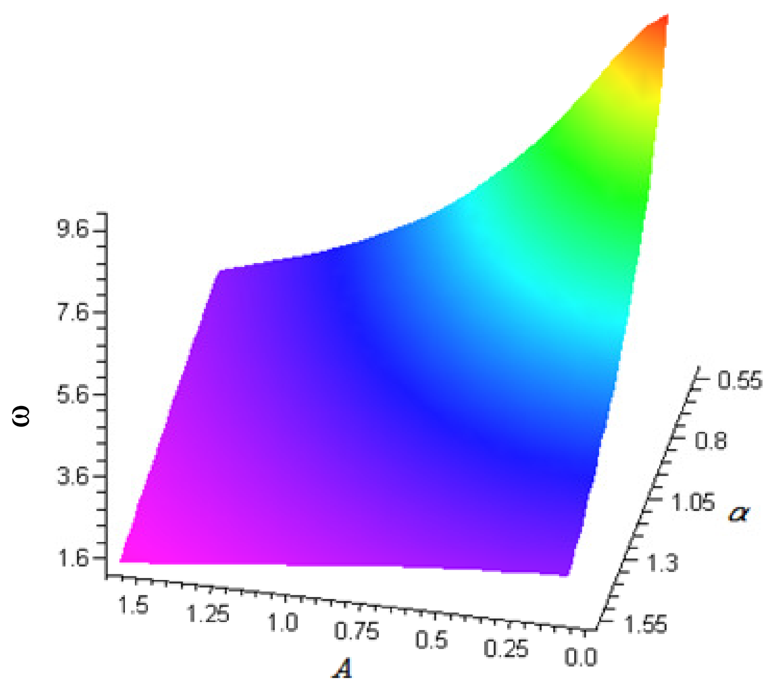

Figure 7. Comparison of frequency corresponding to various parameters of amplitude (A) and α.

Conclusion

Nonlinear oscillators are useful for all oscillators and vibrations in high domain branches of sciences. Energy Balance Method has been utilized on the thin circular cylinder. It has been proved that the Energy Balance Method is clearly effective, convenient and does not require any linearization or small perturbation, and adequately accurate to both linear and nonlinear problems in physics and engineering. It has illustrated that the results of EBM are in an excellent agreement with those obtained by the numerical one. EBM is a fantastic method for the analysis of nonlinear systems. The results indicated that Energy Balance method is extremely speedy, light, with high accuracy. Excellent agreement between approximate solution and the numerical one is demonstrated and discussed. The method can be easily extended to any nonlinear oscillator without any difficulty. Energy Balance Method provides an easy and direct procedure for determining approximations of periodic solutions.

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