

*Full Length Research Paper*

# Tree structured encoding based multi-objective multicast routing algorithm

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**Quality-of-service (QoS) based multicast routing is a major challenge to next generation networks due to the increasing demand of real-time applications which require strict QoS guarantee. In the presented multi-objective multicast routing, the QoS parameters, namely, cost and available bandwidth are represented as objectives, while end-to-end delay and delay jitter are represented as constraints. The optimization is strived using an elitist multi-objective evolutionary algorithm. The topological assisted tree structured encoding was proposed to represent the multicast tree. The individual solution or chromosome was represented as a combination of arrays where each array represents a random route from destination node in multicast group to source node. The effectiveness of the proposed algorithm is tested on various networks, including the network formed using network topology generator BRITE. The best compromise solution is obtained using fuzzy cardinal priority ranking. The performance of this algorithm was compared with weighted sum genetic algorithm. The simulation results demonstrate that the multi-objective optimization with the proposed encoding scheme is effective in providing faster and guaranteed convergence.**

**Key words:** Multicast routing, multi-objective optimization, tree structured encoding, evolutionary algorithm, genetic algorithm.

## INTRODUCTION

Multicasting is the capability of a communication network to accept a single message from a source and to deliver its copies to multiple recipients at different locations. Multicasting is a key requirement of computer networks supporting resource-intensive multimedia applications, such as video conferencing, virtual whiteboard and computer-supported cooperative work. The quality and performance requirements are very high for such resource-intensive applications. A network must minimize the resources consumption while meeting their quality of service (QoS) requirements. The various QoS parameters are cost, bandwidth, end-to-end delay, delay jitter and packet loss. Researchers have been studying for many years to develop efficient multicast routing algorithm based on Steiner tree.

Various heuristics were used to obtain minimum Steiner tree of the network that spans over the given set of nodes with minimum sum of link cost (Takahashi and Matsuyama, 1980; Kou et al., 1981; Smith, 1983). The problem to obtain the minimum cost Steiner tree is NP-complete (Smith, 1983). Kompella et al. (1993) first formulated the delay-constrained minimum Steiner tree problem. The bounded shortest multicast algorithm was used to obtain the minimum cost tree with end-to-end delay constraints (Parsa and Zhu, 1998). Noronha and Tobagi (1994) proposed an algorithm based on integer programming to construct the optimal source-specific delay-constrained minimum Steiner tree. Salama et al. (1997) compared the performance of a shortest path broadcast tree algorithm and a heuristic for tree cost under end-to-end delay and delay jitter bounds. Kumar and Jaffe (1983) presented algorithms for minimum Steiner tree and least delay tree. Rouskas and Baldine (1997) studied the problem of constructing multicast trees with end-to-end delay and delay jitter constraints.

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The evolutionary algorithms (EAs) are the search and optimization algorithms based on the simulated evolutionary process of natural selection, variation and genetics. These algorithms obtain the optimal solution by improving the solution with the progress of iterations. These algorithms have been used for some NP-hard and NP-complete optimization problems, because there are many individuals that can search for multiple good solutions in parallel. The ability to handle complex problems, involving features such as discontinuities and disjointed feasible spaces, reinforces the potential effectiveness of these algorithms for such complex problems.

Kun et al. (2006) used an algorithm based on simulated annealing (SA) to find minimum cost multicast tree by satisfying end-to-end delay and delay jitter. Genetic algorithm (GA) was used in routing for cost, bandwidth and delay optimizations (Zhang and Leung, 1999; Xiang et al., 1999; Zhengying et al., 2001). Zhang et al. (2009) presented least-cost QoS multicast routing by combining GA and SA. Randaccio and Atzori (2007) used GA to the group multicast problem and employed a heuristic procedure to generate a set of possible trees for each session in isolation. Yen et al. (2008) focused on energy consumption efficiency for selecting the node with minimum energy consumption in forming the route. Nesmachnow et al. (2007) evaluated EAs to the shortest path problem.

In solving multi-objective optimization problems, evolutionary algorithms have been adequately applied to demonstrate that multiple and well-spread Pareto optimal solutions can be found in a single simulation run. Various multi-objective evolutionary algorithms (MOEAs) use the concept of Pareto domination to guide the search (Zitzler and Thiele, 1999; Deb, 2001).

Roy and Das (2004) proposed multi-objective genetic algorithm (MOGA) based approach to optimize end-to-end delay, total bandwidth consumption and residual bandwidth utilization. Araújo and Garrozi (2010) considered three objectives for multicast routing. All objectives were combined into one function by using weighting factors depending on the importance of the objectives and used single objective optimization. Ant colony approach algorithm was used to simultaneously optimize cost, end-to-end delay and average delay (Pinto et al., 2005). The strength Pareto evolutionary algorithm (SPEA) was used for routing (Zitzler and Thiele, 1999). Zitzler et al. (2000) found that SPEA2 is better than SPEA, because the number of archived solutions is constant overtime and boundary individuals are protected from being removed. The SPEA2 used a strength value to determine which solution is dominated by the others. Srinivas and Deb (1994) proposed the non-dominated sorting genetic algorithm (NSGA) which combines the ranking approach and diversity mechanism into the fitness assignment. The NSGA-II retains the multi-objective nature of the problem and has an improvement

in rapidly assigning the solutions into several fronts and maintaining the diversity (Deb et al., 2002). After convergence, a set of Pareto optimal solutions was achieved in the first front  $F_1$ . Among these, one solution known as best compromise solution is selected. This solution can be obtained using fuzzy cardinal priority ranking (Klir and Folger, 1993; Dhillon et al., 2002).

In this paper, two objectives, namely, the cost and available bandwidth are simultaneously optimized while satisfying end-to-end delay and delay jitter constraints using the NSGA-II. The approach works in a source-based fashion where the complete knowledge of a network is assumed to be known in advance. The topological assisted tree structured encoding is used to represent the multicast tree. The individual solution or chromosome was represented as a combination of arrays where each array represents a random route from destination node in multicast group to source node. Two consecutive nodes represent a topological connection or link between them. The proposed algorithm is tested on various networks, including the random networks formed using network topology generator BRITE working on Waxman model (Waxman, 1988; Medina et al., 2001). The best compromise solution was obtained using fuzzy cardinal priority ranking. The effectiveness of the developed algorithm is also tested for different sizes of multicast group and varying size of networks. The effectiveness of the multi-objective optimization algorithm was compared with weighted sum genetic algorithm.

## PROBLEM FORMULATION

The network was simply represented as weighted connected graph  $N=(V,E)$ , where  $V$  denotes the set of nodes and  $E$  the set of links. The existence of a link  $e=(u,v)$  from node  $u$  to node  $v$  implies the existence of a link  $e'=(v,u)$  for any  $u,v \in V$ , that is, full duplex in networking terms. Let  $M$  be a subset of  $V$ , that is,  $M \subseteq V$  forms the multicast destination group with each node of  $M$  as a group member. The node  $s \in V$  is a multicast source for multicast group  $M$ . A multicast tree  $T(s,M) \subseteq E$  is a sub-graph of  $N$  that spans all nodes in  $M$ , while it may include non-group member nodes along a path in the tree. Each link  $e=(u,v) \in E$  has its properties, cost  $C(e)$ , available bandwidth  $Aw(e)$  and a delay  $D(e)$  as any real positive value  $R^+$ . The link cost  $C(e)$  may be the monetary cost incurred by the use of the network link or may be some measure of network utilization. The link capacity,  $Cap(e)$ , which represents the maximum data that can be transferred by link, is assumed uniform for all links. The  $Aw(e)$  represents the bandwidth available to meet the current traffic requirement. The delay  $D(e)$  represents the time needed to transmit information through link that includes transmission, queuing and propagation delays. A sample graph of 15-nodes Bellcore topology is as shown in Figure 1 (NT, online).

The QoS provisioning in multicast routing is a challenging task due to the application-specific diverse requirements for end-to-end delay, delay jitter, bandwidth, cost, etc. The problem is to find a tree rooted at the source  $s$  and spanning to all the members of  $M$  such that the total cost of the tree is minimum, the available bandwidth is maximum, the delay from source to each destination is not greater than the specified limit  $\Delta$  and the delay jitter between two destinations is not greater than the specified limit  $\delta$ . Therefore, multi-objective multicast routing problem that minimize the total cost

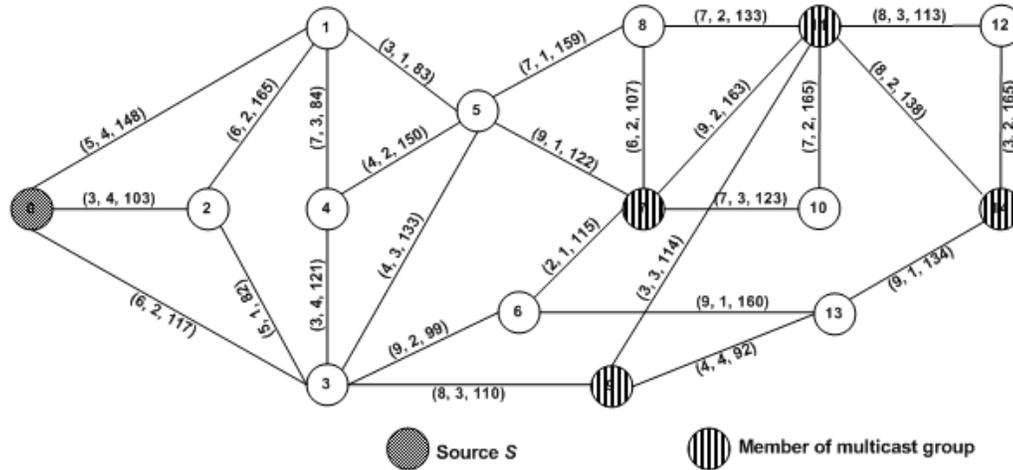


Figure 1. A 15-node Bellcore network topology.

of multicast tree and maximize the available bandwidth while satisfying the end-to-end delay and delay jitter constraints is defined as:

1. Minimize cost of multicast tree  $f_1$ :

$$f_1 = C(T(s, M)) = \sum_{e \in T(s, G)} C(e) \quad (1)$$

2. Maximize available bandwidth  $f_2$ :

$$f_2 = A_v(T(s, M)) = \frac{\sum_{e \in T(s, G)} Aw(e)}{\sum_{e \in T(s, G)} Cap(e)} \quad (2)$$

3. Subjected to end-to-end delay constraint:

$$D(P_T(s, d_i)) = \sum_{e \in P_T(s, d_i)} d(e) \leq \Delta \quad (3)$$

4. Delay jitter bounds:

$$J_{ij} = \left| \sum_{e \in P_T(s, d_i)} d(e) - \sum_{\substack{e \in P_T(s, d_j) \\ i \neq j}} d(e) \right| \leq \delta \quad (4)$$

$P_T(s, d)$  is a path from 's' to 'd' ( $s \rightarrow i \rightarrow j \rightarrow \dots \rightarrow k \rightarrow d$ ).

### MULTI-OBJECTIVE MULTICAST ROUTING USING NSGA-II

The main objective of multi-objective evolutionary algorithm is to find multiple Pareto optimal solutions in single simulation run. The multi-objective multicast routing is based on an elitist non-dominated sorting genetic algorithm, which has the following features suited for multi-objective optimization.

1. It uses fast non dominated sorting techniques to provide the solutions as close as possible to the Pareto optimal solutions.

2. It uses crowding distance techniques to provide diversity in solution.
3. It uses elitist techniques to preserve the best solutions of current population in next generation.

Let  $P_i$  be the parent population,  $Q_i$  is the offspring population and  $R_i$  represent the total population of the generation  $i$ .  $F_k$  is the front  $k$ , where  $k$  is a positive integer. Note that the solutions in front  $F_1$  are better than those of  $F_2$ , etc. Algorithm steps are as follows:

S1: Combine the  $P_i$  and  $Q_i$  to form  $R_i$ , combine the parent and offspring populations. Evaluate the offspring population. Assign  $P_i = P_0$  and  $Q_i = Q_0$ , where  $P_i$  and  $Q_i$  denote the parent and offspring population at  $i^{th}$  generation, respectively. Create a combined population  $R_i = P_i \cup Q_i$  of size  $2N$ .

S2: Assign each population in  $R_i$  to the front ( $F_1, F_2, F_3, \dots, F_n$ ) using fast-non-dominated-sort algorithm.

Fast-non-dominated-sort algorithm divides the population in different fronts. A solution is said to dominate another solution, if it is not worse in all objectives and better in at least one objective. A solution is said to be non-dominated if it is not dominated by any other solution. The solutions in  $R_i$ , which do not dominate each other but dominate all other solutions of  $R_i$ , are kept in the first front, that is, set  $F_1$ . Among the solutions not in  $F = F_1$ , the solutions which do not dominate each other but dominate all other solutions, are kept in the second front, that is, set  $F_2$ . Similarly, among the solutions not belonging to  $F = F_1 \cup F_2$ , the solutions which do not dominate each other but dominate all of the other solutions are kept in the next front, that is, set  $F_3$ . This process is repeated until all solutions in  $R_i$  are assigned one of the front. Subsequently, these generated fronts are assigned corresponding ranks, that is,  $F_1, F_2$  and  $F_3$  are assigned ranks 1, 2 and 3, respectively.

S3: Calculate the crowding distance in each  $F_k$  using crowding distance-assignment algorithm.

The crowding-distance-assignment algorithm was used in providing the diversity in population. To get an estimate of the density of solutions surrounding a particular point in the population, the average distance of the two points on either side of this point along each of the objectives is adopted. The obtained quantity serves as an estimate of the size of the largest cuboid enclosing the point of interest, without including any other point in the population.

S4: Sort the population  $R_i$  (sort by front order ( $F_k$ ) in ascending order and crowding distance in descending order).

S5: Select only first half of the population  $R_i$  and assign to  $P_{(i+1)}$ .

To create parent population  $P_{(i+1)}$  from combined population  $R_i$  for

next (i+1)th generation, initially, the solutions belonging to the set  $F_1$  are considered. If size of  $F_1$  is smaller than  $N$ , then all the solutions in  $F_1$  are included in  $P_{(i+1)}$ . The remaining solutions in  $P_{(i+1)}$  are filled from the rest of the non-dominated fronts in order of their ranks, that is, from  $F_2, F_3$ , etc., until the total number of solutions in  $P_{(i+1)}$  is greater than  $N$ . To make the size exactly equal to  $N$ , some solutions from the last included non-dominated front are discarded from  $P_{(i+1)}$ .

To choose the solutions to be discarded, initially, the solutions of the last included non-dominated front are sorted according to their crowding distances and, subsequently, the solutions having least crowding distances are discarded from  $P_{(i+1)}$ .

S6: Use crossover and mutation to recombine the population  $P_{(i+1)}$  and assign that to  $Q_{(i+1)}$

S7: Increment the iteration counter ( $i = i + 1$ )

S8: Repeat Steps S1 to S7, until the iteration meets with the maximum number of iterations.

**IMPLEMENTING MULTICAST ROUTING**

**Encoding scheme**

An individual or a solution is encoded using  $M$  arrays, where  $M$  represents the number of destinations in multicast group. Each array represents the node numbers forming a random route from a destination node in multicast group to source node. The size of an array varies depending on the number of nodes in each path and it may contain the maximum  $V$  nodes. Two consecutive nodes represent the existence of a link. Therefore, each array has a sequence of nodes from a destination node to the source node that are topologically connected. In this way, a solution is encoded or represented as multi-array arrangement. The loop, free multicast tree was obtained by testing the connectivity of each node in the following array with that of the preceding arrays (Kun et al., 2006). If a destination node is already visited in the preceding arrays, the tree may have less than  $M$  arrays.

With the aforementioned procedure, the population  $P_0$  of  $N$  individuals is created. The structure of two individuals  $I_1$  and  $I_2$  and corresponding loop free trees,  $T_1$  and  $T_2$  is as shown in Figure 2 for a sample 15-node Bellcore network presented in Figure 1. The source node is numbered as '0', whereas multicast group  $M$  members are numbered as 7, 9, 11 and 14. The objective functions  $f_1$  and  $f_2$  were calculated from the resulted trees.

**Creation of offspring**

The offspring population  $Q_0$  of size  $N$  was created using tournament selection, crossover and mutation operator on parent population  $P_0$  through the following two steps:

Step 1: To perform crossover operation, two individuals were selected, randomly from parent population by tournament selection and two randomly selected arrays in the individuals were swapped, and therefore, the two offspring individuals were obtained. The realization of crossover operation in the proposed scheme for offspring generation is as shown in Figure 3 where the arrays at positions 2 and 4 in individuals  $I_1$  and  $I_2$  were swapped to obtain two offspring solutions  $O_1$  and  $O_2$ .

Step 2: For mutation operation, an individual is selected randomly. Among this solution, a node within a randomly selected array is randomly selected. The sub-path from this selected node to the source is replaced by newly generated random sub-path. The realization of mutation operation in the proposed scheme is as shown in Figure 4. Node '5' in 3rd array representing path between destination node '7' and the source was selected randomly from the randomly selected individual  $O_1$ . The new individual  $O_3$  was

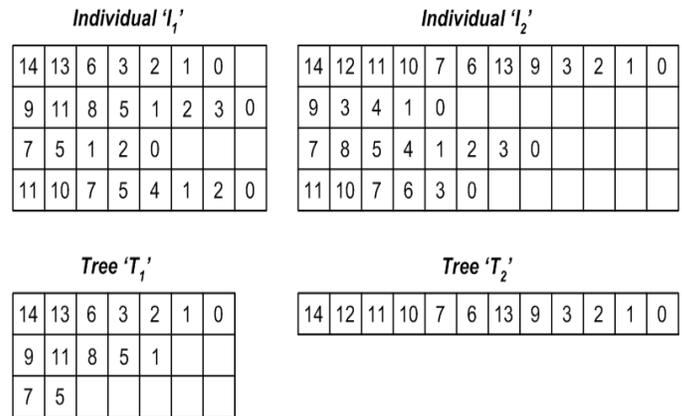


Figure 2. Illustration of solution and tree structure in the proposed tree structured encoding.

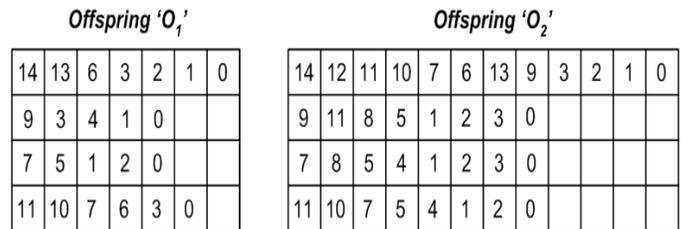


Figure 3. Realizing crossover operation in the proposed scheme.

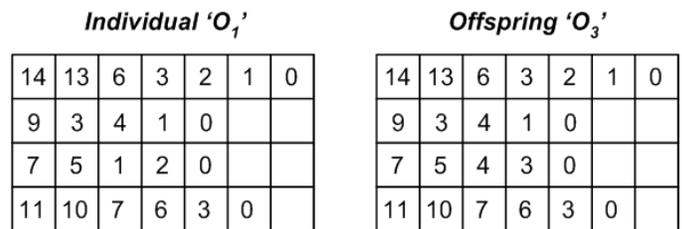


Figure 4. Realizing mutation operation in the proposed scheme.

generated by replacing the path from node '5' to source '0' in the 3rd array by new random path.

**Constraints and dominance check**

Individuals were compared for the dominance and also for the constraints violation. If two solutions are feasible, the winner is decided by the nondominance criteria. If one is feasible and other is infeasible, the feasible solution dominates. If both the solutions are infeasible, then the one with lowest amount of constraint violation dominates the other.

**BEST COMPROMISE SOLUTION**

After convergence of the elitist multi-objective optimization, a set of Pareto optimal solutions is achieved. These are the best solutions and are contained in the first front  $F_1$ . For practical purposes, one

solution among Pareto optimal solutions was selected and is known as the best compromise solution. For this, the normalize membership function  $\beta_k$  that provides the fuzzy cardinal priority ranking of the non-dominated solution was calculated. The solution for which the value of  $\beta_k$  is maximum, is considered as the best compromise solution.  $\beta_k$  is calculated as:

$$\beta_k = \frac{\sum_{i=1}^{N_{obj}} u_i^k}{\sum_{i=1}^{N_{obj}} \sum_{k=1}^M u_i^k} \quad (5)$$

where,

$$u_i^k = \frac{f_i^{\max} - f_i^k}{f_i^{\max} - f_i^{\min}} \quad (6)$$

**MULTI-OBJECTIVE MULTICAST ROUTING USING WEIGHTED-SUM APPROACH**

The multi-objective multicast routing problem for attempting cost minimization and available bandwidth maximization was formulated as a single objective optimization by using weighted sum approach as:

$$F(T) = Wf_1 + 100(1-W)(1 - f_2) + Af_3 + Bf_4 \quad (7)$$

where,

$$f_3 = \max\{delay_{\max} - \Delta, 0\}$$

$$f_4 = \max\{(delay_{\max} - delay_{\min}) - \delta, 0\} \quad (8)$$

Such that:

$$delay_{\max} = \max_{d_i \in M} (D(P_T(s, d_i)))$$

$$delay_{\min} = \min_{d_i \in M} (D(P_T(s, d_i))) \quad (9)$$

weight  $W$  is varied between 0.0 and 1.0.  $A$  and  $B$  are the penalty factors for the violation of delay and delay jitter bounds, and these factors are specified as 100 for the simulation. The encoding scheme to represent the individual and the procedure to generate the offspring is same as discussed in the implementation of multicast routing.

**EXPERIMENTAL RESULTS**

The effectiveness of the developed algorithm has been tested for various networks. The effects of change in multicast group size have also been studied. The Pareto optimal solutions and correspondingly, the best compromise solution have been obtained. The link parameters like cost and available bandwidth have been

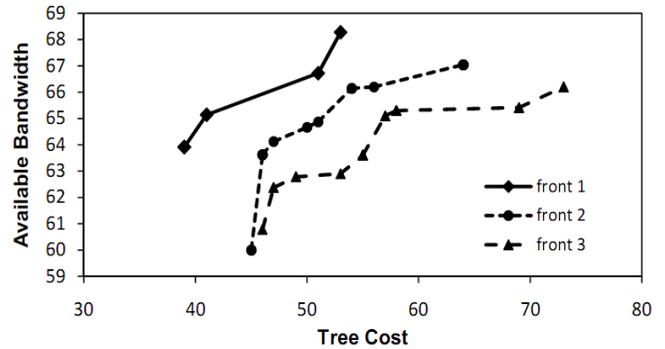


Figure 5. Initial Pareto fronts for 15-node Bellcore network.

assigned randomly in specified range. The link cost is assigned random integer value between 2 and 10, whereas the available bandwidth was assigned, randomly, integer between 70 and 170 Kbps. The capacity of each link is taken as 200 Kbps. Simulation was executed for 100 iterations with crossover and mutation rates as 0.9 and 0.10, respectively.

**15-node Bellcore network**

For the 15-node Bellcore network as shown in Figure 1, the link cost and available bandwidth were assigned according to the earlier discussed procedure. The link delay is randomly assigned integer values between 1 and 5. The optimization has been attempted for the source node as '0' and four destination nodes as 7, 9, 11 and 14. The limits on end-to-end delay and delay jitter were considered as 15 and 4, respectively. The initial population size  $N$  has been considered as 25. The Pareto fronts obtained from the initial population is as shown in Figure 5. The ranks of "front 1", "front 2" and "front 3" are 1, 2 and 3, respectively. The tree cost and available bandwidth for these fronts are summarized in Table 1.

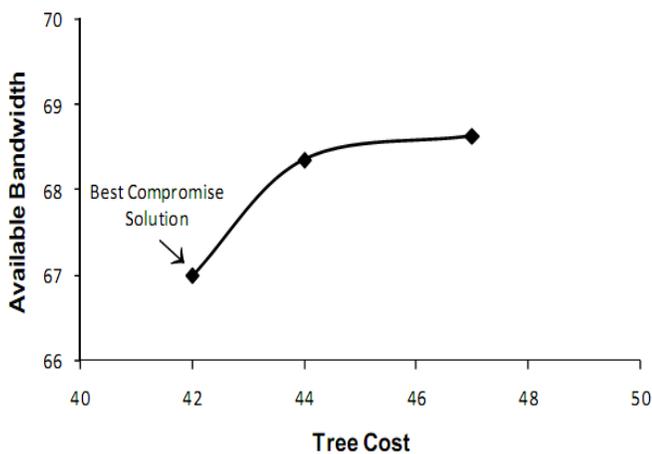
The Pareto optimal front at the convergence is as shown in Figure 6. The best compromise solution is also marked as shown in Figure 6. Correspondingly, the solution structure and the encoded tree are as shown in Figure 7a and corresponding optimal multicast tree on the network topology is as shown in Figure 7b.

**100-node network**

A random network of 100 nodes is generated by the BRITE network topology generator using the Waxman model (Medina et al., 2001; Waxman, 1988). The network spread was assumed in the area of 500 Km x 500 Km. The parameters that are controlling the edge density  $\beta$  and the density of short edges with respect to longer ones  $\alpha$  were specified as 0.9 and 0.7, respectively. The connection of each new node has been considered with

**Table 1.** Cost and available bandwidth for Initial Pareto fronts for 15-node Bellcore network.

Front 1		Front 2		Front 3	
Tree cost	Available bandwidth	Tree cost	Available bandwidth	Tree cost	Available bandwidth
39	63.92	45	60.00	46	60.79
41	65.14	46	63.63	47	62.38
51	66.72	50	64.67	49	62.78
53	68.28	51	64.88	53	62.90
-	-	54	66.15	55	63.61
-	-	56	66.20	57	65.10
-	-	64	67.05	58	65.30
-	-	-	-	69	65.42
-	-	-	-	73	66.20

**Figure 6.** Pareto optimal front for 15-node Bellcore network.

two other nodes. The delay was calculated as  $2/3$  of light velocity multiplied by the distance between the communications nodes. The study was carried out with delay and delay jitter bounds as 100 and 25, respectively. The population size  $N$  is taken as 100.

Simulation is attempted for the random multicast group size of 10%. Correspondingly, the initial population was classified into various Pareto fronts. Some initial Pareto fronts are as shown in Figure 8. The cost and available bandwidth for these fronts were summarized as shown in Table 2. The ranks of the Pareto fronts are same as the number of front. The “front 1” is closest to the vertices, whereas the “front 3” is farthest to the vertices.

The Pareto optimal front obtained in this study is as shown in Figure 9. For comparison, the initial “front 1” is also represented as shown in Figure 9. As expected, the Pareto optimal front is the closest to the vertices. Correspondingly, the cost and available bandwidth are as shown in Table 3.

The effect of the change in the multicast group size on the convergence time has been studied with both the

multi-objective optimization and the weighted genetic algorithm. The 100-node network described earlier has been considered for the study. Correspondingly, the results were summarized as shown in Table 4 and are depicted as shown in Figure 10. In this study, the multicast group size varied and the destination nodes forming the multicast group were selected, randomly. The other parameters are kept unchanged. The convergence time increases as the size of the multicast group increases and they exhibits near linear relationship. The significantly high convergence time is needed with the weighted genetic algorithm as compared to multi-objective optimization for all sizes of the multicast groups. As the size of multicast group increases, the difference of the convergence time needed by two algorithms also increase.

The effect on the convergence time for varying sizes of the random networks has been studied and the results are summarized in Table 5 and are as shown in Figure 11 for the optimization carried out by NSGA-II and weighted genetic algorithm. Random networks were obtained using Waxman model as per the procedure explained for 100-node network. For each, the multicast group size is kept as 20%. The link parameters cost, delay and available bandwidth were assigned as explained during the study of 15-node Bellcore network. The delay and delay jitter are taken as 100 and 25, respectively. The convergence time increases exponentially with the network size and the time needed by weighted approach is significantly high as compared to multi-objective optimization NSGA-II.

## Conclusion

The multi-objective multicast routing using an elitist multi-objective evolutionary algorithm was presented in this paper to simultaneously optimize cost and available bandwidth, while considering end-to-end delay and delay jitter as constraints. The topological assisted tree structured encoding has been used to represent the

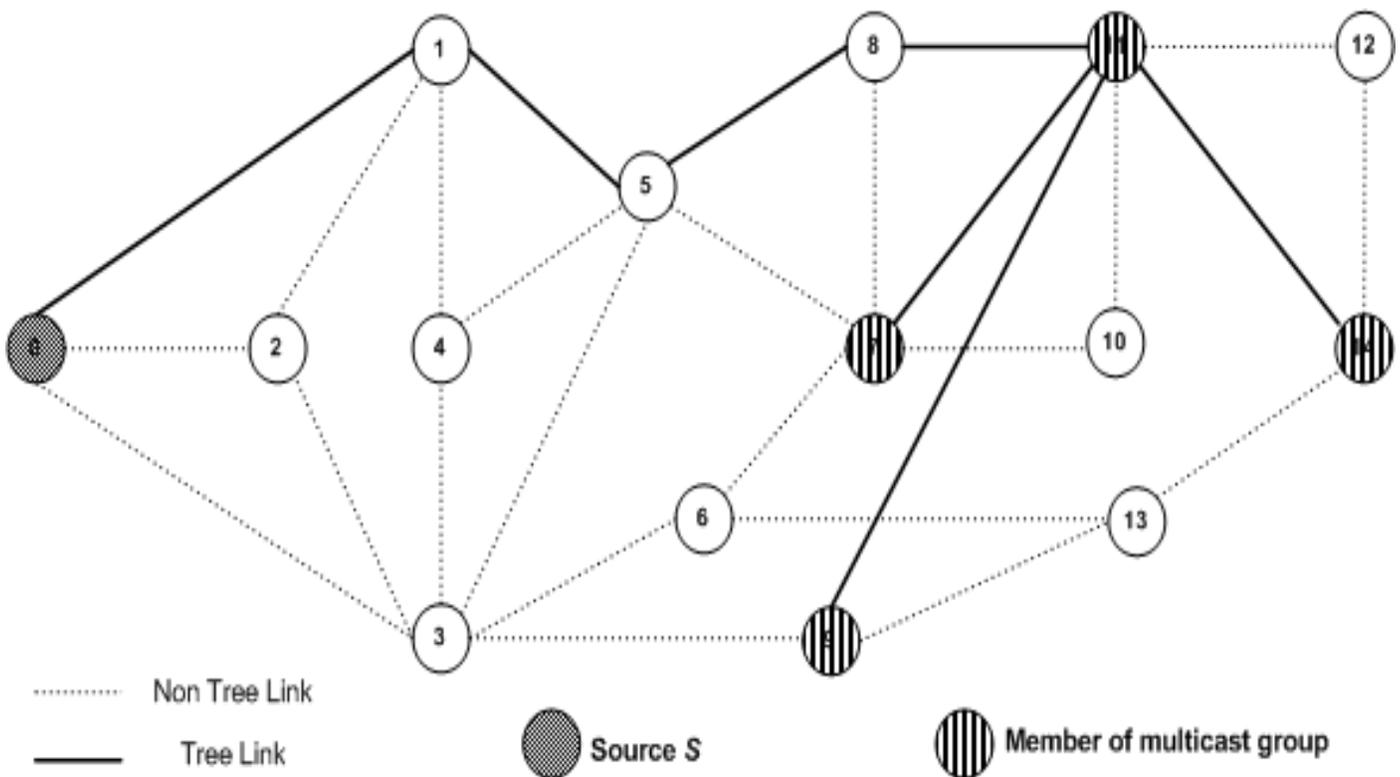
### Best Compromise Solution Structure

7	11	8	5	1	0		
9	11	10	7	5	1	2	0
11	10	7	5	1	2	0	
14	11	9	3	2	0		

### Best Compromise Tree Structure

7	11	8	5	1	0
9	11				
14	11				

(a)



(b)

**Figure 7.** Representation of optimal compromise solution and multicast tree: (a) encoded best compromise solution and tree, (b) best compromise optimal tree on network topology.

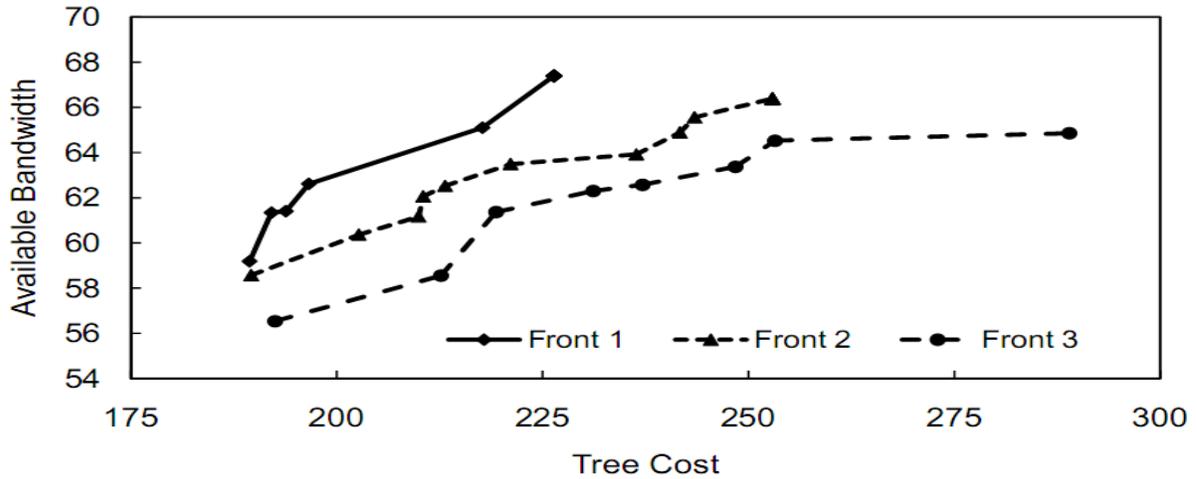


Figure 8. The initial Pareto fronts for 100-node network.

Table 2. Cost and available bandwidth for initial Pareto fronts for 100-node random network.

Front 1		Front 2		Front 3	
Tree cost	Available bandwidth	Tree cost	Available bandwidth	Tree cost	Available bandwidth
189.42	59.19	189.61	58.57	192.51	56.54
192.07	61.34	202.68	60.37	212.65	58.56
193.82	61.41	209.94	61.17	219.34	61.37
196.62	62.62	210.51	62.06	231.14	62.30
217.70	65.10	213.16	62.53	237.13	62.58
226.36	67.39	221.12	63.49	248.42	63.37
-	-	236.38	63.92	253.22	64.53
-	-	241.67	64.89	288.96	64.86
-	-	243.46	65.56	-	-
-	-	252.92	66.39	-	-

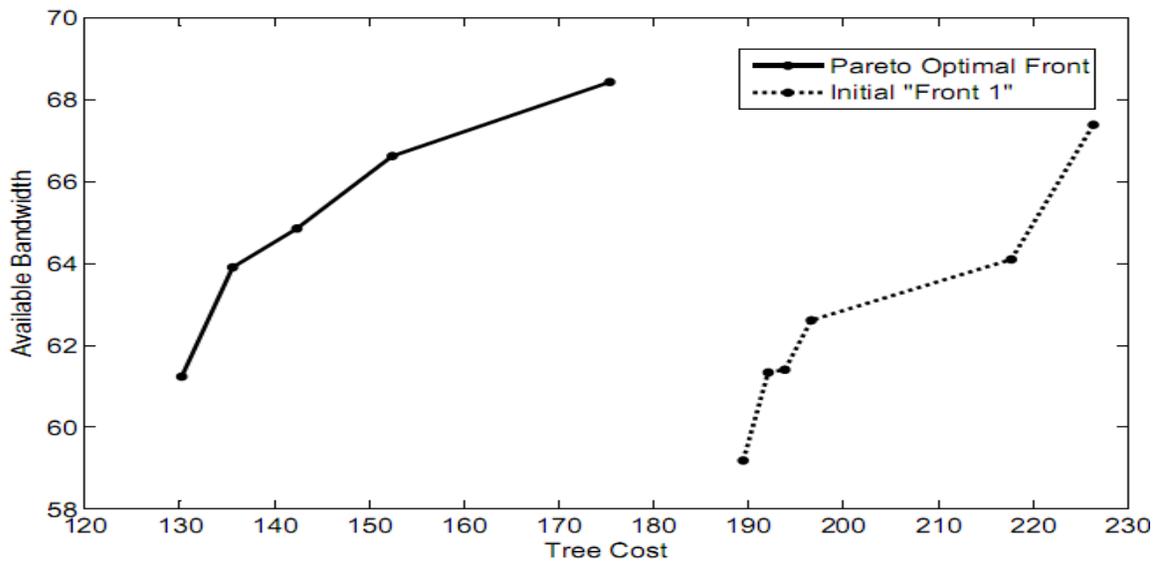


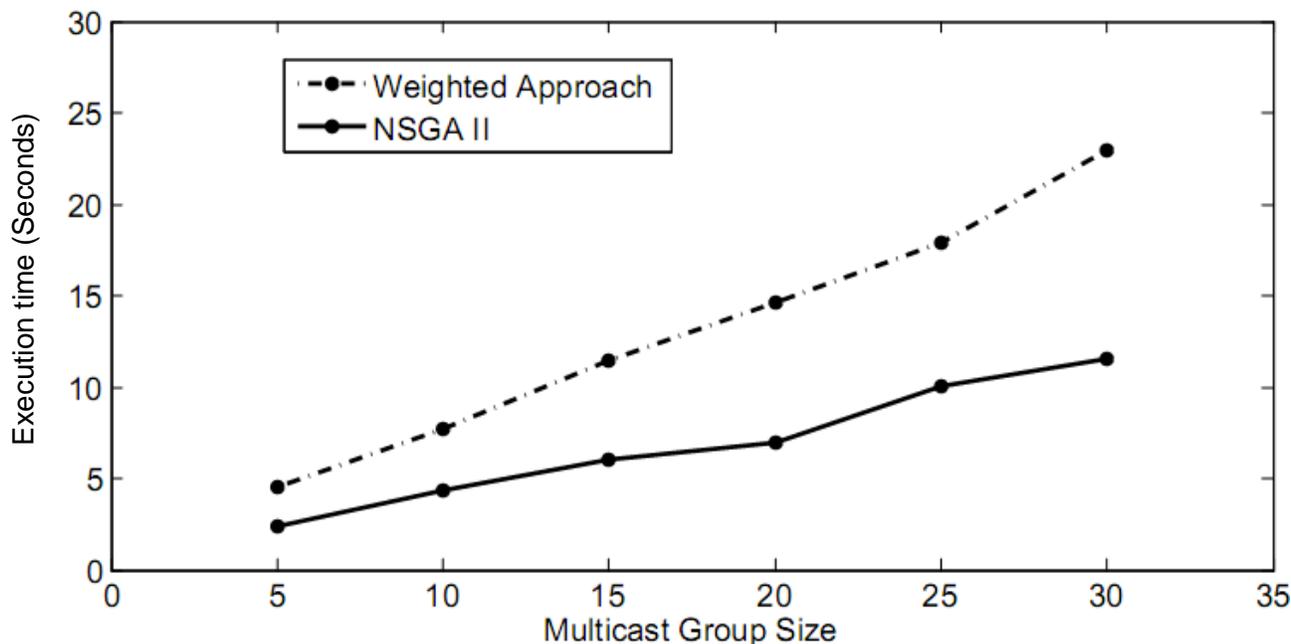
Figure 9. Representation of Pareto "optimal front" and initial Pareto "front 1" for 100-node network.

**Table 3.** Summary of Pareto optimal (final) front and initial Pareto “front 1” for 100-node random network.

Pareto optimal front		Initial Pareto “front 1”	
Tree cost	Available bandwidth	Tree cost	Available bandwidth
130.21	61.24	181.90	58.02
135.57	63.91	189.42	59.19
142.36	64.86	192.07	61.34
152.41	66.63	193.82	61.41
175.35	68.44	196.62	62.62
175.35	68.44	217.70	64.10
175.35	68.44	226.36	67.40

**Table 4.** Summary of convergence time for different sizes of multicast group in 100-node network.

Multicast group size	Weighted GA	NSGA-II
5	4.50	2.34
10	7.74	4.32
15	11.43	6.04
20	14.66	6.98
25	17.88	10.05
30	22.95	11.57

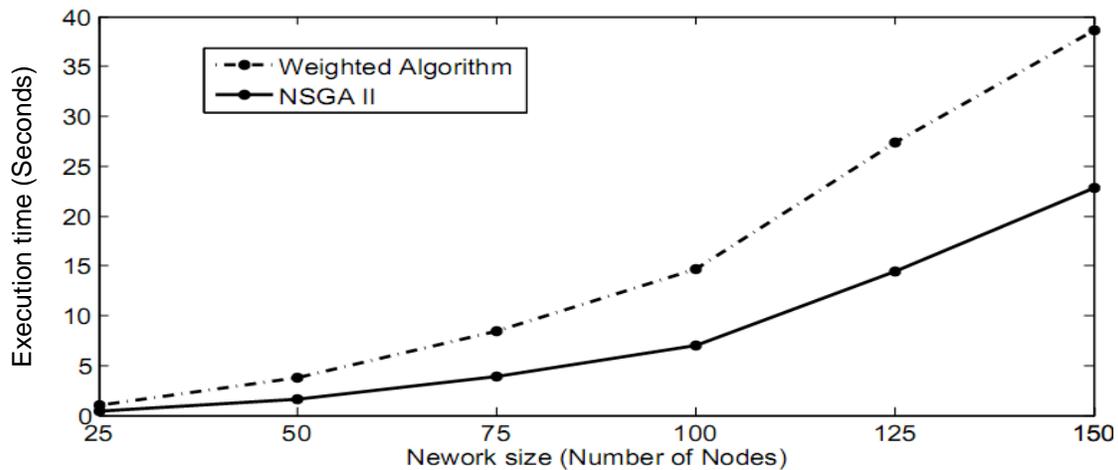
**Figure 10.** Effect of multicast group on the convergence time on 100-node network.

multicast tree. The individual solution, which is a multicast tree, has been represented as a combination of arrays where each array represents a random route from destination node in multicast group to source node. The tree structure was preserved during the crossover and

mutation operations. The effectiveness of the proposed algorithm has been tested on various networks and the compromise solution was obtained using fuzzy cardinal priority ranking. The performance of this algorithm has been compared with weighted sum genetic algorithm.

**Table 5.** Summary of convergence time for different sizes of random networks.

No. of nodes in the network	Time (s) weighted GA	Time (s) NSGA-II
25	1.01	0.44
50	3.74	1.57
75	8.47	3.89
100	14.66	6.98
125	27.42	14.43
150	38.65	22.80

**Figure 11.** The effect of network size on convergence time.

The convergence time increases with increase in the size of the multicast group and the size of the networks. The experimental study demonstrates that the multi-objective optimization with the proposed encoding scheme is effective in providing faster and guaranteed convergence.

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