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Full Length Research Paper

Soft ordered Abel-Grassman's groupoid (AG-groupoid)

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In 1999, Molodtsov developed the theory of soft sets involving enough parameters, which is relatively free from complexities when dealing with uncertainties. Most of the applications of soft sets towards algebraic structures stress on associativity of the binary operations (e.g., semigroups, groups, modules and rings etc.). In this study, we aim to apply Molodtsov's notion of soft sets to a class of non-associative algebraic structures and derive various related properties.

Key words: Ordered, Abel-Grassman's groupoid (AG-groupoid), soft sets, soft ordered AG-groupoids.

INTRODUCTION

For our urgent and straight understanding, the real world is multifaceted. Many problems in different disciplines such as engineering, social sciences, medical sciences, physics, computer sciences and artificial intelligence are usually not specific. We construct "models" of reality that are simplifications of aspects of the real world. Unluckily, these mathematical models are too intricate and we cannot find the precise solutions. There are always many uncertainties mixed up in the data. The traditional tools used to deal with these uncertainties are applicable only under certain environment. These may be owing to the uncertainties of natural environmental phenomena, of human awareness about the real world or to the confines of the means used to measure objects. For example, elusiveness or uncertainty in the boundary between states or between urban and rural areas or the exact growth rate of population in a country's rural area or making decision in a machine based environment using database information. Thus, the classical set theory, which is based on crisp and exact case, may not be fully suitable for conduct such problems of uncertainty.

Recently, many theories have been developed to deal with uncertainties, for example, theory of fuzzy sets (Zadeh, 1965), theory of intuitionistic fuzzy sets (Atanassov, 1986), theory of vague sets, theory of interval mathematics (Atanassov, 1994; Gorzalzany, 1987) and theory of rough sets (Pawlak, 1982). Though many techniques have been developed as a result of these theories, yet difficulties seem to be there. The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory as it was mentioned by Molodtsov (Molodtsov, 1999). He initiated the concept of soft set theory as a new mathematical tool which is free from the problems aforementioned. In his paper (Molodtsov, 1999), he presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc. A soft set is a collection of approximate description of an object. He also showed how soft set theory is free from parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly in these years.

Maji et al. (2002, 2003) presented an application of soft sets in decision making problems that are based on the reduction of parameters to keep the optimal choice objects. Chen (2005) presented a new definition of soft set parameterization reduction and a comparison of it with attributes reduction in rough set theory. Pie and Miao (2005) showed that soft sets are a class of special information systems. Kong et al. (2008) introduced the notion of normal parameter reduction of soft sets and its

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use to investigate the problem of sub-optimal choice and added parameter set in soft sets. Zuo and Xiao (2008) discussed soft data analysis approach. Ali et al. (2009) defined some new operations on soft sets and some old operations are redefined. Application of soft set theory in algebraic structures was introduced by Aktaş and Çağman (2007). They discussed the notion of soft groups and derived some basic properties. They also showed that soft groups extend the concept of fuzzy groups. Jun (2008), and Jun and Park (2008) investigated soft BCK/BCI-algebras and its application in ideal theory. Shabir et al. (2009) studied soft semigroups and soft ideals over a semi group which characterizes generalized fuzzy ideals and fuzzy ideals with thresholds of a semigroup.

An Abel-Grassmann's groupoid (Protic et al., 1995), abbreviated as AG-groupoid, is a groupoid S whose elements satisfy the left invertive law: $(ab)c = (cb)a$ It is also called as a left almost semigroup (Kazim and Naseeruddin, 1972). An AG-groupoid is a midway structure between a groupoid and a commutative semigroup. It is a useful non-associative structure and due to its peculiar characteristics, it has wide applications in theory of flocks and in theory of automata. Shah et al. (2010) [shrinkage reducing admixture (SRA)] have discussed the ordering of AG-grouped. Later Shah et al. (2010) discussed M -systems in ordered AG-groupoids and investigated various related properties.

The main purpose of this paper is to extend the study on soft ordered semigroups initiated by Jun et al. (2010), which are defined over an initial universe with a fixed set of parameters. We generalize most of results of Jun et al. (2010), by applying the notion of soft sets by Molodtsov to ordered AG-groupoids.

SOFT SETS

Definition 1

Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \to P(U)$ (Molodtsov, 1999).

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε –approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2

For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

(a) $A \subseteq B$ and

(b) for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations (Maji et al., 2003).

We write $(F, A) \tilde{\subseteq} (G, B)$. (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by $(F, A) \tilde{\supseteq} (G, B)$.

Definition 3

Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) (Maji et al., 2003).

Definition 4

Let $E = \{e_1, e_2, e_3, \ldots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\sim E$ is defined by $\sim E = \{\sim e_1, \sim e_2, \sim e_3, \ldots, \sim e_n\}$, where $\sim e_i =$ not all i (Maji et al., 2003).

The following results are obvious.

Proposition 1

1. $\sim(\sim A) = A$;
2. $\sim(A \cup B) = \sim A \cup \sim B$;
3. $\sim(A \cap B) = \sim A \cap \sim B$ (Maji et al., 2003).

Definition 5

The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \sim A)$ where, $F^c: \sim A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\sim \alpha)$, for all $\alpha \in \sim A$. (Maji et al., 2003).

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

Definition 6

A soft set (F, A) over U is said to be a $NULL$ soft set denoted by Φ if for all $\varepsilon \in A$, $F(\varepsilon) = \phi$ (null set) (Maji et al., 2003).

Definition 7

A soft set (F, A) over U is said to be $absolute$ soft set denoted by $\tilde{\tilde{A}}$ if for all $\varepsilon \in A$, $F(\varepsilon) = U$. Clearly $\bar{A}^c = \phi$

and $\phi^c = \bar{A}$ (Maji et al., 2003).

Definition 8

If (F, A) and (G, B) are two soft sets then, (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is defined by, where $H((\alpha, \beta)) = F(\alpha) \cap G(\beta)$, for all $(\alpha, \beta) \in A \times B$ (Maji et al., 2003).

Definition 9

If (F, A) and (G, B) are two soft sets then "(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by $(F, A) \vee (G, B) = (O, A \times B)$ where, $O((\alpha, \beta)) = F(\alpha) \cup G(\beta)$ for all $(\alpha, \beta) \in A \times B$ (Maji et al., 2003).

Proposition 2

1. $((F, A) \vee (G, B))^c = (F, A)^c \wedge (G, B)^c$
2. $((F, A) \wedge (G, B))^c = (F, A)^c \vee (G, B)^c$ (Ali et al., 2009).

Definition 10

Union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & if\ e \in A - B \\ G(e) & if\ e \in B - A \\ F(e) \cup G(e) & if\ e \in A \cap B \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$ (Maji et al., 2003).

Definition 11

The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$ (Maji et al., 2003)

ORDERED AG-GROUPOIDS

Here, we recall some definitions and results from Shah et al. (2010).

Definition 12

Let S be a nonempty set, $\cdot$ a binary operation on S and $\leq$ a relation on S. $(S, \cdot, \leq)$ is called an ordered AG-groupoid if $(S, \cdot)$ is a AG-groupoid, $(S, \leq)$ is a partially ordered set and for all $a, b, x \in S$, $a \leq b$ implies that $ax \leq bx$ and $xa \leq xb$. This structure is a generalization of AG-groupoids and ordered semigroups. The following theorem follows by Theorem 1 in Shah et al. (2010) and definitions of ordered AG-groupoids and ordered semigroups (Shah et al., 2010).

Theorem 1

An ordered AG-groupoid S is an ordered semigroup if and only if $a(bc) = (cb)a$ for all $a, b, c \in S$ (Shah et al., 2010).

For $H \subseteq S$, let $(H] = \{t \in S \mid t \leq h$ for some $h \in H\}$. Following lemma is similar to the case of ordered semigroups.

Lemma 1

Let S be an ordered AG-groupoid and A, B subsets of S. The following statements hold:

(1) If $A \subseteq B$, then $(A] \subseteq (B]$.
(2) $(A](B] \subseteq (AB]$.
(3) $((A](B]] \subseteq (AB]$. (Shah et al., 2010)

Definition 13

A nonempty subset A of an ordered AG-groupoid S is called a left ideal of S if $(A] \subseteq A$ and $SA \subseteq A$ and called a right ideal of S if $(A] \subseteq A$ and $AS \subseteq A$. A nonempty *subset* A *of* S *is called an ideal* of S if A is both left and right ideal of S .(Shah et al., 2010).

Proposition 3

Let S be an ordered AG-groupoid with left identity (Shah et al., 2010). Then every right ideal of S is a left ideal of S .

Lemma 2

Let S be an ordered AG-groupoid with left identity and $A \subseteq S$ (Shah et al., 2010). Then $S(SA) = SA$ and $S(SA] \subseteq (SA]$.

Lemma 3

Let S be an ordered AG-groupoid with left identity and $a \in S$. Then $< a >= (Sa]$ (Shah et al., 2010)

SOFT ORDERED AG-GROUPOIDS

Definition 14

Let $S = (S, \leq)$ be an ordered set and A a nonempty set. We define mapping $F : A \to P(S)$ as $F(x) = \{y \in S$ such that $xRy\}$ for all $x \in A$, where $R \subseteq A \times S$ is a binary relation between the elements of A and S. Then we call the pair (F, A) as a soft set over S.

Definition 15

Let the pair (F, A) be a soft set over ordered AG-groupoid $(S, \cdot, \leq)$. Then (F, A) is called a soft ordered AG-groupoid over S if for all $x \in A$, $F(x) \neq \phi$ imply that $F(x)$ is a subAG-groupoidof S .

Example 1

Let $S = \{a, b, c, d\}$. Define the multiplication and the order as

$\cdot$	a	b	c	d
a	a	b	c	d
b	d	c	c	c
c	c	c	c	c
d	b	c	c	c

$\leq = \{(a,a),(a,c),(a,d),(b,b),(b,c),(b,d),(c,c),(c,d),(d,d)\}$. Then $(S, \cdot, \leq)$ is an ordered AG-groupoid. Let (F, A) be a soft set over S where $A = S \setminus \{d\}$ is the set of parameters and $F : A \to P(S)$ is a mapping defined by $F(a) = \{b, c, d\}$, $F(b) = \{b, c\}$ and $F(c) = S$. Then (F, A) is a soft ordered AG-groupoid over S.

Example 2

Consider a set $S = \{a, b, c, e, f\}$ with the following multiplication "$\cdot$" and order relation $\leq$

$\cdot$	a	b	c	e	f
a	a	b	c	e	f
b	b	b	b	e	f
c	c	b	f	e	f
e	f	f	f	b	e
f	e	e	e	f	b

$\leq = \{(a,a),(a,c),(a,e),(a,f),(b,b),(b,e),(b,f),(c,c),(c,f),(e,e),(e,f),(f,f)\}$, then $(S, \cdot, \leq)$ is an ordered AG-groupoid. Let (F, A) be a soft set over S where $A = \{x, y, z\}$ is the set of parameters and $F : A \to P(S)$ is the set-valued mapping given by $F(x) = \{b, e, f\}$, $F(y) = \{a, b\}$ and $F(z) = S$. Then it is not hard to see that (F, A) is a soft ordered semigroup over S.

Remark 1

Every fuzzy subAG-groupoid may be considered as the soft ordered AG-groupoid $(F, [0,1])$. Indeed, if f is a fuzzy subAG-groupoid of S, that is, $f(xy) \geq$ min $\{f(x), f(y)\}$, for all $x, y \in f$. We may consider the family of α -level sets of mapping f as follows: $F(\alpha) = \{x \in S \mid f(x) \geq \alpha\}$, where $\alpha \in [0,1]$. Obviously $F(\alpha)$ is a subAG-groupoid of S. If we know the family F, then by using $f(x) =$ sup $\{\alpha \in [0,1] \mid x \in F(\alpha)\}$, we can find the mapping $f(x)$.

Lemma 4

If (F,A) and (G,B) are soft ordered AG-groupoids over S and $A\cap B\neq\phi$, then any intersection $(F,A)\cap(F,B)$ is a soft ordered AG-groupoid over S.

Proof

By definition 8, we have $(F,A)\cap(F,B)=(H,C)$, where $C=A\cap B$ and $H(x)=F(x)\cap G(x)$ for all $x\in C$. Since $H:C\to P(S)$ is a mapping it follows that (H,C) is a soft set over S. By hypothesis, as (F,A) and (G,B) are soft ordered AG-groupoids over S, so it imply that $H(x)$ is a subAG-groupoid of S for all $x\in C$. Thus $(F,A)\cap(F,B)=(H,C)$ is a soft ordered AG-groupoid over S.

Lemma 5

Let (F,A) and (G,B) be soft ordered AG-groupoids over S. If $A\cap B=\phi$, then union $(F,A)\cup(F,B)$ is a soft ordered AG-groupoid over S.

Proof

Using definition 10, $(F,A)\cup(F,B)=(H,C)$, where $C=A\cup B$ and

$$H(x)=\begin{cases} F(x) & \text{if } x\in A-B \\ G(x) & \text{if } x\in B-A \\ F(x)\cup G(x) & \text{if } x\in A\cap B \end{cases}$$

Given that A and B are disjoint, hence either $x\in A-B$ or $x\in B-A$ for all $x\in C$. Let $x\in A-B$. Then $H(x)=F(x)$ is a subAG-groupoid of S. If $x\in B-A$, then $H(x)=G(x)$ is a subAG-groupoid of S. Hence $(F,A)\cup(F,B)=(H,C)$ is soft ordered AG-groupoid over S.

Lemma 6

If (F,A) and (G,B) are soft ordered AG-groupoids over S, then $(F,A)\wedge(G,B)$ is a soft ordered AG-groupoid·

Proof

By definition 11, $(F,A)\wedge(G,B)=(H,A\times B)$, where $H((x,y))=F(x)\cap G(y)$, for all $(x,y)\in A\times B$. As (F,A) and (G,B) are subAG-groupoids so there intersection is also a subAG-groupoid. It follows that $H((x,y))$ is a subAG-groupoid of S for all $(x,y)\in A\times B$. Hence $(F,A)\wedge(G,B)=(H,A\times B)$ is a soft ordered AG-groupoid over S.

In the following, we define whole soft ordered AG-groupoid parallel to definition 4.10 in Jun et al. (2010).

Definition 16

A soft ordered AG-groupoid (F,A) is called a whole if $F(x)=S$ for all $x\in A$.

Example 3

1. We take the same ordered AG-groupoid $(S,\cdot,\leq)$ as given in Example 1. Let $F:A\to P(S)$ is a set-valued mapping defined by $F(x)=\{y\in S$ such that $xy=x\}$ for all $x\in A$ and let $A=\{c\}$ be a set of parameters. It is easy to observe that $F(c)=S$. Hence (F,A) is a whole soft ordered AG-groupoid.
2. From Example 2, if we take $A=\{f\}$ and define $F:A\to P(S)$ by $F(x)=\{y\in S \mid (xy)y=x\}$ for all $x\in A$, then it is not hard to see that $F(f)=S$. Hence (F,A) becomes another example of whole soft ordered AG-groupoid.

Definition 17

An ordered AG-groupoid S is said to be a poe-AG-groupoid if S contains a greatest element and we denote it by T.

Definition 18

Let S be poe-AG-groupoid. A soft ordered AG-groupoid

(F, A) over S is said to be trivial if $F(x) = \{T\}$ for all $x \in A$.

Example 4

Again consider the ordered AG-groupoid $(S,\cdot,\leq)$ given in Example 2. As f is the greatest element in S, so $(S,\cdot,\leq)$ is a poe-AG-groupoid. Let $B = \{a, f\}$ and interestingly if we take the same mapping defined for the case of whole soft ordered AG-groupoid in Example 3(b), that is, $F : B \to P(S)$ is defined by $F(x) = \{y \in S \mid (xy)y = x\}$ for all $x \in B$. Then $F(a) = F(f) = \{f\}$. It follows that $F(B)$ is a trivial soft ordered AG-groupoid over S.

Let $f : S \to T$ be a mapping of ordered AG-groupoids. If (F, A) is soft set over S, then $(f(F), A)$ will be a soft set over T, where the mapping $f(F): A \to P(T)$ is defined as $f(F)(x) = f(F(x))$ for all $x \in A$.

Definition 19

A mapping $f : S \to T$ of ordered AG-groupoids is called a homomorphism if it satisfies:

(a) $f(xy) = f(x)f(y)$,
(b) $x \leq y \Rightarrow f(x) \leq f(y)$, for all $x, y \in S$.

Lemma 7

Let $f : S \to T$ be a homomorphism of ordered AG-groupoids. If H is a subAG-groupoid of S, then $f(H)$ is a subAG-groupoid of T.

Proof

Let $a, b \in f(H)$. Then for some $x, y \in H$, we have $f(x) = a$ and $f(y) = b$. Since f is a homomorphism and H is a subAG-groupoid of S, it imply that $ab = f(x)f(y) = f(xy) \in f(H)$. Hence $f(H)$ is a subAG-groupoid of T.

Lemma 8

Let $f : S \to T$ be a homomorphism of ordered AG-groupoids. If (F, A) is soft ordered AG-groupoid over S, then $(f(F), A)$ is a soft ordered AG-groupoid over T.

Proof

Since $F(x)$ is a subAG-groupoid of S and its homomorphic image, that is, $f(F(x))$ is also a subAG-groupoid of T. It imply that for all $x \in A$, $f(F)(x) = f(F(x))$ is a subAG-groupoid of T. Hence $(f(F), A)$ is a soft ordered AG-groupoid over T.

Theorem 2

If $f : S \to T$ is a homomorphism from an ordered AG-groupoid S to a poe-AG-groupoid T and if (F, A) is soft ordered AG-groupoid over S, then the following assertions are true:

(1) If for all $x \in A$, $F(x) \subseteq \ker(f)$, then $(f(F), A)$ is a trivial soft ordered AG-groupoid over T.
(2) If (F, A) is whole, then $(f(F), A)$ is a whole soft ordered AG-groupoid over T.

Proof

(1) Let for all $x \in A$, $F(x) \subseteq \ker(f)$. Then $f(F)(x) = f(F(x)) = \{T\}$ for all $x \in A$. Thus from Definition 18 and Lemma 8, it follows that $(f(F), A)$ is a trivial soft ordered AG-groupoid over T.
(2) If (F, A) is a whole, then by definition for all $x \in A$ we have $F(x) = S$, and so $f(F)(x) = f(F(x)) = f(S) = T$ for all $x \in A$. Hence $(f(F), A)$ is a whole soft ordered AG-groupoid over T by Definition 16 and Lemma 8·

Definition 20

Let (F, A) and (G, B) be soft ordered AG-groupoids over S. Then (G,B) is called soft ordered subAG-groupoid of (F,A), denoted by $(G,B)\leq(F,A)$, if following conditions are satisfied.

(a) $B\subseteq A$.

(b) For all $x\in B$, $G(x)$ is a subAG-groupoid of $F(x)$.

We illustrate this definition by the following example.

Example 5

In the following Cayley table, let reconsider the ordered AG-groupoid S as given in Example 2, where $S=\{a,b,c,e,f\}$.

$\cdot$	a	b	c	e	f
a	a	b	c	e	f
b	b	b	b	e	f
c	c	b	f	e	f
e	f	f	f	b	e
f	e	e	e	f	b

Let (F,A) be a soft set over S where $A=S$ is the set of parameters and $F:A\to P(S)$ is a mapping defined by $F(x)=\{y\in S\mid(xx)y\in\{b,e,f\}\}$, for all $x \in A$. It follows that for all $x\in A$,

$$F(x)=\begin{cases} S & if\ x\in A-\{a\} \\ S-\{a\} & if\ x=a. \end{cases}$$

Obviously S and $S-\{a\}$ are subAG-groupoids of S. Thus (F,A) is a soft ordered AG-groupoid over S. Now let we take the set of parameters as $B=\{e,f\}$ and define a mapping $G:B\to P(S)$ by $G(e)=\{b,c,e,f\}$ and $G(f)=\{b,e,f\}$. Then $G(e)$ and $G(f)$ are subAG-groupoids of $F(e)$ and $F(f)$ respectively and hence (G,B) is a soft ordered subAG-groupoid of (F,A).

Proposition 4

If (F,A) and (G,A) are soft ordered AG-groupoids over a poe-AG-groupoid S, then the following assertion are true:

Theorem 3

(1) If for all $x\in A$, $F(x)\subseteq G(x)$, then $(F,A)\leq(G,A)$.

(2) If $B=\{T\}$ and if (H,B) and (H,S) are soft ordered AG-groupoids over S, then $(H,B)\leq(H,S)$.

Proof: Straightforward.

Proposition 5

Let (F,A) be a soft ordered AG-groupoids over S and let (G_1,B_1) and (G_2,B_2) be soft ordered subAG-groupoids of (F,A). Then

(1) Intersection of (G_1,B_1) and (G_2,B_2) is a soft ordered subAG-groupoids of (F,A).

(2) If B_1 and B_2 are distinct, then union of (G_1,B_1) and (G_2,B_2) is a soft ordered subAG-groupoids of (F,A).

Proof

(1) From Definition16, it follows that $(G_1,B_1)\cap(G_2,B_2)=(G,B)$, where $B=B_1\cap B_2$ and for all $x\in B$, $G(x)=G_1(x)\cap G_2(x)$. Let $x\in B$, then obviously $G(x)=G_1(x)$ or $G(x)=G_2(x)$. Since by hypothesis $(G_1,B_1)\leq(F,A)$ and $(G_2,B_2)\leq(F,A)$, it imply that $G(x)$ is a subAG-groupoid of $F(x)$. Hence $(G_1,B_1)\cap(G_2,B_2)=(G,B)\leq(F,A)$.

(2) Let $B_1\cap B_2=\phi$. Using definition, we have $(G_1,B_1)\cup(G_2,B_2)=(G,B)$, where $B=B_1\cup B_2$ and for all $x\in B$,

$$G(x)=\begin{cases} G_1(x) & if\ x\in B_1-B_2 \\ G_2(x) & if\ x\in B_2-B_1 \\ G_1(x)\cup G_2(x) & if\ x\in B_1\cap B_2. \end{cases}$$

By hypothesis $(G_1,B_1)\leq(F,A)$ and $(G_2,B_2)\leq(F,A)$. Now $B=B_1\cup B_2\subseteq A$ and for all $x\in B$, $G_1(x)$ and $G_2(x)$ subAG-groupoids of $F(x)$. Since $B_1\cap B_2=\phi$, it implies that for all $x\in B$, $G(x)$

is a subAG-groupoid of $F(x)$. Hence $(G_1, B_1) \cup (G_2, B_2) = (G, B) \le (F, A)$.

Lemma 9

Let $f : S \to T$ be a homomorphism of ordered AG-groupoids and let (F, A) and (G, B) be soft ordered AG-groupoids over S. If $(F, A) \le (G, B)$, then $(f(F), A) \le (f(G), B)$.

Proof

Let $x \in A$. Now if $(F, A) \le (G, B)$, then by definition $A \subseteq B$ and $F(x)$ is a subAG-groupoid of $G(x)$. Also since f is a homomorphism, $f(F(x)) = f(F)(x)$ is a subAG-groupoid of $f(G(x)) = f(G)(x)$ and hence $(f(F), A) \le (f(G), B)$.

Definition 21

Let (F, A) be a soft AG-groupoid over S. A soft set (G, I) over S is called a soft left ideal of (F, A) if (i) $I \subseteq A$ and (ii) for all $x \in I$, $G(x)$ is a left ideal of $F(x)$.

A soft set (G, I) over S is called soft right ideal of (F, A) if (i) $I \subseteq A$ and (ii) for all $x \in I$, $G(x)$ is a right ideal of $F(x)$. And (G, I) is called a soft ideal of (F, A). if it is both a soft left ideal and a soft right ideal of (F, A).

Example 6

Reconsidering the Example 2 and Example 5 for ordered AG-groupoid and for soft ordered AG-groupoid, respectively. Let $I = \{c, e, f\} \subset A$ and define a mapping $G : I \to P(S)$ by $G(x) = \{y \in S \mid x(xy) \in \{b, e, f\}\}$, for all $x \in I$. Then $G(c) = G(e) = G(f) = \{a, b, c, e, f\}$ which obviously is both a left ideal and a right ideal of $F(x)$.

Theorem 4

Let (F, A) be a soft ordered AG-groupoid over S and let (G_1, I_1) and (G_2, I_2) be any soft sets over S. If $I_1 \cap I_2 \ne \phi$, then we have:

(1) If (G_1, I_1) and (G_2, I_2) are soft left ideals of (F, A), then $(G_1, I_1) \cap (G_2, I_2)$ is also a soft left ideal of (F, A).

(2) If (G_1, I_1) and (G_2, I_2) are soft right ideals of (F, A), then $(G_1, I_1) \cap (G_2, I_2)$ is also a soft right ideal of (F, A).

Proof

(1) From Definition 16, we have $(G_1, I_1) \cap (G_2, I_2) = (G, I)$, where $I = I_1 \cap I_2$ and for all $x \in I$, $G(x) = G_1(x) \cap G_2(x)$. It is not hard to see that $I \subset A$ and $G : I \to P(S)$ is a mapping. It follows that (G, I) is a soft set over S. Since (G_1, I_1) and (G_2, I_2) are soft left ideals of (F, A) and also for all $x \in I$, $G(x) = G_1(x)$ and $G(x) = G_2(x)$ are left ideal and right ideal of $F(x)$ respectively. Hence $(G_1, I_1) \cap (G_2, I_2) = (G, I)$ is a soft left ideal of (F, A). Likewise we can prove that $(G_1, I_1) \cap (G_2, I_2) = (G, I)$ is a soft right ideal of (F, A).

Remark 2

As a Remark, we can say that if (F, A) is a soft ordered AG-groupoid over S and if (G, I) and (H, J) be any soft sets over S, then

(i) If (G, I) and (H, J) are soft left ideals of (F, A), then $(G, I) \cap (H, J)$ is also a soft left ideal of (F, A).

(ii) If (G, I) and (H, J) are soft right ideals of (F, A), then $(G, I) \cap (H, J)$ is also a soft right ideal of (F, A).

Theorem 5

Let (F, A) be a soft ordered AG-groupoid over S and if (G, I) and (H, J) are any soft sets over S such that $I \cap J = \phi$, then we have:

(1) If (G, I) and (H, J) are soft left ideals of (F, A),

then $(G,I)\cup(H,I)$ is also a soft left ideal of (F,A).
(2) If (G,I) and (H,J) are soft right ideals of (F,A), then $(G,I)\cup(H,I)$ is also a soft right ideal of (F,A).

Proof

(1) Using definition10, we have $(G,I)\cup(H,J)=(L,K)$, where $K=I\cup J$ and for all $x\in K$,

$$L(x)=\begin{cases} G(x) & if\ x\in I-J \\ H(x) & if\ x\in J-I \\ G(x)\cup H(x) & if\ x\in I\cap J. \end{cases}$$

As I and J are disjoint, it follows that either $x\in I-J$ or $x\in J-I$ for all $x\in K$. Let $x\in I-J$, then we have $L(x)=G(x)$, which obviously a left ideal of $F(x)$ because by hypothesis (G,I) is a soft left ideal of (F,A). On the other hand if $x\in J-I$, then $L(x)=H(x)$, which by hypothesis, is also a left ideal of $F(x)$. Thus it follows that for all $x\in K$, $L(x)$ is a left ideal of $F(x)$ and hence $(G,I)\cup(H,J)=(L,K)$ is a soft left ideal of (F,A). Likewise we prove the second part.

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