

Short Communication

Mass, energy and momentum of photon in medium

Adnan Salih

Department of Physics, College of Science for Women, Baghdad University, Iraq.

Accepted 13 May, 2013

When photon travels in a medium where the velocity is lower than the speed of light, the photon possesses both the kinetic (E_k) and the potential (E_p) energy. We proved that the energy of photon ($E = nE_k$), where (n) is the refractive index, and the effective mass of photon in medium ($m' = nm$), where (m) is the effective mass of photon in vacuum. Also we proved that the momentum of photon ($p = E/c$) inside medium.

Key word: Mass, momentum, energy.

INTRODUCTION

Suppose that a photon, having momentum $\hbar k$ in vacuum, enters a transparent medium with index of refraction $n > 1$. What is the photon's new momentum? Remarkably, there is still no definite answer. In 1908, the German Hermann Minkowski derived one possible, yet surprising answer. The photon momentum should actually increase and take the value $n\hbar k$. In effect, Minkowski started from Einstein's earlier suggestion that a photon's energy is given by $E = hu$. Assuming a velocity c/n and $p = h/\lambda$, one observes that $p = n\hbar k$. One year later, the German physicist Max Abraham proposed a different answer. Abraham argued that the photon inside the medium would have a lower velocity and lower momentum. Abraham's momentum is $p = \hbar k/n$ (Campbell et al., 2005).

For nearly 100 years physicists and mathematicians have been debating the correct form of the energy momentum tensor required to describe the behavior of light at the interface between two dielectric materials of different refractive indices. The two main 'competing' theories during this time have been those proposed by Minkowski (1908) and Abraham (1909). The dilemma is whether the momentum of a photon in a medium at the single-photon level, either to multiplying or dividing the free-space value (h/λ) by the refractive index (n). In dispersive media, the situation is a bit more complicated

in that we need to discriminate between phase and group indices (Milonni and Boyd, 2005; Loudon et al., 2005; Garrison and Chiao, 2004). The debate that this work started has continued till the present day, punctuated by the occasional publication of 'decisive' experimental demonstrations supporting one or other of these values (Phil. Trans. R. Soc, 2010), Stephen Barnett of the University of Strathclyde in the UK has concluded that both formulations are in fact correct, with the difference essentially boiling down to whether one considers the wave or particle nature of light (Barnett and Stephen, 2010).

A 2010 study suggested that both equations are correct, with the Abraham version being the kinetic momentum and the Minkowski version being the canonical momentum, and claims to explain the contradicting experimental results using this interpretation (Barnett and Stephen, 2010).

The experiment of Ashkin and Dziedzic showed that the action of light on the surface of a liquid was also consistent with the Minkowski momentum, although this interpretation is far from unambiguous (Gordon, 1973). The experiments of Walker et al. (1975) provide evidence that is no less convincing in favour of the Abraham form. These early experiments and the conclusions derived from them are discussed at greater length in Brevik

(1979) and Pfeifer et al. (2007).

Until now there are no articles that discussed the relation between the energy, and mass of photon with refractive index and while the momentum of photon depends on the energy and mass. So why the refractive index effect is on the momentum, but there are not effects on the energy and mass of photon in the medium as indicated in current publications.

The aim of this work is to study the effect of the refractive index on energy, and effective mass of photon in medium.

ENERGY OF PHOTON IN MEDIUM

When photon travels in the transparent medium, the energy of photon can be divided into the kinetic and the potential energy of photon. It is complying with the principle of energy conservation in transparent medium. The same photon traveling in the water medium has higher kinetic energy than in the diamond medium and vice versa for the potential energy.

When the photon is traveling at the speed of light where the M and E vectors of photon are perpendicular, the photon possesses full kinetic (E_k) energy, and when a photon is brought to rest, the total energy of photon is transformed to the potential energy (E_p). This is also called the rest energy of mass. Completely stopped, all energy is transferred to matter in agreement with the experimental evidence of Hau and coworkers (Lene Hau', 2007).

The energy of photon can be further sub-divided into two portions. There are the kinetic and potential energy of photon. The energy equation of photon is described below:

$$E_t = E_k + E_p \quad (1)$$

$$E_k = pv \quad (2)$$

$$E_p = \tau \phi \quad (3)$$

$$E = pv + \tau \phi \quad (4)$$

Where p = momentum of photon, v = traveling speed of photon, τ = torque, angular force between electric and magnetic component, ϕ = deform angle of M and E components

$$E = hv \quad v/c + hv \quad (n-1)/n \quad (5)$$

Where $\tau = 2 h v / \pi$, and $\phi = \pi / 2(1-1/n)$ (www.greatians.com/physics).

From Equation (5), The kinetic energy of photon is:

$$E_k = hv \quad /n \quad (6)$$

And the potential energy is:

$$E_p = hv \quad (n-1)/n \quad (7)$$

From Equation (6) and equation (7)

$$E_p = E_k \quad (n-1) \quad (8)$$

From Equations (1 and 8):

$$E_t = n E_k \quad (9)$$

let a (1064 nm) photon with energy (1.8×10^{-19} J) enter into medium with refractive index (1.5), according to equations (6, 8):

$$\begin{aligned} E_k &= 1.8 \times 10^{-19} / 1.5 = 1.2 \times 10^{-19} \text{ J} \\ E_p &= 1.2 \times 10^{-19} (1.5-1) = 0.6 \times 10^{-19} \text{ J} \\ E_t &= 1.2 \times 10^{-19} + 0.6 \times 10^{-19} = 1.8 \times 10^{-19} \text{ J} \end{aligned}$$

These results prove that Equations (6, 8 and 9) are correct.

MASS OF PHOTON IN MEDIUM

Let (m) is the effective mass of photon in vacuum, and (m') is the effective mass in medium.

From Equation (9),

$$mc^2 = n m'v^2 \quad (10)$$

Where $E = mc^2$ in vacuum, and $E_k = m'v^2$ inside medium
Dividing equation (10) by c^2 ;

$$m = nm' / n^2 \quad (11)$$

$$m' = nm \quad (12)$$

$$E_k = nmv^2 \quad (13)$$

According to example,

$$\begin{aligned} E &= mc^2 \\ 1.8 \times 10^{-19} &= m \cdot 9 \times 10^{16} \\ m &= 0.2 \times 10^{-35} \text{ Kg} \end{aligned}$$

From Equations (13 and 8)

$$\begin{aligned} E_k &= 1.5 \cdot 0.2 \times 10^{-35} \cdot 4 \times 10^{16} \\ E_k &= 1.2 \times 10^{-19} \text{ J} \\ E_p &= 1.2 \times 10^{-19} \cdot (1.5-1) \\ E_p &= 0.6 \times 10^{-19} \text{ J} \\ E_t &= 1.2 \times 10^{-19} + 0.6 \times 10^{-19} \\ E_t &= 1.8 \times 10^{-19} \text{ J} \end{aligned}$$

These results prove that Equations (12 and 13) are correct.

MOMENTUM OF THE PHOTON IN MEDIUM

When photon travels in a medium where the velocity is lower than the speed of light, the photon possesses both the kinetic and the potential energy. The momentum of the photon depends on the kinetic energy only. Therefore,

$$P = E_k/v \quad (14)$$

According to Equation (6);

$$P = hv/nV \quad (15)$$

$$P = hv/C \quad (16)$$

Equation (16) represent the momentum of photon in medium which is equal to the momentum of photon in vacuum ($P = E/C$)

Minkowski described the momentum of light as a wave,

$$P = h/\lambda$$

$$P = hv/V \quad (17)$$

$$P = nE/c \quad (18)$$

In Equation (18), E is the total energy in medium, the momentum depends on kinetic energy, therefore,

$$E_k = E_t - E_p$$

$$E_k = hv - hv(n-1)/n \quad (19)$$

By using Equation (19) in (18),

$$P = n(hv - hv(n-1)/n)/c \quad (20)$$

$$P = hv/C \quad (21)$$

The result in Equation (21) is the same as in Equation (16)

According to the results in Equations (16 and 21) the momentum of photon in medium is equal to the momentum outside of medium.

Now if we suppose that the light is a particle according to Abraham, then;

$$P = m'v \quad (22)$$

From equation (12)

$$P = nmv \quad (23)$$

$$P = mC \quad (24)$$

$$P = hv/C \quad (25)$$

Abraham described the momentum of light as a particle;

$$P = mv = Ec^2/cn = hv/nc \quad (26)$$

By using Equation (12) in (26)

$$P = n hv/nc$$

$$P = hv/C \quad (27)$$

According to the results in Equations (25 and 27) the momentum of photon in medium is equal to the momentum outside of medium.

CONCLUSIONS

A simple analysis was done to prove that:

- 1- The total energy of photon inside medium is $E = n E_k$,
- 2- The effective mass of photon inside medium $m' = nm$,
- 3- The momentum of photon inside medium $P = /C$.

REFERENCES

- Abraham M (1909). Zur Elektrodynamik bewegter Krper. Rend. Circ. Mat. Palermo. 28:1-28. (doi:10.1007/BF03018208).
- Barnett S (2010-02-07). "Resolution of the Abraham-Minkowski Dilemma". Phys. Rev. Lett. 104(7):070401.
- Brevik I (1979). Experiments in phenomenological electrodynamics and the electromagnetic energy-momentum tensor. Phys. Rep. 52:133-201. (doi:10.1016/0370-1573(79)90074-7).
- Campbell GK, Leanhardt AE, Mun J, Boyd M, Streed EW, Ketterle W, Pritchard DE (2005). Photon recoil momentum in dispersive media. Phys. Rev. Lett. 9:170403. (doi:10.1103/PhysRevLett.94.170403).
- Garrison JC, Chiao RY (2004). Canonical and kinetic forms of the electromagnetic momentum in an *ad hoc* quantization scheme for a dispersive dielectric. Phys. Rev. A 70:053826, (doi:10.1103/PhysRevA.70.053826)
- Gordon JP (1973). Radiation forces and momenta in dielectric media. Phys. Rev. A. 8:14-21. (doi:10.1103/PhysRevA.8.14).
- Lene Hau' (2007). <http://www.harvard.edu/gazette/2007/02.08/99-hau.html>.
- Loudon R, Barnett SM, Baxter C (2005). Radiation pressure and momentum transfer in dielectrics: the photon drag effect. Phys. Rev. 71:063802. (doi:10.1103/PhysRevA.71.063802).
- Milonni PW, Boyd RW (2005). Recoil and photon momentum in a dielectric. Laser Phys. 15:1432-1438.
- Minkowski H, Nachr G, Wiss G (1908). Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern. Nachr. Königl. Ges. Wiss. Göttingen, pp. 53-111.
- Pfeifer RNC, Nieminen TA, Heckenberg NR, Rubinsztein-Dunlop H (2007). Colloquium: Momentum of an electromagnetic wave in dielectric media. Rev. Mod. Phys. 79:1197-1216. (doi:10.1103/RevModPhys.79.1197). Phil. Trans. R. Soc. A. 368:1914 927-939 (2010).
- Walker GB, Lahoz DG, Walker G (1975). Measurement of the Abraham force in a barium titanate specimen. Can. J. Phys. 53: 2577-2586. www.greatians.com/physics/wave/energy%20of%20photon.htm.