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Lehmann Type II weighted Weibull distribution

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We upgrade on the method used on weighted Weibull model proposed by Azzalini (1985) using the logit of Beta function by Jones (2004) to have Lehmann Type II weighted Weibull model with a view to obtaining a distribution that is more better than both weighted Weibull and Weibull distribution in terms of estimate of their characteristics and their parameter. The weighted Weibull distribution is proposed by slightly modifying the method of Azzalini (1985) on weighted distribution with additional shape (β) and scale (λ) parameters. Some basic properties of the newly proposed distribution including moments and moment generating function, survival rate function, hazard rate function, asymptotic behaviours, and the estimation of parameters have been studied.

Key words: Asymptotic, hazard rate, Lehmann Type II, moments, weighted-Weibull.

INTRODUCTION

The Weibull distribution is a well known common distribution and has been a powerful probability distribution in reliability analysis, while weighted distributions are used to adjust the probabilities of the events as observed and recorded. The Weibull distribution can also be used as an alternative to Gamma and Log-normal distribution in reliability engineering and life testing. Numerous authors/researchers have been worked on Weibull and exponential distribution in literature. In a nut shell, Shahbaz et al. (2010) applied Azzalini's method with the Weibull distribution that produced a new class of weighted Weibull distribution in which they obtained the probability density and cumulative density function as given below:

$$f(x) = \frac{\alpha+1}{\alpha}\lambda\beta x^{\beta-1}e^{-\lambda x^{\beta}}\left(1-e^{-\alpha\lambda x^{\beta}}\right);\quad \alpha,\beta,\lambda,\ x>0 \quad (1)$$

The graph of the probability density function of Equation (1) with $\alpha = 4, \beta = 3$ is given Figure 1.

$$F(x) = \frac{\alpha+1}{\alpha}\left[\left(1-e^{-\lambda x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^{\beta}}\right)\right] \quad (2)$$

In the literature, researchers/authors on Lehmann Type II are very few. Figure 1 is the pdf graph of the weighted Weibull by Shabarz (2010). The aim of this article is to introduce this distribution and study on its statistical properties. This article presents the proposed distribution Lehmann Type II weighted Weibull distribution. Thereafter, moments and moment generating function is studied. This is followed by a critical discussion of the estimation of parameters showing the empirical distribution of data and the study was concluded.

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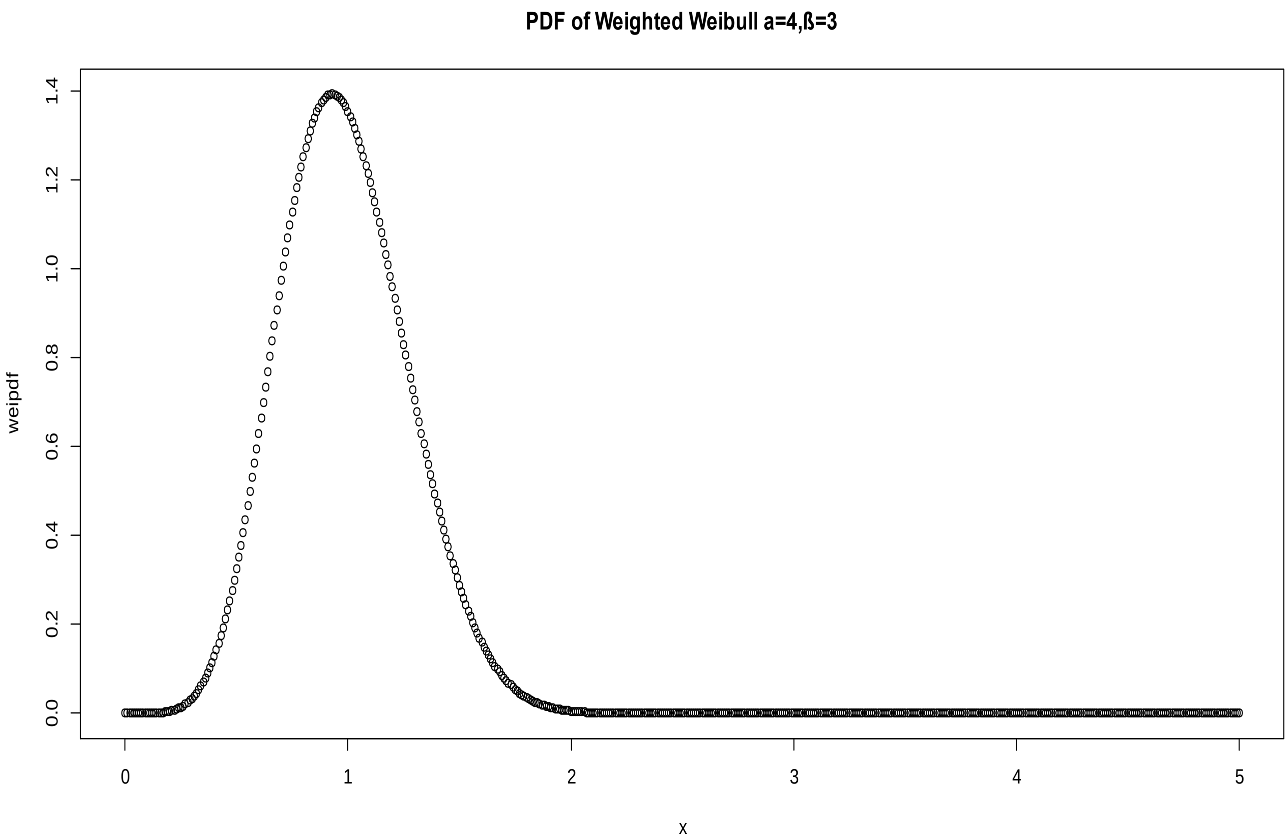

Figure 1. The probability density function of weighted Weibull distribution with $\alpha = 4, \beta = 3$ and the cumulative density function from Equation (1) is given in Equation (2).

MATERIALS AND METHODS

The proposed Lehmann Type Ii weighted Weibull distribution

The logit of beta function (the link function of the beta generalized distribution) is introduced by Jones (2004), since then extensive work has been done using the logit of beta distribution in literature. For instance, Gupta and Kundu (1990) proposed a generalized exponential distribution that provides an alternative to exponential and Weibull distribution. Lee et al. (2007), Eugene and Famoye (2002) and Nadarajah and Kotz (2005) also used the logit of beta distribution and they provided an extension on exponential distribution. The logit of beta distribution used by Famoye and Olugbenga (2005) to introduce the Beta-Weibull distribution alongside its major properties. The transformation $Exp^{c}F$ is called the Lehmann Type I distribution. There is a dual transformation defined by $Exp^{c}(1-F)$ referred to as the Lehmann Type II distribution introduced by Cordeiro et al. (2011) among others. Figures 2, 3 and 4 shows pdf, cdf and hazard rate graphs at several values of parameters. Figure 2 clearly shows that it is rightly skewed, Figure 3 rise upward to right and Figure 4 shows the level at which the hazard rate by rightly upward breaking points.

Now, let x be a random variable form of the distribution with parameters and defined (1) and (2) using the logit of beta by Jones (2004), we then have

$$f_{LWW}{}^{(x)} = \frac{1}{B(a,b)}\,[F(x)]^{a-1}\,[1-F(x)]^{b-1}\,f(x) \qquad (3)$$

Both probability density and cumulative density function (pdf and cdf) of Lehmann Type II weighted Weibull distribution is obtained by substituting pdf and cdf of weighted Weibull distribution in Equations (1) and (2) into Equation (3) and set $a = \lambda = 1,$ we then obtain

$$g_{LWWD}^{(x)} = b\left[1 - \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}\right]^{b-1}$$
$$\times\ \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^{\beta}}\left(1 - e^{-\alpha x^{\beta}}\right) \qquad (4)$$

Where, $a = 1, b > 0, \lambda = 1, \beta > 0, \alpha > 0 \text{ and } x > 0 \text{ such that } x \sim LWWD(1, b, 1, \alpha, \beta)$. Equation (5) is the pdf of Lehmann Type II weighted Weibull distribution.

setting $w(x) = \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}$ (5)

$$\frac{\partial w}{\partial x} = \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^{\beta}}\left(1 - e^{-\alpha x^{\beta}}\right)\frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}$$

substituting dbx into Equation (4), we get

$$g_{LWWD}^{(x)} = b\left[1 - \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}\right]^{b-1} dw$$
$$w = \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\} \qquad (6)$$

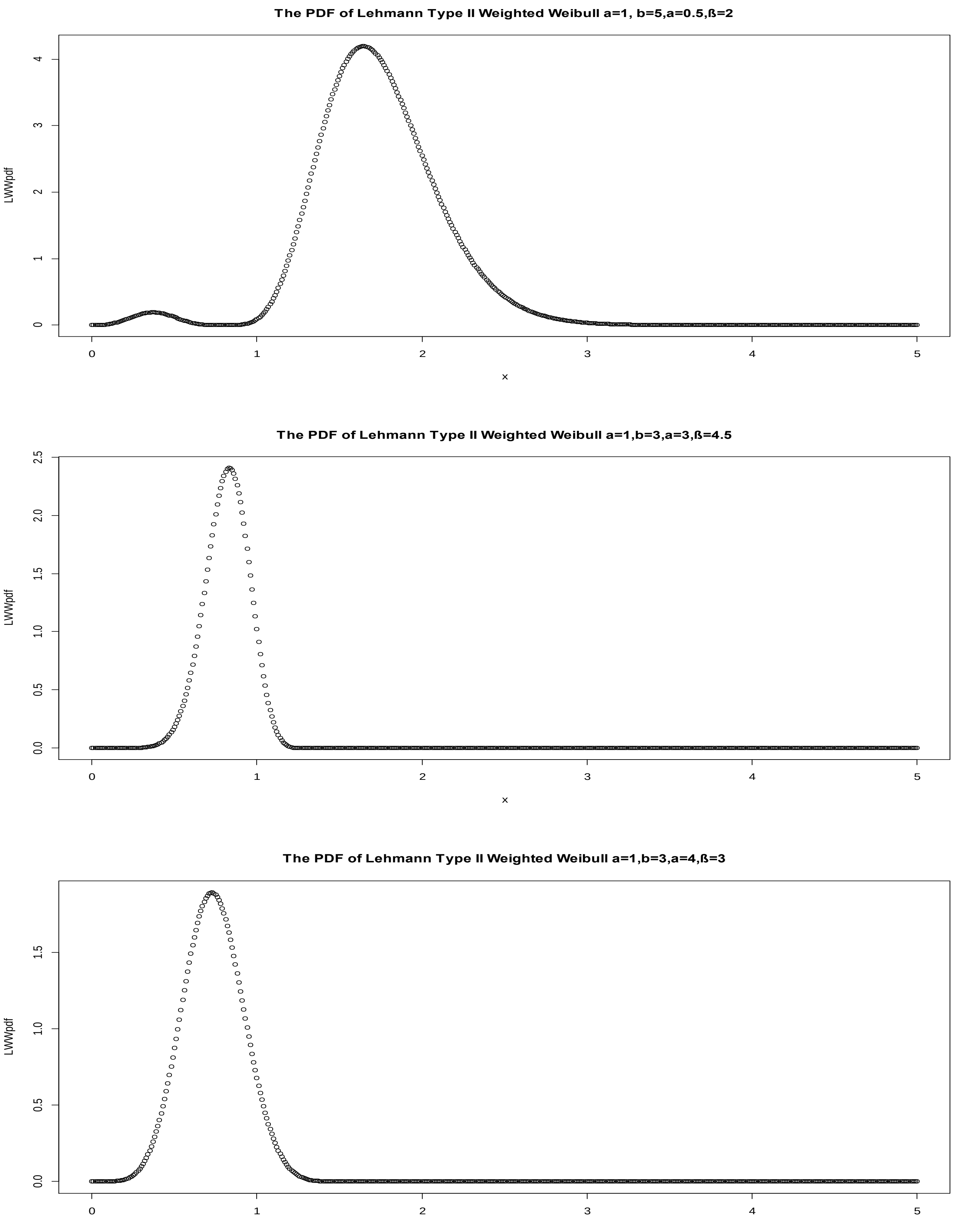

Figure 2. The pdf of LWW distribution at various values of the parameters and it is clear that indeed it is rightly skewed.

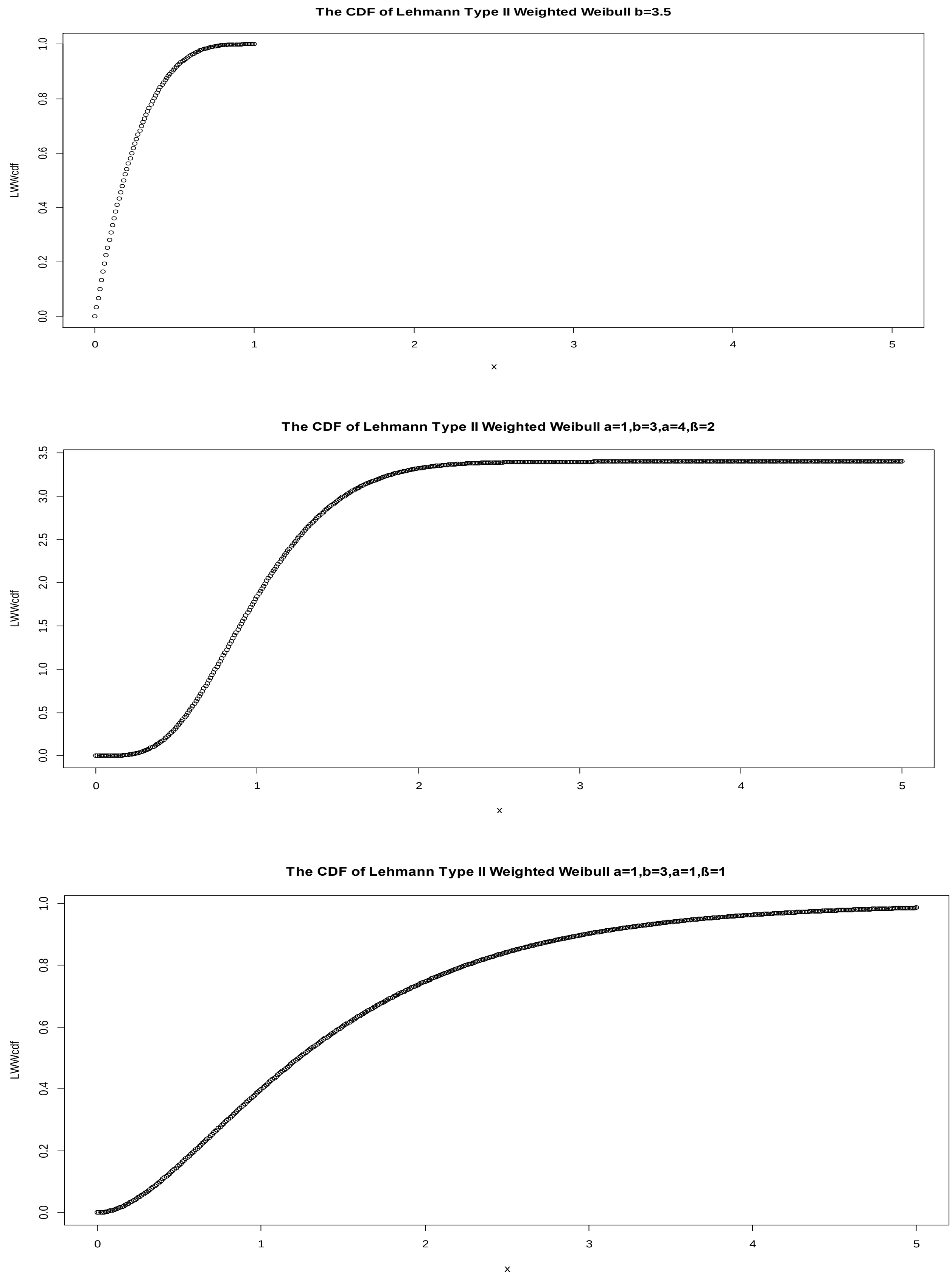

Figure 3. The CDF of LWW with a=1, b=3, $\alpha = 1, \beta = 1$. However, at given various values of the parameters, the graphs give the same pattern and shape.

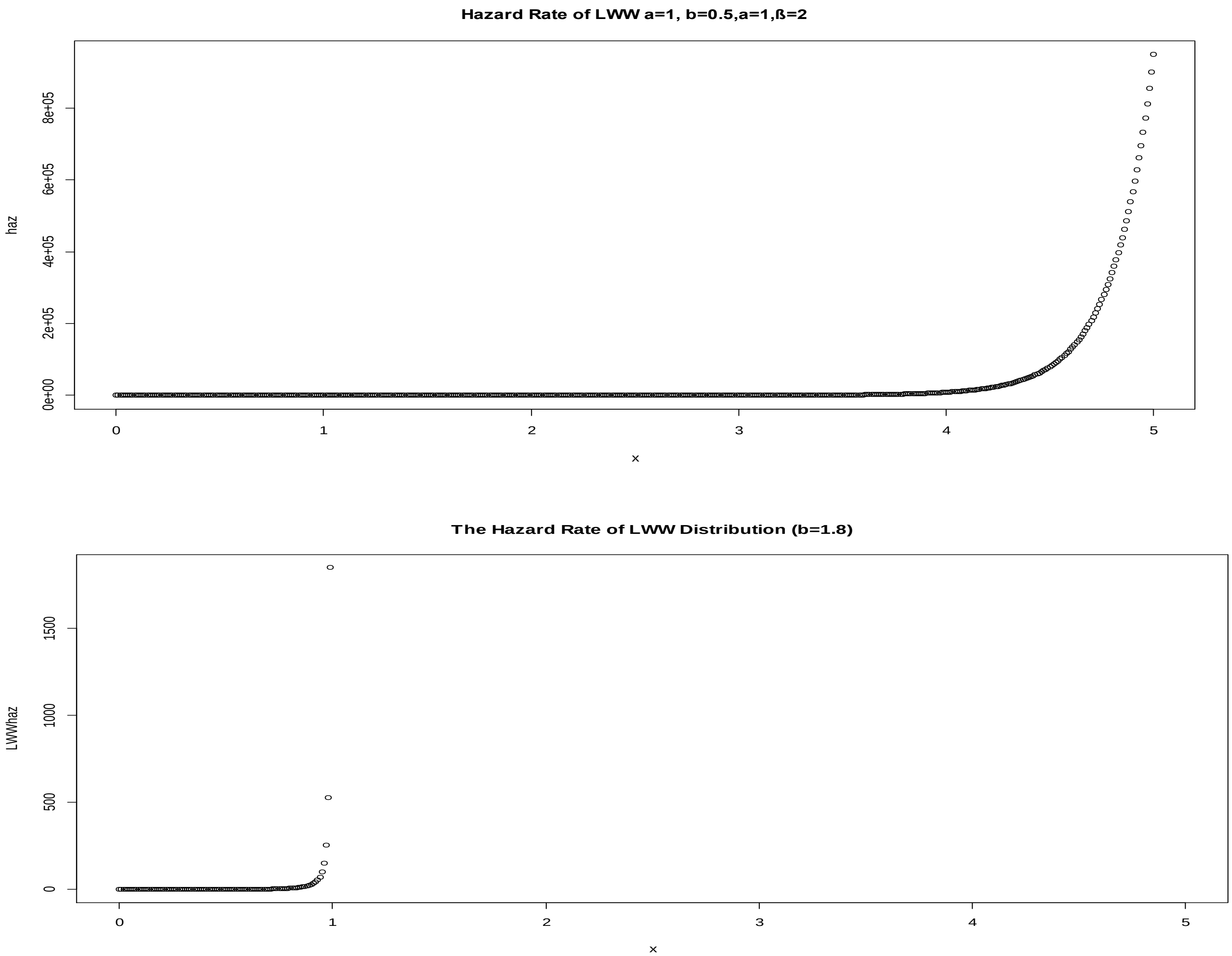

Figure 4. The hazard rate function of Lehmann Type II weighted Weibull distribution with several values of the parameter.

Therefore, Equation (4) can be re-written as

$$g_{LWW}{}^{(x)} = b[1 - W]^{b-1}\frac{dw}{dx} \tag{7}$$

From logit of beta link function, we show that Equation (7) equal to one

$$g_{LWWD}{}^{(x)} = b\left[1 - \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}\right]^{b-1} \times \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^{\beta}}\left(1 - e^{-\alpha x^{\beta}}\right)$$

Then, recall that $w = \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}$ and using the differential equation method on $\frac{\partial w}{\partial x}$. Therefore, we have

$$b\int_0^1 (1 - w)^{b-1}\, dw = 1$$

Figure 2 show the graph of the Lehmann Type II weighted Weibull distribution at different values of the parameters.

Cumulative density function (CDF)

We have defined the probability density function (pdf) of $LWWD\,(1, b, 1, \alpha, \beta)$ in Equation (6), then Equation (6) can be given as

$$G_{LWWD}{}^{(x)} = P(X \le x) = \int_0^x f(w)dw$$

$$= \int_0^x b\left[1 - \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^{\beta}}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^{\beta}}\right)\}\right]^{b-1}\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^{\beta}}\left(1 - e^{-\alpha x^{\beta}}\right)\, dw \tag{8}$$

$$G_{LWWD}{}^{(x)} = P(X \le x) = \int_0^x b[1 - W]^{b-1}\, dw$$

$= b\int_0^x [1 - W]^{b-1}\, dw$ and cdf is given as

$$G_{LWWD}{}^{(x)} = [1 - (1 - x)^b] \tag{9}$$

The graph of cumulative distribution of Lehmann Type II Weighted Weibull is shown Figure 3.

The survival rate function

The survival rate function of the $LWWD\,(1, b, 1, \alpha, \beta)$ is defined by

$$S_{LWWD}(x) = 1 - G_{LWWD}(x) = 1 - \int_0^x f(w)\,dw$$
$$= 1 - b\int_0^x [1 - W]^{b-1}\,dw$$
$$S_{LWWD}(x) = (1 - x)^b \qquad (10)$$

The hazard rate function

The hazard rate function of a random variable x with the pdf and cdf is given by

$$h_{LWWD}(x) = \frac{g_{LWWD}(x)}{1 - G_{LWWD}(x)}$$

However, $LWWD\,(1, b, 1, \alpha, \beta)$ with $g_{LWWD}(x)$ and $G_{LWWD}(x)$ respectively defined in Equations (4) and (9), the hazard rate function can be expressed as:

$$= \frac{b(1-w)^{b-1}w'}{1-[1-(1-x)^b]} \quad \text{or} \quad = \frac{b(1-w)^{b-1}w'}{1 - G_{LWWD}(x)} \qquad (11)$$

Where W is the expression in Equation (5)

To show that $\lim_{x\to\infty} h_{LWWD}(x) = 0 \text{ and } \lim_{x\to 0} h_{LWWD}(x) = 0$, we get the following

$$\lim_{x\to\infty} h_{LWWD}(x) = \lim_{x\to\infty} \frac{b(1-w)^{b-1}w'}{1-[1-(1-x)^b]}$$

where, $w' = \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^\beta}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^\beta}\right)\}$

=x

$$\lim_{\to\infty} \frac{b\left[1-\frac{\alpha+1}{\alpha}\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\}\right]^{b-1}\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)}{1-[1-(1-x)^b]}$$

For simplification, we take the limit of the following:

When $x \to \infty = 0 \text{ and } x \to 0 = 0$

$$= \lim_{x\to\infty} \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1 - e^{-\alpha x^\beta}\right) \quad = \lim_{x\to\infty} \frac{\alpha+1}{\alpha}\lambda\beta\infty^{\beta-1}e^{-\infty^\beta}\left(1 - e^{-\alpha\infty^\beta}\right)$$
$$= 0$$
$$= \lim_{x\to 0} \frac{\alpha+1}{\alpha}\beta 0^{\beta-1}e^{-0^\beta}\left(1 - e^{-\alpha 0^\beta}\right) = 0$$

$As\ x \to \infty = 0 \text{ and } x \to 0 = 0$, Equation (11) above tends to $\infty \text{ and } 0$ and equal to zero (Figure 4).

Asymptotic behaviours

Following the steps in "Hazard Rate Function" above, by taking the $\lim_{x\to\infty} g_{LWWD}(x) \text{ and } \lim_{x\to 0} g_{LWWD}(x)$ of the $LWWD\,(1, b, 1, \alpha, \beta)$ distribution which is investigated as follows. Now from Expression (4), we have

$$\lim_{x\to\infty} g_{LWWD}(x) = b\left[1 - \frac{\alpha+1}{\alpha}\{\left(1 - e^{-x^\beta}\right) - \frac{1}{\alpha+1}\left(1 - e^{-(1+\alpha)x^\beta}\right)\}\right]^{b-1} \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1 - e^{-\alpha x^\beta}\right)$$

taking the limit

$$= \lim_{x\to\infty} \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1 - e^{-\alpha x^\beta}\right)$$
$$= \lim_{x\to\infty} \frac{\alpha+1}{\alpha}\beta\infty^{\beta-1}e^{-\infty^\beta}\left(1 - e^{-\alpha\infty^\beta}\right) = 0$$
$$= \lim_{x\to 0} \frac{\alpha+1}{\alpha}\beta 0^{\beta-1}e^{-0^\beta}\left(1 - e^{-\alpha 0^\beta}\right) = 0$$

This shows that the asymptotic behaviours when $x \to \infty = 0 \text{ and } x \to 0 = 0$, and it is sure that the distribution has at least a mode.

MOMENTS AND MOMENT GENERATING FUNCTION

Shittu and Adepoju (2013) described that when a random variable following a generalized beta generated distribution that is, $x \sim GBG(f, 1, b, c)$ then $\mu'_r = E[F^{-1}\,U^{\frac{1}{c}}]^r$ where $U \sim B(1, b), c$ is a constant and $F^{-1}(x)$ is the inverse of CDF of the weighted Weibull distribution, since $LWWD\,(1, b, 1, \alpha, \beta)$ distribution is a special form when a=c=1.We then derive the moment generating function (mgf) of the proposed distribution $m(t) = E(e^{tx})$ and the general rth moment of a beta generated distribution is defined by

$$\mu'_r = b\int_0^1 [F^{-1}(x)]^r\,[1 - x]^{b-1}\,dx \qquad (12)$$

Also, using the taylor series expansion around the point $E(x_f) = \mu_f$ to obtain

$$\mu'_r = \sum_{u=0}^{r}\binom{r}{u}[F^{-1}(\mu)]^{r-u}[F^{-1(1)}(\mu_f)]^u \sum_{u=0}^{n}(-1)^i\binom{r}{i} \qquad (13)$$

Cordeiro et al. (2011) described an alternative series expansion in their paper for μ'_r in terms of $r(r, z) = E(Z^r\,F(Z)^v)$ where k follows the parent distribution then for z = 0, 1, …

$$\mu'_r = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\,r(r, i-1)$$

They further discussed another mgf of x for generated beta distribution

$$M(t) = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\,\rho(t, i-1) \qquad (14)$$

Where, $\rho(t,\ r) = \int_{-\infty}^{\infty} e^{tx}[F(x)]^m\,f(x)\,dx$

Therefore,

$$M_x^{\ (t)} = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int_{-\infty}^{\infty} e^{tx}[F(x)]^{(i+1)-1}\,f(x)\,dx \qquad (15)$$

Substituting both pdf and cdf (F(x) and f(x)) of the weighted Weibull distribution into Equation (15), we obtain

$$M_{LWWD\,(x)}^{(t)} = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\int e^{tx}\left[\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}\right]^{(i+1)-1} \times \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)dx \quad (16)$$

Equation (17) becomes the mgf of Lehmann Type II weighted Weibull distribution and by setting b=1 and i = 0, the same expression (17) is reduced to become the parent distribution. To obtain the rth moment of $LWWD\,(1,b,1,\alpha,\beta)$, the weighted Weibull distribution by Shahbaz et al. (2010) is given by

$$M_{(x)}^{(t)} = \int_0^\infty e^{tx}\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)dx$$

$$=\sum_{j=0}^{\infty}\frac{t^j}{j!\alpha^\beta}\{1+\alpha-(1+\alpha)^{-\frac{j}{\beta}}\}\Gamma(1+\frac{j}{\beta}) \quad (17)$$

Equation (16) can be re-written as

$$M_{LWWD\,(x)}^{(t)} = b\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(-1)^i\binom{b-1}{i}\frac{t^j}{j!\alpha^\beta}\{1+\alpha-(1+\alpha)^{-\frac{j}{\beta}}\}\Gamma(1+\frac{j}{\beta})\} \quad (18)$$

Moreover, the rth moment of the proposed distribution can also be written from the above equation (18) as given below:

$$\mu'_{LWWD\,(r)} = E(X^r) =$$

$$b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\frac{t^r}{r!\alpha^\beta}\{1+\alpha-(1+\alpha)^{-\frac{j}{\beta}}\}\Gamma(1+\frac{j}{\beta})\} \quad (19)$$

Again, setting b = 1 in Equation (19) leads to the rth moment of the weighted Weibull distribution by Shahbaz et al. (2010) and is given by

$$\mu'_r = E(X^r) = \frac{r}{\alpha}\{1+\alpha-(1+\alpha)^{-\frac{j}{\beta}}\}\Gamma(1+\frac{j}{\beta})\}$$

While the rth moment of the proposed distribution is

$$\mu'_{LWWD\,(r)} = E(X^r) =$$

$$b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\frac{r}{\alpha}\{1+\alpha-(1+\alpha)^{-\frac{j}{\beta}}\}\Gamma(1+\frac{j}{\beta})\} \quad (20)$$

From Equation (20), one can easily obtain the mean about the origin e.g when r = 1and the second moment when r = 2, etc. The first moment of $LWW\,(1,b,\lambda,\beta,\alpha)$ is obtain

$$\mu'_{LWWD(1)} = E(1,b,\alpha,\beta)^{(x)} = E(X^{'}) = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\frac{1}{\alpha}\{1+\alpha-(1+\alpha)^{-\frac{1}{\beta}}\}\Gamma(1+\frac{1}{\beta}) \quad (21)$$

The second moment can also be obtained as follows:

$$\mu'_{LWWD\,(2)} = V(1,b,\alpha,\beta)^{(x)} = K_1 - K_2 \quad (22)$$

where,

$$K_1 = E(1,b,\lambda,\beta,\alpha)^{(x^2)} = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\frac{2}{\alpha}\{1+\alpha-(1+\alpha)^{-\frac{2}{\beta}}\}\Gamma(1+\frac{2}{\beta})$$

$$K_2 = E(1,b,\lambda,\beta,\alpha)^{(x)} = b\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}[\frac{1}{\alpha}\{1+\alpha-(1+\alpha)^{-\frac{1}{\beta}}\}\Gamma(1+\frac{1}{\beta})]^2$$

We can also get the standard deviation (SD) as

$$SD_{LWWD}(1,b,\alpha,\beta)\quad(X) = \sqrt{K_1-K_2} \quad (23)$$

Others like coefficient of variation, skewness and kurtosis can easily be obtained.

ESTIMATION OF PARAMETER

The maximum likelihood estimate (MLEs) of the parameter of $LWWD\,(1,b,\alpha,\beta)$ distribution following Cordeiro et al (2011) given the log-likelihood function for $\tau = (1,b,c,\theta)$, where $\theta = (\alpha,\beta)$. Shittu and Adepoju (2013) in their study discussed that by setting τ to be a vector of parameter and given by

$$L(\tau) = nlogc - nlogb + \sum_{i=1}^{n}\log[f(x;\theta)] + (b-1)\sum_{i=1}^{n}\log[1-F(x;\theta)] \quad (24)$$

Recall that, a = c = 1 (24) becomes $\tau = (1,b,1,\theta)$

$$L(\tau) = -nlog\,(b) + \sum_{i=1}^{n}\log[f(x;\theta)] + (b-1)\sum_{i=1}^{n}\log\,[1-F(x;\theta)] \quad (25)$$

where, $f(x;\theta) = \frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)$ and

$$F(x;\theta) = \frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}$$

$$L_{LWWD}^{(\tau)} = -nlog\,(b) + \sum_{i=1}^{n}\log\left[\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)\right] + (b-1)\sum_{i=1}^{n}\log\left[1-\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}\right] \quad (26)$$

For determining the MLE of b,β,α, we took the partial derivative of Equation (26) with respect to (b,β,α) as follows:

$$\frac{L_{LWWD}^{(\tau)}}{\partial b} = -nlog(1,b) + (b-1)\sum_{x=1}^{n}\log\left[1-\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}\right] \quad (27)$$

$$\frac{L_{LWWD}^{(\tau)}}{\partial\alpha} = \sum_{x=1}^{n}\log\left[\frac{\frac{\partial}{\partial\alpha}\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)}{\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)}\right] + (b-1)\sum_{x=1}^{n}\log\left[\frac{\frac{\partial}{\partial\alpha}\left(1-\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}\right)}{1-\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}}\right] \quad (28)$$

$$\frac{L_{LWW}^{(\theta)}}{\partial\beta} = \sum_{x=1}^{n}\log\left[\frac{\frac{\partial}{\partial\beta}\left[\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)\right]}{\frac{\alpha+1}{\alpha}\beta x^{\beta-1}e^{-x^\beta}\left(1-e^{-\alpha x^\beta}\right)}\right] + (b-1)\sum_{x=1}^{n}\log\left[\frac{\frac{\partial}{\partial\beta}\left(1-\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}\right)}{1-\frac{\alpha+1}{\alpha}\left\{\left(1-e^{-x^\beta}\right)-\frac{1}{\alpha+1}\left(1-e^{-(1+\alpha)x^\beta}\right)\right\}}\right] \quad (29)$$

The above equations can be solved using Newton Ralphson method to obtain the $\hat{b},\hat{\alpha},\hat{\beta}$ the MLE of (b,α,β) respectively.

Taking second derivatives of Equations 27, 28 and 29 with respect to the parameters above we can derive the interval estimate and hypothesis tests on the model parameter and inverse of fisher's information matrix needed.

RESULTS AND DISCUSSION

Application to life time data set

Here we used a data set studied by Shahbaz et al (2010) on life components in years to compare the Lehmann Type II weighted Weibull and weighted Weibull distribution. The data contains Life (Grouped data say (0 – 1.0, 1.0 – 2.0,…,> 5.0) and frequency all together is 123619. R software is used to determine the maximum likelihood estimates and the log-likelihood for the Lehmann Type II weighted Weibull distribution are: $\hat{b} = 7.90434$, $\hat{\beta} = 8.44282$, $\hat{\alpha} = 8.70033$ and $lg_{LWW} = 454.4$ while the maximum likelihood estimates and the log-likelihood for the weighted Weibull distribution are: $\hat{\beta} = 8.649918$, $\hat{\alpha} = 8.041522$ and $lg_{WW} = 452.4337$, where lg_{LWW} and lg_{WW} denote log-likelihood of both Lehmann Type II weighted Weibull distribution and weighted Weibull distribution. The asymptotic covariance matrix of the maximum likelihood estimates for the Lehmann Type II weighted Weibull distribution, which is generated from the inverse of fisher's information matrix and is given by

$$\begin{pmatrix} 0.4754645 & 0.0000000 & 0.0000000 \\ 0.0000000 & 0.5330965 & 0.0000000 \\ 0.0000000 & 0.0000000 & 0.5674899 \end{pmatrix}$$

Following the result above, this shows that the new proposed distribution can take care even more skew data than the weighted Weibull distribution because of the additional shape parameter b, and with the hope that the model will be wider application to many areas of research e.g economics, finance, environmental, biomedical among others.

Conclusion

In a nut shell, we studied the statistical properties of the proposed distribution e.g moments, moment generating function, estimation of parameters with R software are presented in this paper. We also upgraded with an additional parameter to the existing two parameters in the weighted Weibull distribution and with the method of maximum likelihood the parameters of the model were estimated in order to give way for the derivation of fisher information matrix. The data used indicates that Lehmann Type II weighted Weibull distribution has a better representation of data and more flexible than weighted Weibull distribution.

Conflict of Interests

The author(s) have not declared any conflict of interests.

REFERENCES

Azzalini A (1985). A class of distributions which include the normal ones, Scand. J. Stat. 12:171-178.

Shahbaz S, Shahbaz MQ, Butt NZ (2010). A class of weighted Weibull distribution in Pakistan J. Statistics Operation Res. VI(1):53-59.

Cordeiro GM, Alexandra C, Ortega MM, Edwin Sarabia JM (2011). Generalized Beta Generated distributions. ICMA Centre. Discussion Papers in Finance DP 2011-05. pp. 1-29.

Gupta RD, Kundu D (1990). The role of weighted distribution in stochastic modeling. Communications in statistics, 19:3147-3162.

Eugene NC, Famoye F (2002). Beta-Normal distribution and its applications. Communications in Statistics – Theory and Methods 31:497-512.

Famoye F, Lee C, Olugbenga O (2005). The beta-Weibull distribution. J. Statistical Theory Applications, 4(2):121-138.

Jones MC (2004). Families of distributions arising from distributions of order statistics test 13:1-43.

Lee C, Famoye F, Olumolade O (2007). Beta-Weibull distribution: Some properties and applications to censored data. J. modern Appl. statistical methods, 6:173-186.

Nadarajah S, Kotz S (2005). The beta exponential distribution. Reliability Eng. Syst. Safety, 91:689-697.

Shittu OI, Adepoju AK (2013). On the Beta-Nakagami Distribution. Progress in Appl. Maths. 5(1):49-58.