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Some classifications on $\alpha-$Kenmotsu manifolds

Saadet DOĞAN* and Müge KARADAĞ

Department of Mathematics 44069, Faculty of Arts and Science, Inonu University, Malatya/Turkey.

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In this paper, we investigate some curvature problems of $\alpha-$Kenmotsu manifolds satisfying some certain conditions and we reach some classicifications. We consider φ-recurrent $\alpha-$Kenmotsu manifolds and we show that φ-recurrent $\alpha-$Kenmotsu manifolds are also η-Einstein manifolds. Next, we study φ -Ricci symmetric $\alpha-$Kenmotsu manifolds and we find this manifolds are Einstein manifolds too. In addition, we examine locally φ-symmetric α-Kenmotsu manifolds. Later we investigate this type manifold with quasi-conformally curvature tensor and concircular curvature tensor. In addition to these, we construct an example of $\alpha-$Kenmotsu manifolds and we see that this example is a locally $\varphi-$ symmetric $\alpha-$ Kenmotsu manifold.

Key words: $\alpha-$Kenmotsu manifold, φ-recurrent, φ-Ricci symmetric, locally φ-symmetric, concircular curvature tensor, quasi-conformally curvature tensor, η-Einstein manifolds, Einstein manifolds.

INTRODUCTION

Janssens and Vanhecke (1981) define $\alpha-$Kenmotsu manifolds. These are trans-sasakian of type (0, α) in J. A. Oubina's sense (Oubina, 1985). Öztürk et al. (2010) study about $\alpha-$Kenmotsu manifolds satisfying some curvature conditions. Dileo (2011) write paper named "A classification of certain almost $\alpha-$Kenmotsu manifolds". On the other hand De (2014) study globally $\varphi-$quasi-conformally symmetric $\alpha-$Kenmotsu manifold and give some examples 3-dimensional $\alpha-$Kenmotsu manifolds. We generally have interest on conditions about curvature tensor, because curvature tensors play important role in geometry and physics. For example; concircular transformation transforms every geodesic circle of a Riemannian manifold *M* into a geodesic circle. An interesting invariant of a concircular transformation is the concircular curvature tensor (Yano, 1940). In this paper, we study φ-recurrent $\alpha-$ Kenmotsu manifolds. In additon to this, we investigate $\varphi-$ricci symmetric $\alpha-$ Kenmotsu manifolds and show that $\varphi-$ ricci symmetric $\alpha-$ Kenmotsu manifolds are Einstein manifolds. In differential geometry and mathematical physics, an Einstein manifold is a Riemannian or pseudo-Riemannian manifold whose Ricci tensor is

*Corresponding author. E-mail: saadetdoganmat@gmail.com

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proportional to the metric. They are named after Albert Einstein because this condition is equivalent to saying that the metric is a solution of the vacuum Einstein field equations (Besse, 1987). Next, we deal with locally $\varphi-$symmetric α Kenmotsu manifolds and we prove some theorems about the scalar curvature of the manifolds. In addition to these, we consider quasi-conformally flat condition on this type manifolds. We find interesting results when we investigate concircularly flat condition on locally $\varphi-$symmetric α-Kenmotsu manifolds.

MATERIALS AND METHODS

Let *(M; g)* be an *(2n + 1)*-dimensional Riemannian manifold. We denote by ∇ the covariant differentiation with respect to the Riemannian metric *g*. The Ricci tensor of *M* are defined by

$$S(X,Y)=\sum_{i=1}^{2n+1}R(e_i,X,Y,e_i) \tag{1}$$

where $\{e_1,e_2,...,e_{2n+1}\}$ is a locally orthonormal frame and *X, Y* are vector fields on *M.* The Ricci operator *Q* is a tensor field of type (1,1) on *M* defined by

$$g(QX,Y)=S(X,Y) \tag{2}$$

for all vector fields on *TM.*

Let *M* be an *(2n+1)*- dimensional C^∞ manifold and $\chi(M)$ the Lie algebra of C^∞ vector fields on *M.* An almost contact structure on *M* is defined by (1,1) tensor field φ, a vector field ξ and a 1-form η on *M.* If (φ,ξ,η) satisfy the following condition then (φ,ξ,η) is said to be almost contact structure,

$$\eta(\xi)=1, \quad \varphi^2=-I+\eta\otimes\xi \tag{3}$$

$$\varphi\xi=0, \quad \eta\circ\varphi=0 \tag{4}$$

where *I* denotes the identity transformation of the tangent space T_pM at the point of *p*. Then *M* equipped with (φ,ξ,η) almost contact manifold. *M* with metric tensor *g* and with a triple (φ,ξ,η) such that

$$g(\varphi X,\varphi Y)=g(X,Y)-\eta(X)\eta(Y) \tag{5}$$

and

$$g(X,\xi)=\eta(X) \tag{6}$$

where $X,Y\in\chi(M)$, is an almost contact metric manifold.

Let $(M^{2n+1},\varphi,\xi,\eta,g)$ be an almost contact metric manifold and $\Phi(X,Y)=g(X,\varphi Y)$ is the fundamental 2- form of *M. M* is called almost α-Kenmotsu manifold, if the 1- form η and the 2-form Φ satisfy the following conditions:

$$d\eta=0, \qquad d\Phi=2\alpha\eta\wedge\Phi \tag{7}$$

α being a non-zero real constant (Janssens and Vanhecke, 1981).

We have known that an almost contact metric manifold $(M^{2n+1},\varphi,\xi,\eta,g)$ is said to be normal if the Nijenhius tensor

$$N_\varphi(X,Y)=[\varphi X,\varphi Y]-\varphi[\varphi X,Y]-\varphi[X,\varphi Y]+\varphi^2[X,Y]+2d\eta(X,Y)\xi$$

vanishes for any $X,Y\in\chi(M)$. Remarking that a normal almost α-Kenmotsu manifold is said to be α-Kenmotsu manifold $(\alpha\neq 0)$ (Janssens and Vanhecke, 1981). Moreover, if the manifold *M* satisfies the following relations

$$(\nabla_X\varphi)Y=\alpha\{-g(X,\varphi Y)\xi-\eta(Y)\varphi X\} \tag{8}$$

and

$$\nabla_X\xi=\alpha(X-\eta(X)\xi) \tag{9}$$

then $(M^{2n+1},\varphi,\xi,\eta,g)$ is called α-Kenmotsu manifold (Pitiş, 2007).

A Riemannian manifold *(M,g)* is called a $\varphi-$recurrent Riemannian manifold, if the curvature tensor *R* satisfies the following condition:

$$\varphi^2((\nabla_W R)(X,Y)Z)=A(W)R(X,Y)Z \tag{10}$$

where *A* is 1-form (De et al., 2009; Yıldız et al., 2009).

A Riemannian manifold *(M,g)* is called $\varphi-$Ricci symmetric, if its Ricci tensor *S* satisfies the following condition:

$$\varphi^2[(\nabla_X Q)Y]=0 \tag{11}$$

for all vector fields *X* and *Y* in *TM* (Shukla and Shukla, 2009). A Riemannian manifold M is said to be locally $\varphi-$symmetric, if

$$\varphi^2[(\nabla_W R)(X,Y)Z]=0 \tag{12}$$

for all vector fields *X,Y,Z,W* orthogonal to ξ. This notion was introduced by Takahashi (Binh et al., 2002), for a Sasakian manifold.

A Riemannian manifold *(M,g)* is called quasi-conformally flat if its quasi-conformal curvature tensor $\overline{C}$,

$$\overline{C}(X,Y)Z=aR(X,Y)Z+b\begin{Bmatrix}S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX\\-g(X,Z)QY\end{Bmatrix} -\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)[g(Y,Z)X-g(X,Z)Y] \tag{13}$$

satisfies $\overline{C}=0$, where *r* is the scalar curvature of *(M,g)*.

A Riemannian manifold *(M,g)* is called concircularly flat if its concircular curvature tensor *Z*,

$$Z(X,Y)W=R(X,Y)W-\frac{r}{2n(2n+1)}\{g(Y,W)X-g(X,W)Y\}$$

satisfies *Z=0* , where *r* is the scalar curvature of *(M,g)*.

On an $\alpha-$ Kenmotsu manifold *M*, the following relations are held (Janssens and Vanhecke, 1981):

$$S(X,\xi)=-2n\alpha^2\eta(X) \quad (14)$$

$$R(\xi,X)Y=\alpha^2[-g(X,Y)\xi+\eta(Y)X] \quad (15)$$

$$R(X,Y)\xi=\alpha^2[\eta(X)Y-\eta(Y)X] \quad (16)$$

$$S(\varphi X,\varphi Y)=S(X,Y)+\alpha^2 2n\eta(X)\eta(Y) \quad (17)$$

$$(\nabla_X\eta)Y=\alpha[g(X,Y)-\eta(X)\eta(Y)] \quad (18)$$

φ-RECURRENT $\alpha-$ KENMOTSU MANIFOLDS

Here, we find that a $\varphi-$recurrent $\alpha-$Kenmotsu manifold is an $\eta-$Einstein manifold.

Theorem

A $\varphi-$recurrent $\alpha-$Kenmotsu manifold is an $\eta-$Einstein manifold (Dogan, 2014).

Proof

Let (M,φ,ξ,η,g) be a $\varphi-$recurrent $\alpha-$Kenmotsu manifold. In this case; Riemannian curvature tensor of *M* satisfy the following equation for all *X,Y,Z* and *W* in *TM:*

$$\varphi^2[(\nabla_W R)(X,Y)Z]=A(W)R(X,Y)Z$$

From Equation (3), we get

$$(\nabla_W R)(X,Y)Z+\eta[(\nabla_W R)(X,Y)Z]\xi=A(W)R(X,Y)Z \quad (19)$$

for all X,Y,Z,W in *TM.* If we take the inner product of Equation (19) with $U\in\chi(M)$, we find

$$A(W)g(R(X,Y)Z,U)=-g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) \quad (20)$$

for all *X,Y,Z,W,U* in *TM.* Then the sum for $1\le i\le 2n+1$ of the relation (20) with $X=U=e_i$ fields

$$A(W)S(Y,Z)=-(\nabla_W S)(Y,Z)+\eta[(\nabla_W R)(\xi,Y)Z]. \quad (21)$$

If we write ξ instead of *Z*, we get

$$A(W)S(Y,\xi)=-(\nabla_W S)(Y,\xi)+\eta[(\nabla_W R)(\xi,Y)\xi]. \quad (22)$$

From Equations (9), (14) and (16), we get

$$-2n\alpha^2 A(W)\eta(Y)=(2n+1)\alpha^3 g(W,Y)+\alpha S(Y,W) -\alpha^3\eta(W)\eta(Y). \quad (23)$$

If we write φY and φW instead *Y* and *W*, respectively, we find

$$0=(2n+1)\alpha^3 g(\varphi W,\varphi Y)+\alpha S(\varphi Y,\varphi W), \quad (24)$$

From Equations (5) and (17), we have

$$S(Y,W)=-(2n+1)\alpha^2 g(Y,W)+\alpha^2\eta(Y)\eta(W) \quad (25)$$

for all *Y,W* in *TM.* Then , *M* is an $\eta-$ Einstein manifold.

$\varphi-$ RICCI SYMMETRIC $\alpha-$ KENMOTSU MANIFOLDS

Here, we find that a $\varphi-$Ricci symmetric $\alpha-$Kenmotsu manifold is an Einstein manifold.

Theorem

Let (M,φ,ξ,η,g) be a $\varphi-$Ricci symmetric $\alpha-$Kenmotsu manifold. Then *M* is an Einstein manifold.

Proof

Suppose that (M,φ,ξ,η,g) is a $\varphi-$Ricci symmetric $\alpha-$Kenmotsu manifold. In this case; Ricci operator of *M* satisfy the following condition:

$$\varphi^2[(\nabla_X Q)Y]=0$$

for all *X,Y* in *TM.* Then, we find

$$-(\nabla_X Q)Y+\eta[(\nabla_X Q)Y]\xi=0 \ . \quad (26)$$

From this last equation, we have

$$-\nabla_X QY + Q\nabla_X Y + \eta(\nabla_X QY)\xi - \eta(Q\nabla_X Y)\xi = 0 \quad (27)$$

for all vector fields *X,Y* in *TM.* If we take the inner product of Equation (27) with $\xi \in \chi(M)$, then we find

$$-g(\nabla_X QY, \xi) + g(Q\nabla_X Y, \xi) + \eta(\nabla_X QY) - \eta(Q\nabla_X Y)\xi = 0 \quad (28)$$

and we continue the process, we get

$$\begin{aligned} &S(\nabla_X Y, \xi) - \eta(Q\nabla_X Y) = 0 \\ &-2n\alpha^2\eta(\nabla_X Y) = \eta(Q\nabla_X Y) \\ &g(-2n\alpha^2\nabla_X Y, \xi) = g(Q\nabla_X Y, \xi) \end{aligned} \quad (29)$$

for all *X,Y* in *TM*. From Equations (2) and (29)

$$Q = -2n\alpha^2$$

and

$$QX = -2n\alpha^2 X$$

for all *X* in *TM.* In this case, we have

$$\begin{aligned} S(X,Y) &= g(QX,Y) \\ &= g(-2n\alpha^2 X, Y) \\ &= -2n\alpha^2 g(X,Y) \end{aligned}$$

for all *X,Y* in *TM.* Then the proof is complete.

LOCALLY $\varphi-$SYMMETRIC $\alpha-$KENMOTSU MANIFOLD

Here, we prove that locally $\varphi-$symmetric $\alpha-$Kenmotsu manifolds have constant scalar curvature. In addition to if this type manifolds are quasi-conformal flat, then the manifold is Einstein manifold. On the other hand, we find that if locally $\varphi-$symmetric $\alpha-$Kenmotsu manifolds are concircular flat, then these manifolds have constant curvature and their curvature is given $\frac{r}{2n(2n+1)}$ (*r* is scalar curvature of *M)*.

Lemma 1

Let (M,φ,ξ,η,g) be a locally $\varphi-$symmetric $\alpha-$Kenmotsu manifold. Then scalar curvature of *M* is constant .

Proof

Suppose that (M,φ,ξ,η,g) is a locally $\varphi-$symmetric $\alpha-$Kenmotsu manifold. That is; Riemannian curvature tensor of *M* satisfy the following equation

$$\varphi^2[(\nabla_W R)(X,Y)Z] = 0$$

where *X,Y,Z* and *W* are orthogonal to ξ. If we continue the process, we obtain

$$-(\nabla_W R)(X,Y)Z + \eta[(\nabla_W R)(X,Y)Z]\xi = 0 \quad (30)$$

for all *X,W,Z* orthogonal to ξ. Then the sum for $1 \le i \le 2n+1$ of the relation (30), we get

$$(\nabla_W S)(X,Z) + \eta((\nabla_W R)(X,\xi)Z) = 0 .$$

In this case;

$$(\nabla_W S)(X,Z) + \eta\begin{bmatrix} \nabla_W R(X,\xi)Z - R(\nabla_W X,\xi)Z \\ -R(X,\nabla_W \xi)Z - R(X,\xi)\nabla_W Z \end{bmatrix} = 0$$

for all *X,W,Z* orthogonal to ξ. So, using Equations (9) and (15), we obtain,

$$\begin{aligned} &(\nabla_W S)(X,Z) + \alpha^2 g(X,Z)\eta(\nabla_W \xi) - \alpha^2\eta(Z)\eta(\nabla_W X) \\ &+ \alpha^2 g(\nabla_W X, Z) - \alpha^2\eta(Z)\eta(\nabla_W X) - \alpha\eta(R(X,W)Z) \\ &+ \alpha\eta(R(X,W)Z) - \alpha\eta(W)\eta(R(X,\xi)Z) - \alpha\eta(W)\eta(R(X,\xi)Z) \\ &- \eta(R(X,\xi)\nabla_W Z) = 0. \end{aligned}$$

If we continue the process, we get

$$\begin{aligned} (\nabla_W S)(X,Z) = &-\alpha^2 g(\nabla_W X, Z) + \alpha^2 g(\nabla_W Z, X) + \alpha^3\eta(W)g(X,Z) \\ &-\alpha^3\eta(W)\eta(X)\eta(Z) - \alpha^2\eta(X)\eta(\nabla_W Z). \end{aligned} \quad (31)$$

M is locally $\varphi-$symmetric, so

$$\eta(X) = \eta(Y) = \eta(W) = \eta(Z) = 0 .$$

Then we find

$$(\nabla_W S)(X,Z) = -\alpha^2 g(\nabla_W X, Z) + \alpha^2 g(\nabla_W Z, X). \quad (32)$$

If we write $X = Z = e_i$ and we take the sum for $1 \le i \le 2n+1$ of the relation (32), we obtain

$$dr(W) = 0$$

for all vector fields W in TM. Then, the proof is complete.

Theorem

Let (M,φ,ξ,η,g) be a locally $\varphi-$symmetric $\alpha-$Kenmotsu manifold. If M is quasi-conformally flat, then M is Einstein manifold.

Proof

Suppose that (M,φ,ξ,η,g) is a locally $\varphi-$symmetric $\alpha-$Kenmotsu manifold. Then $\overline{C}(X,Y)Z$ quasi-conformal curvature tensor of M vanishes for any X,Y,Z in TM. That is,

$$aR(X,Y)Z+b[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY]-\frac{r}{2n(2n+1)}\left(\frac{a}{2n}+2b\right)[g(Y,Z)X-g(X,Z)Y]=0 \qquad (33)$$

for all X,Y,Z in TM. If we write ξ instead of X and Z and later we take the inner product of Equation (33) with $W\in\chi(M)$, then we get

$$S(Y,W)=\frac{1}{b}\left\{a\alpha^2+2nb\alpha^2+\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\right\}g(Y,W)+\frac{1}{b}\left\{-a\alpha^2-4nb\alpha^2-\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\right\}\eta(Y)\eta(W). \qquad (34)$$

If we use Lemma 1 and we consider locally $\varphi-$symmetric then r is constant and $\eta(Y)=\eta(W)=0$ since Y and W orthogonal to ξ. So we have

$$S(Y,W)=\lambda g(Y,W)$$

($\lambda=\frac{1}{b}\left\{a\alpha^2+2nb\alpha^2+\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\right\}$). In this case, M is Einstein Manifold.

Theorem

Let M be a locally $\varphi-$symmetric $\alpha-$Kenmotsu manifold. If M is concircularly flat then M has got constant curvature and its curvature is $\frac{r}{2n(2n+1)}$.

Proof

Suppose that M is a locally $\varphi-$symmetric $\alpha-$Kenmotsu manifold. If M is concircularly flat then we obtain

$$R(X,Y)W=\frac{r}{2n(2n+1)}\{g(Y,W)X-g(X,W)Y\}. \qquad (35)$$

If we consider Lemma 1 and the Equation (35), then we complete the proof.

Example

$M=\{(x,y,z)\in R^3,(x,y,z)\neq(0,0,0)\}$, where (x,y,z) are the standard coordinates in R^3. The vector fields

$$e_1=\alpha z\frac{\partial}{\partial x}\qquad e_2=\alpha z\frac{\partial}{\partial y}\qquad e_3=-\alpha z\frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be Riemannian metric defined by

$$g=\frac{dx^2+dy^2+dz^2}{\alpha^2z^2}.$$

Then we find

$$g(e_1,e_3)=g(e_1,e_2)=g(e_2,e_3)=0$$
$$g(e_1,e_1)=g(e_2,e_2)=g(e_3,e_3)=1.$$

Let η be the 1-form defined by $\eta(X)=g(X,e_3)$ for any $X\in\chi(M)$. Let φ be a $(1,1)$ tensor field defined by $\varphi(e_1)=-e_2$, $\varphi(e_2)=e_1$, $\varphi(e_3)=0$. If we define $\xi=e_3$, $\eta(X)=g(X,e_3)$ for all vector fields X in TM and use the linearity of φ and g, then we find

$$\eta(\xi)=1,\quad \varphi^2X=-X+\eta(X)\xi,\quad g(\varphi X,\varphi Y)=g(X,Y)-\eta(X)\eta(Y)$$

for all vector fields X, Y in TM. In this case, (M,φ,ξ,η,g) is an almost contact metric manifold. Suppose that ∇ is Levi-Civita connection with respect to the metric g. For all $f\in C(R^3,R)$, we get

$$\begin{aligned}[e_1,e_2]f&=e_1(e_2(f))-e_2(e_1(f))\\&=e_1(\alpha zf_y)-e_2(\alpha zf_x)\\&=0\end{aligned}$$

$$
\begin{aligned}
[e_1,e_3]f &= e_1(e_3(f))-e_3(e_1(f)) \\
&= e_1(-\alpha z f_z)-e_3(\alpha z f_x) \\
&= \alpha z(-\alpha z f_{zx})+\alpha z(\alpha z f_{xz}+\alpha f_x) \\
&= \alpha e_1(f)
\end{aligned}
$$

and

$[e_2,e_3]f=\alpha e_2(f)$. In this case, from Kozsul's Formula, we find

$$
\begin{array}{lll}
\nabla_{e_1}e_1=-\alpha e_3 & \nabla_{e_2}e_1=0 & \nabla_{e_3}e_1=0 \\
\nabla_{e_1}e_2=0 & \nabla_{e_2}e_2=-\alpha e_3 & \nabla_{e_3}e_2=0 \\
\nabla_{e_1}e_3=\alpha e_1 & \nabla_{e_2}e_3=\alpha e_2 & \nabla_{e_3}e_3=0.
\end{array}
$$

Let $X=ae_1+be_2+c\xi$ and $Y=\overline{a}e_1+\overline{b}e_2+\overline{c}\xi$ be vector fields in *TM* (Where *a,b,c,* $\overline{a},\overline{b},\overline{c}\in R$). Then we get $\varphi Y=\overline{b}e_1-\overline{a}e_2$. In this case;

$$
\begin{aligned}
(\nabla_X\varphi)Y &= \nabla_X\varphi Y-\varphi\nabla_X Y \\
&= \nabla_{ae_1+be_2+c\xi}(\overline{b}e_1-\overline{a}e_2)-\varphi(\nabla_{ae_1+be_2+c\xi}(\overline{a}e_1+\overline{b}e_2+\overline{c}\xi)) \\
&= \alpha\{-(a\overline{b}-b\overline{a})\xi-\overline{c}(-ae_2+be_1)\} \\
&= \alpha\{-g(X,\varphi Y)\xi-\eta(Y)\varphi X\}
\end{aligned}
$$

for all vector fields *X,Y* in *TM.* Hence (M,φ,ξ,η,g) is an $\alpha-$Kenmotsu manifold. With the help of above results we can find the following:

$$
\begin{aligned}
g(R(e_1,X)Y,e_1) &= -\alpha^2(b\overline{b}+c\overline{c}) \\
g(R(e_2,X)Y,e_2) &= -\alpha^2(a\overline{a}+c\overline{c}) \\
g(R(e_3,X)Y,e_3) &= -\alpha^2(a\overline{a}+b\overline{b})
\end{aligned}
$$

and

$$
\begin{aligned}
S(X,Y) &= \sum_{i=1}^{2n+1}R(e_i,X,Y,e_i) \\
S(X,Y) &= -\alpha^2 g(X,Y)-\alpha^2\eta(X)\eta(Y).
\end{aligned}
$$

Hence, *M* is an $\eta-$Einstein manifold. Now, we take *X,Y,Z* and *W* orthogonal to ξ. Then we can write

$$
\begin{aligned}
X &= ae_1+be_2 \\
Y &= \overline{a}e_1+\overline{b}e_2 \\
Z &= \tilde{a}e_1+\tilde{b}e_2 \\
W &= \hat{a}e_1+\hat{b}e_2.
\end{aligned}
$$

In this case, if we compute $(\nabla_W R)(X,Y)Z$, we find

$$
\begin{aligned}
(\nabla_W R)(X,Y)Z &= \nabla_W R(X,Y)Z-R(\nabla_W X,Y)Z-R(X,\nabla_W Y)Z-R(X,Y)\nabla_W Z \\
&= 0.
\end{aligned}
$$

Then,

$$\varphi^2(\nabla_W R)(X,Y)Z=0$$

for all vector fields *X,Y,Z* and *W* orthogonal to ξ. In this case, this manifold is a locally $\varphi-$ symmetric $\alpha-$Kenmotsu manifold. In Lemma 1, we show that scalar curvature of a locally $\varphi-$ symmetric $\alpha-$Kenmotsu manifold is constant. Actually, if we compute scalar curvature for all vector fields *X,Y* orthogonal to ξ, we see that

$$
\begin{aligned}
S(X,Y) &= -\alpha^2 g(X,Y) \\
r &= \sum_{i=1}^{3}S(e_i,e_i)=-3\alpha^2.
\end{aligned}
$$

Conflict of Interest

The authors have not declared any conflict of interest.

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