

*Full Length Research Paper*

# Fractal behavior of the sales restaurant in a great tourism hotel

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**The tourist industry can be considered as a complex system because it is made up of many elements (tourist companies) that interact with each other in a non-linear way. In turn, each company can also be considered by itself as a complex system because its behavior is non-linear with time, so that some aspects of tourism companies can be characterized by a non-linear mathematical tool called fractal analysis. In this paper fractal analysis is used to characterize the dynamics of the sales of a restaurant in a Great Tourism hotel (GTs). The purpose of applying this fractal analysis is to characterize the patterns which rule out the sales dynamic in a restaurant hotel, in order to improve the planning and the customer services in the restaurant area.**

**Key words:** Hotel industry, complex system, fractal geometry, invariance scale.

## INTRODUCTION

Tourism is recognized as one of the largest world level industries (Torres, 2003). Several countries trust in their tourism industry dynamics to generate revenues (Rodenburg, 1980) employments (Clever et al., 2007) and infrastructure development (Gee and Fayos, 1997). The economic importance of tourist companies resides in the impulse, creation and development of other industries (construction, banking, trade) through the multiplying effect that characterizes the tourism industry (Figuera, 2001). Tourism contributes to the 8.2% of Gross Domestic Product and generates one out of eleven direct and indirect jobs in Mexico. The hosting services present an average occupancy of 51.36%. One of the most important tourism destinations is Mexico City (SECTUR, 2008) because of its businesses, which are defined by professional activities.

The study of hospitality services has been approached by using different techniques to cope with distinctive problems such as improving forecast, modeling demand, cost efficiency and analysis of supply chain using qualitative and quantitative methods. According to Song and Li (2008), it is possible to classify these application methods in: a) time series models, b) econometrics models, and c) other models.

The time series methods, explain a variable with regard to its own past and a random disturbance term. These methods range from the very simple extrapolation, to more complex time-series techniques like the Autoregressive Integrated Moving Average (ARIMA), typically applied to forecast a demand of a destiny (Kim and Moosa, 2001; Kulendran and Witt, 2003; Chu, 2008a). Coshall (2009) used ARIMA to develop an estimated demand data added social events that disestablish tourism, thus the author describes this like a superior combination. Meanwhile the Autoregressive Moving Average with Exogenous Variable (ARMAX) (Akal, 2004) is a method to outperform the simple econometric cause-effect technique in terms of accuracy, and the ARAR algorithm (Chu, 2008b) to forecast multiple demands.

There are some examples of techniques adaptable to the tourism relations to generate findings, in any case. The successful development of these methods is linked with the quality input data. However, for applying these techniques, it is necessary to fit data to the models relation, which means it is not necessary to know the exact dynamic behavior of the data. However, some models consider the data dynamic like a process, for example sales as linear behavior, in which case the historical data are related by a normal distribution. Nevertheless, the dynamics of a studied real process could be non-linear; in such a case, the analysis would

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be incorrect.

The econometric models deal with casual relations between the study phenomenon and its influence variables. The main econometric methods are: the Autoregressive Distribution like lag model (ADLM), the Time Varying Parameter (TVP) (Song and Witt, 2000), and the Vector Autoregressive model (VAR) (Song and Witt, 2006). The most outstanding study was applied by Wong et al. (2006), which was worked with a Bayesian focus Bayesian Vector Auto-regression (BVAR). The results showed better adjustment to tourism data and the authors made it to improve the forecast model. However, it does not take into account possible long-run or co-integration relationships between the variables. On the other hand, in the Error Correction Model (ECM), Dritsakis (2004) gave an evidence supporting co-integration, which suggests the existence of an equilibrium in a long-run relationship among important variables. Therefore, the author demonstrated no single forecasting method can be consistently ranked as best across different situations (origin-destination country, forecasting horizons, accuracy measures etc.). Thus, the emergence of combined methods has been more successful. Nevertheless, these methods currently are limited to the number of variables and possible combinations; meanwhile the same condition gives them the possibility to discriminate the data.

There are models composed by diverse methods, which (have been using a combination of algorithms to analysis tourism. These models include chaos theory (Zahra and Ryan, 2007; Russell et al., 1999), probabilistic models (Rodriguez and Estéves, 2007), dynamic models (Muñoz, 2006; Muñoz and Mattin, 2007) and generated algorithms (Hernandez and Cáceres, 2007; Chen and Wang, 2007). In a particular study, Chen and Soo (2007) constructed a cost function to determinate the relation cost in hotel services. Nevertheless, the authors recognize that there are other factors affecting the outcome of a hotel's performance such as management style, market orientation, hotel image, employee skills and other operating characteristics. These elements can be considered in the neuronal net techniques (Palmer, 2006). These methods are flexible instruments for researchers interested in forecasting the behaviors which occur in the field of tourism (Bloom, 2005; Pattie and Snyder, 1996; Tsaur et al., 2002). These are commonly used in forecasting, since they do not depend on the statistical conditions, such as the type of relation between variables or the type of data distribution. However, the values of the parameters obtained by these methods do not have a practical interpretation, so it is not possible analyze the role played by each input variable in the forecasting carried out.

According to McKercher (1999) tourism is described by many variables; it behaves as a non-linear system and displays the dynamics of a complex system (Runyan and Wu, 1979) Complex systems science offers enormous

possibilities for new research in tourist companies (Farrel and Ward, 2004). It is a tool compatible with their development and allows a holistic understanding of the non-linear nature of the interactions between various factors affecting the tourism industry. The fractal analysis provides a quantitative tool to characterize complex systems, such as the factors of supply and demand that affect tourist companies. This method is more robust, so it is possible to know the behavior of the phenomena in different scales, which means that the change over time is associated with a power law that characterizes the system structure.

The work developed in a hotel restaurant is very complex not only because it integrates many sequential tasks, but also because the order of the services depends on the consumer behavior. The uniformity of services is difficult to establish because every production unit can be different from other. Changes come from the consumer requirements and provoke hard possibilities to predict the sales level, requirements materials and inventory needs. Nevertheless, the managers try to obtain the appropriated information to design the planning program according to the sales capacity, the inventory costs and the revenue of expected volume. The vital information is constituted by how many units of material are needed to develop the service processes and how much time must be considered right for planning and scheduling these services.

The use of appropriate instruments for the optimal planning of production capacity is important, since they allow reducing cost, increasing profits and supporting the services expectations. By the other hand, the application of traditional tools is limited to cope with the complex behavior of these sort of systems. Therefore, the present work aims to carry out a fractal analysis that characterizes the dynamics of the fluctuations sales in a restaurant system located into Great Tourism hotels. This type of analysis is appropriate because the restaurant manifests a high degree of complexity due to their service schedules, the specification materials, the food offered and the productive considerations associated with their services.

The purpose of applying the fractal analysis is to characterize the emergence of patterns due to the non-linear interaction of these and other factors, in a global manner that can be used to improve the planning services and scheduling of the restaurant.

The research begins by gathering the sales data of the restaurant system, in order to determine the global dynamics of the system, the statistical distribution of the data, and the self-affine methods to be used. These are contrasted with the time series of the data to obtain the Hurst exponent, which is finally used to establish the fractal characterization of the system under study.

The organization of this paper is as follows: First is an introduction of tourism and the literature review of analyzed methods applied to this industry. Next is a

presentation of basic aspects related to complex systems and fractal geometry. This is followed by an application of the fractal analysis to the case study, presentation of the results, discussions, and then conclusions.

### Complexity science

Complex systems contain many constituents interacting non-linearly in such a way that their behavior is often a balance between non-stationary and stationary components. Accordingly, a complex system is capable of emergent behavior that usually is responsible for power-laws that are universal and independent of the microscopic details of the phenomenon, e.g., atoms in a solid, cells in a living organism, or traders in a financial market (Auyang, 2001). A complex system (physical, biological, chemical, or financial, just to mention a few) can be defined as a system with a large number of degrees of freedom.

A number of different tools for studying complex, non-linear dynamic systems have been developed in the last few decades: phase transition, self-organization, growth models, cellular automata, hierarchical models, co-adaptation, strange attractors, emergence, computability, recursion, and fractal geometry, among others.

The scientific study of complex systems within a fractal analysis framework, in general, consists of three major approaches: theoretical, experimental, and computational. The goal is to have, on the one hand, the simplest and most parsimonious description of the phenomena under study and, on the other hand, the most faithful representation of the observed characteristics.

Fractal mathematics has proven to be a useful tool in quantifying the structure of a wide range of idealized and naturally-occurring objects, from pure mathematics, through physics and chemistry, to biology, medicine, sociology, and economics (Klonowski, 2000).

The term fractal (from Latin *fractus* –irregular, fragmented) applies to objects in space or fluctuations in time, which possess a form of self-similarity and they cannot be described within a single absolute scale of measurement. Fractals are recurrently irregular in space or time, with themes repeated at different levels or scales like the layers of an onion. Fragments of a fractal object or sequence are exact or statistical copies of the whole created by shifting and stretching. *Fractal geometry* has evoked a fundamentally new view of how both non-living and living systems result from the coalescence of spontaneous, self-similar fluctuations over many orders of time and how systems are organized into complex, recursively-nested patterns over multiple levels of space (Klonowski, 2000).

In a strict sense, most time series are one dimensional, since the values of the considered observable are measured in homogeneous time intervals. Hence, unless there are missing values, the fractal dimension of the support is  $D(0) = 1$ . However, there are rare cases where most of the values of a time series are very small or even zero, causing a dimension  $D(0) < 1$  of the support. Even if the fractal dimension of support, the information dimension  $D(1)$  and the correlation dimension  $D(2)$  can be studied.  $D(2)$  is in fact explicitly related to all exponents studied in monofractal time series analysis. However, usually a slightly different approach is employed based on the notion of self-affinity. Here, one takes into account that the time axis and the axis of the measured value  $x(t)$  are not equivalent. Hence, a rescaling of time  $t$  by a factor  $a$  may require rescaling of the series value  $x(t)$  by a different factor  $a^H$  in order to obtain a statistically similar (that is, self-similar) picture.

$$x(t) \rightarrow a^H x(at) \quad (1)$$

In this case the scaling relation, holds for an arbitrary factor  $a$ ,

describing the data as self-affine (Feder, 1988). The Hurst exponent  $H$  (after the hydraulic engineer H. E. Hurst (1951) characterizes the type of self-affinity).

The trace of random walk (Brownian motion) is characterized by  $H = 0.5$ , implying that the position axis must be rescaled by a factor of 2 if the time axis is rescaled by a factor of 4.

Time series is one dimensional array of numbers  $x(i), i = 1 \dots, N$  representing values of an observable  $x$  usually measured equidistant in time. The series analysis allows verification of macroscopic models of complex evolution on the basis of data analysis (Olemskoi, 2002). The expected relationship between the value of a series at time  $t$  and its value at time  $\tau + t$  is a measure of the correlation present in the series.

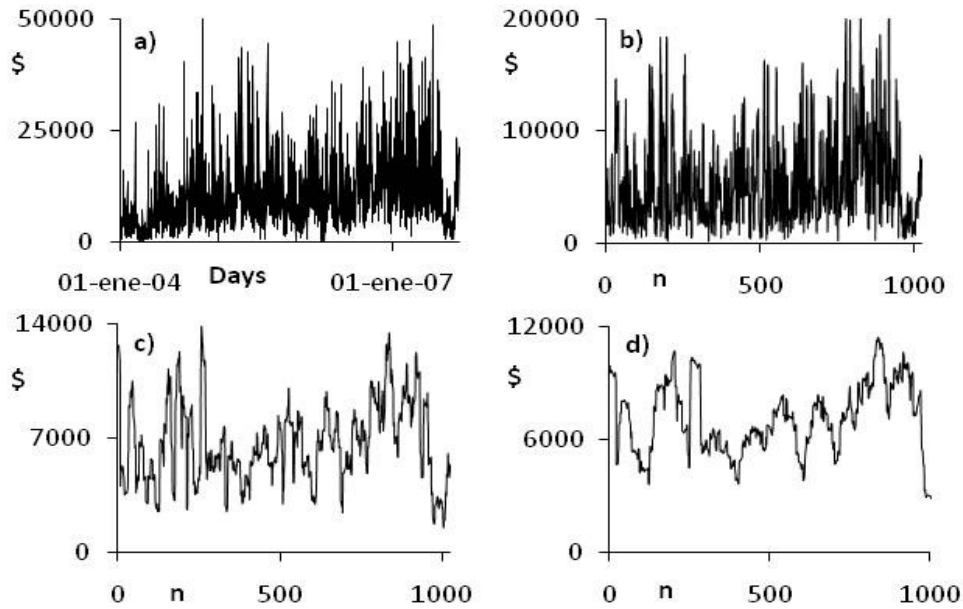
The Hurst exponent  $H$  can be used to determine whether a time series possesses statistical auto-affine invariance (such as the fractional Brownian motion); it can also tell us what kind of correlation is presented in the time series. If the Hurst exponent is less than 0.5, the time series displays “anti-persistence” or negative correlations: positive increments are more likely to be reversed and, therefore, the next period’s performance is likely to be below average. If the Hurst exponent is greater than 0.5, the process displays “persistence” or positive correlations: positive increments are more likely to remain above average. If the Hurst exponent is equal to 0.5, the process is completely random (no correlation): the data do not display any memory, and positive increments are thus equally likely to be followed by above-average or below-average performance (Morales et al., 2010).

### Self-affine traced methods

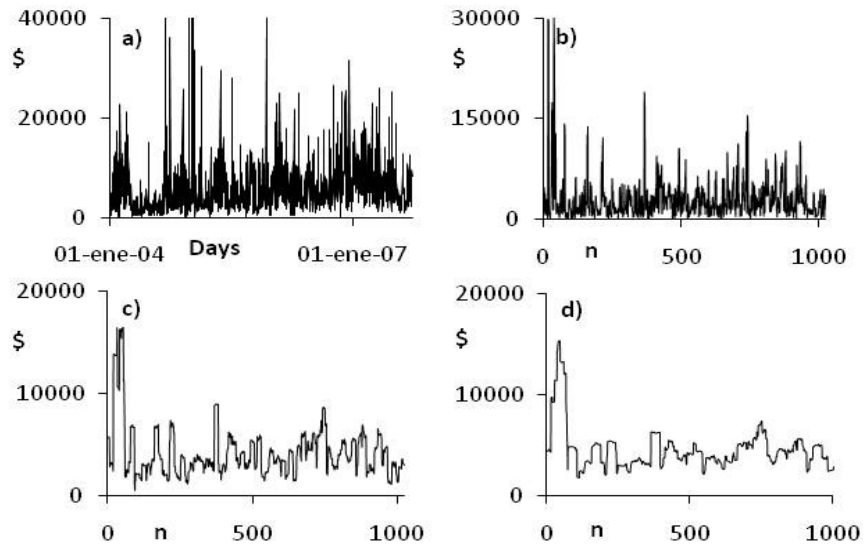
In order to observe fractal scaling behavior in time series, several quantitative tools have been developed. In this paper were applied four self-affine traced methods to determine the Hurst ( $H$ ) exponent: 1) the rescaled-range analysis (R/S), 2) the roughness-length, (R/L), 3) the variogram (VG), and 4) the wavelets (WV) method. The rescaled-range analysis is one of the oldest and best-known methods for determining  $H$ . This method was proposed by Mandelbrot and Wallis (1969) and it is based on previous hydrological studies of Hurst (1951). The *rescaled-range*  $R/\sigma$  is defined as the ratio of the maximal range of the integrated signal ( $R$ ) normalized to its standard deviation. For time series characterized by long-range correlations, the expected value of rescaled-range scales as  $R/(\sigma \propto \tau^H)$ . If the time record possesses only short-range correlations, then the log-log plot of  $R/\sigma$  is also a straight line, with slope 0.5. The roughness-length and variogram methods are based on the scaling behaviour of the standard deviation  $\sigma_n \propto (\tau)^{Hn}$  and the semivariance  $Var \propto \tau^{2H}$ . Finally, wavelets offer an alternative method for analysis of complex time series (Struzik, 2001). The *wavelets* method is based on the fact that wavelet transforms of self-affine traces have self-affine properties. This method is appropriate for analysis of non-stationary series, that is, those in which the variance does not remain constant with increasing length of the data set. The aim of the wavelet transform is to express an input signal into a series of coefficients of specified “energy”. The discrete numbers associated with each coefficient contain all the information needed to completely describe the series provided one knows which analyzing wavelet was used for the decomposition. Wavelet transform applies scaling functions that have the properties of being localized in both time and frequency. A scaling coefficient  $w \propto a^{Hn+1/2}$ , where  $a$  denotes a scale parameter, characterizes and measures the width of a wavelet (Balankin et al., 2004).

### APPLICATION OF SYSTEM SCIENCE TO THE CASE STUDY

Time series of complex systems exhibit fluctuations on a wide



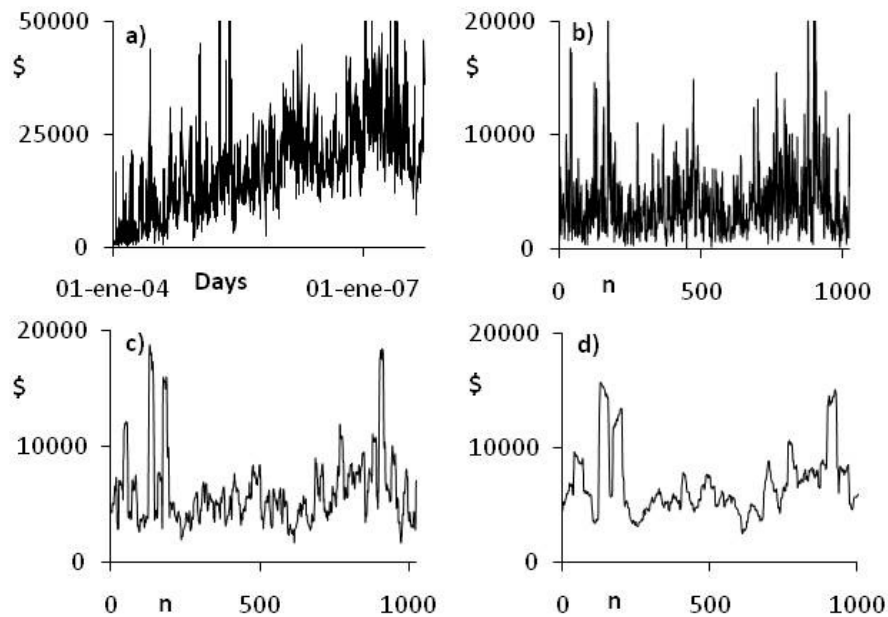
**Figure 1.** (a) Time records of breakfast sales in the 2007 constant Mexican pesos; and (b)-(d) realized sales volatility for the period of 1,024 business days for different horizons: (b)  $n = 3$ , (c)  $n = 14$  and (d)  $n = 32$  business days. All time series correspond to the period from 7 December 2004 to 26 September 2007.



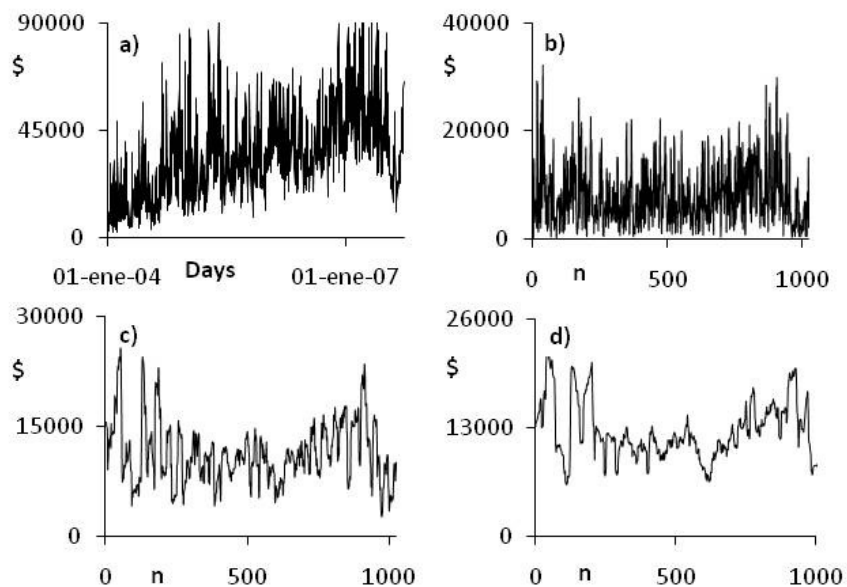
**Figure 2.** (a) Time records of lunch sales in the 2007 constant Mexican pesos; and (b)-(d) realized sales volatility for the period of 1,024 business days for different horizons: (b)  $n = 3$ , (c)  $n = 14$  and (d)  $n = 32$  business days. All time series correspond to the period from 7 December 2004 to 26 September 2007.

range of time scales and/or broad distribution of the values. In both equilibrium and non equilibrium situations, the natural fluctuations are often found to follow a scaling relation over several orders of magnitudes. Such scaling behavior allows for a characterization of the data and the generating complex system by fractal (or multifractal) scaling exponents, which can serve as characteristic finger spring of the system in comparison with other systems and

with models. So that to quantify the scaling dynamics of the Great Tourism category hotels in this work, we studied the daily records of the sales  $S(t)$  and the sale  $V_n(S)$  fluctuations (or volatility) from the daily sales of meals (breakfast, lunch, and dinner) provided by a Great Tourism Category hotel located in the Mexico City downtown. Specifically, the breakfast (Figure 1(a)), lunch (Figure 2(a)), dinner (Figure 3(a)), and total (Figure 4(a)) sales were analyzed in



**Figure 3.** (a) Time records of dinner sales in the 2007 constant Mexican pesos; and (b)-(d) realized sales volatility for the period of 1,024 business days for different horizons: (b)  $n = 3$ , (c)  $n = 14$  and (d)  $n = 32$  business days. All time series correspond to the period from 7 December 2004 to 26 September 2007.

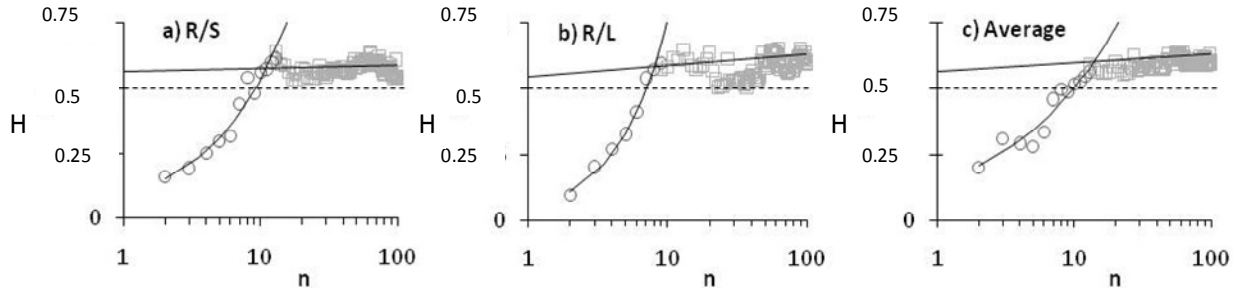


**Figure 4.** (a) Time records of total sales in the 2007 constant Mexican pesos; and (b)-(d) realized sales volatility for the period of 1,024 business days for different horizons: (b)  $n = 3$ , (c)  $n = 14$  and (d)  $n = 32$  business days. All time series correspond to the period from 7 December 2004 to 26 September 2007.

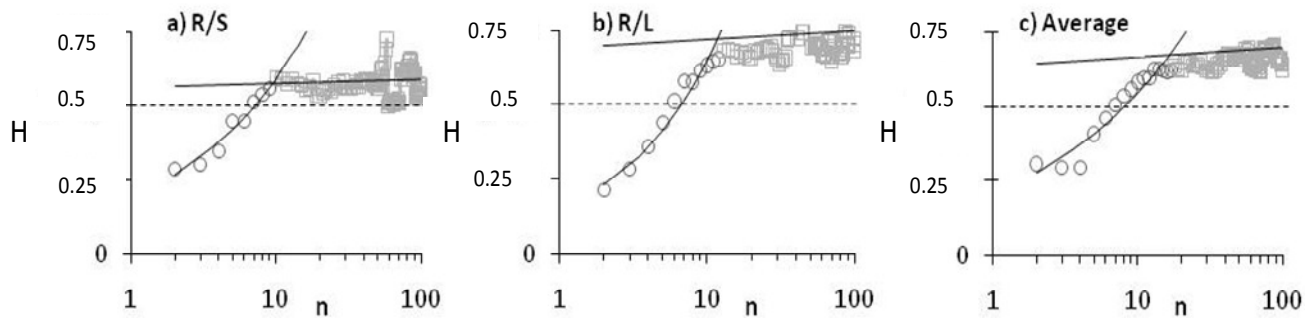
constant 2007 Mexican pesos over the period from 1<sup>st</sup> January 2004 to 27 September 2007, representing 1,365 observations for each time series. Then there were constructed 311 time series of realized volatility  $V_n(S)$  for each time series (because in finance is

typically characterized the price volatility in terms of the standard deviation of prices at particular time scale),

$$V_n(\tau) = \sqrt{\langle S^2(t) \rangle_n - \langle S(t) \rangle_n^2} \tag{2}$$



**Figure 5.** Horizon dependence of the Hurst exponent (values of  $H_n$ ) in the semilog coordinates (time scale in business days, circles and squares: experimental data of the breakfast sales volatility, solid line: data fitting by a power-law). The values of  $H_n$  are obtained from: (a) the rescaled-range method, (b) the roughness length method, and (c) the averaged from four methods.



**Figure 6.** Horizon dependence of the Hurst exponent (values of  $H_n$ ) in the semilog coordinates (time scale in business days, circles and squares: experimental data of the lunch sales volatility, solid line: data fitting by a power-law). The values of  $H_n$  are obtained from: (a) the rescaled-range method, (b) the roughness length method, and (c) the averaged from four methods.

Length  $T = 1,024$  business days (the last 1,024 records of each time series) for different time horizons  $n = 2, 3, \dots, 311$  (from two business days to three business years), where  $t$  is the business time,  $\langle \dots \rangle_n$  denotes the business time average within a window of size  $n$ . In this study, all records of volatility (Figures 2(b)-2(d), 3(b)-3(d), 4(b)-4(d), and 5(b)-5(d)) correspond to the period from 7 December 2004 to 26 September 2007. One can see that the sale volatility changes day to day in such a way that time series of volatilities  $V_n(S)$  realized at different time intervals  $n$  look similar.

**Fractal analysis**

To quantify the intensity of long range correlations, the local Hurst exponents of each time record  $V_n(t)$  were determined by four self-affinity traced methods adopted, from the Benoit 1.3 Software: the rescaled-range analysis, the roughness length, the variogram, and the wavelet methods.

On the other hand, the statistical distributions of the daily sales of meals were analyzed with the help of @Risk Software, which ranks the fitted distributions using three test statistics; Chi-Square, Anderson-Darling, and Kolmogorov-Smirnov statistics (Conover, 1980).

**RESULTS AND DISCUSSION**

We find that the realized volatilities (Figures 1(b)-1(d),

2(b)-2(d), 3(b)-3(d), and 4(b)-4(d)) possess self-affine invariance within wide ranges of business time scale characterized by well defined Hurst exponent  $H_n$  for each horizon  $n$  (Figures 5(a)-5(c), 6(a)-6(c), 7(a)-7(c), and 8(a)-8(b), and Table 1):

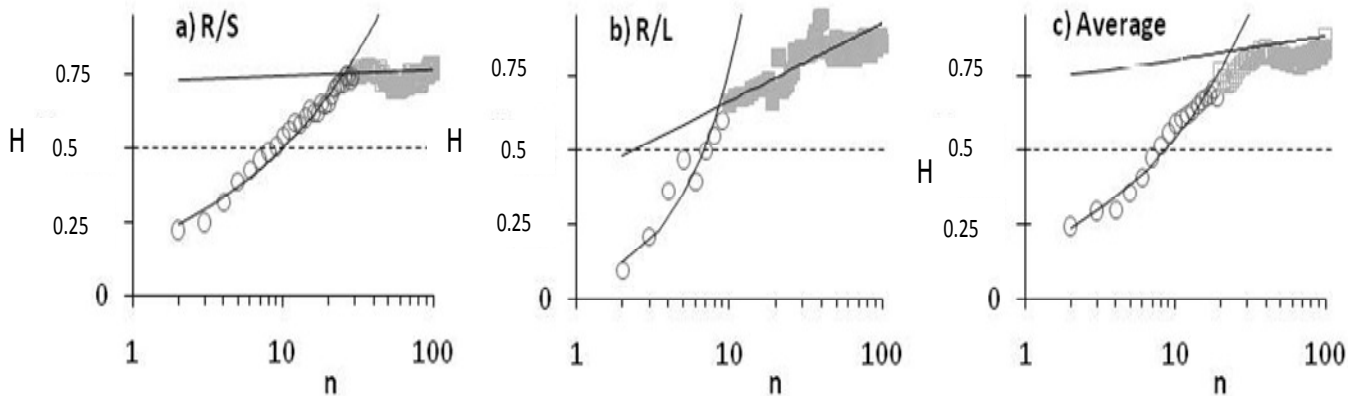
$$H_n = 0.58 \mp 0.02 \quad \text{for breakfast sales volatility} \quad (3)$$

$$H_n = 0.62 \mp 0.06 \quad \text{for lunch sales volatility} \quad (4)$$

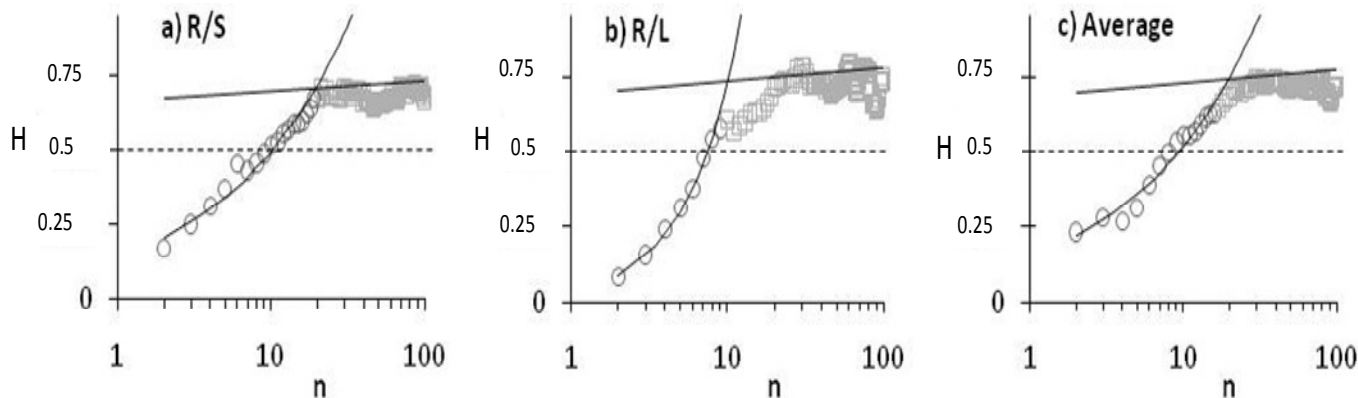
$$H_n = 0.80 \mp 0.05 \quad \text{for dinner sales volatility} \quad (5)$$

$$H_n = 0.73 \mp 0.03 \quad \text{for total sales volatility} \quad (6)$$

Our findings mean that the long horizon realized volatilities ( $n > 13$  for breakfast,  $n > 12$  for lunch,  $n > 20$  for dinner, and  $n > 16$  for total) are persistent, that is, volatility increments are positively correlated in business days (a large value is usually followed by a large value and a small value is followed by small value), whereas the short-horizon volatilities ( $n < 13$  for breakfast,  $n < 12$  for lunch,  $n < 20$  for dinner, and  $n < 16$  for total) display negative correlations in business days (a large value is usually followed by a small value and a small value is



**Figure 7.** Horizon dependence of the Hurst exponent (values of  $H_n$ ) in the semilog coordinates (time scale in business days, circles and squares: experimental data of the dinner sales volatility, solid line: data fitting by a power-law). The values of  $H_n$  are obtained from: (a) the rescaled-range method, (b) the roughness length method, and (c) the averaged from four methods.



**Figure 8.** Horizon dependence of the Hurst exponent (values of  $H_n$ ) in the semilog coordinates (time scale in business days, circles and squares: experimental data of the total sales volatility, solid line: data fitting by a power-law). The values of  $H_n$  are obtained from: (a) the rescaled-range method, (b) the roughness length method, and (c) the averaged from four methods.

**Table 1.** The best fitted values of Hurst exponents for the sale volatilities.

Method	Breakfast	Lunch	Dinner	Total
R/S	0.58	0.56	0.75	0.70
R-L	0.57	0.68	0.84	0.75
Average	0.60	0.66	0.80	0.73

usually followed by a large value).

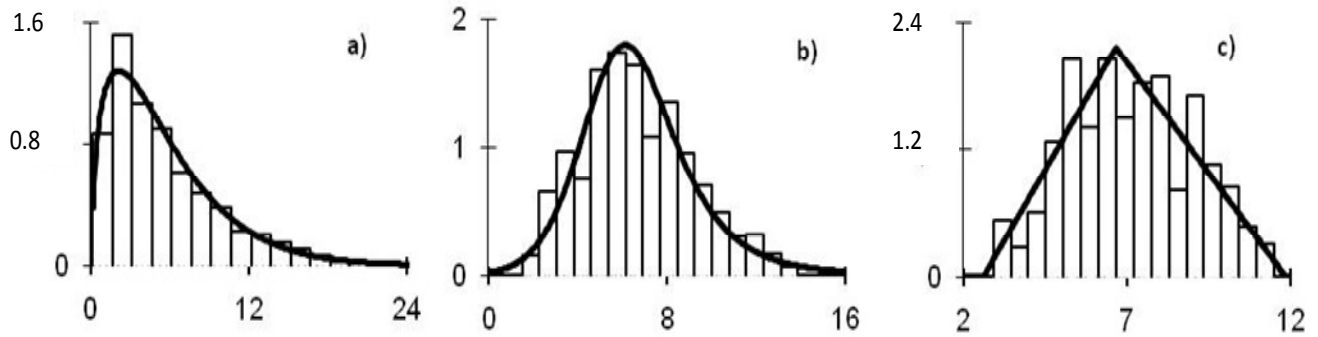
The crossover (change point of the scaling exponent value) from anti-persistent to persistent behavior indicates the existence of intrinsic horizon scale of sales volatility,  $n \approx 14$ . To get a deeper insight into the sales volatility dynamics, there was also performed the statistical analysis of the time series volatilities.

We find for short time horizons,  $n \leq 13$ , that the

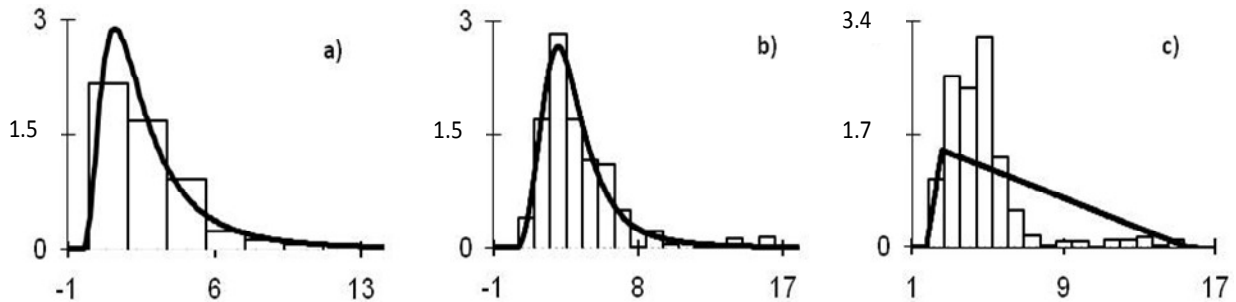
conditional probability of realized volatilities is best fitted by the light-tailed Beta General (for breakfast and total sales) and Pearson (for lunch and dinner sales) distributions (Figures 10(a), 11(a), 12(a), and 13(a)); that is, there are correlations that decay sufficiently fast that they can be described by a characteristic correlation time scale.

At the same time, for larger horizons,  $14 \leq n < 32$ , the conditional probability of realized volatilities is the heavy-tailed Log-logistic distribution for breakfast (Figure 9(b)), lunch (Figure 10(b)), dinner (Figure 11(b)), and total sales (Figure 12(b)), that is, there are correlations that decay sufficiently slow that a characteristic correlation time scale cannot be defined.

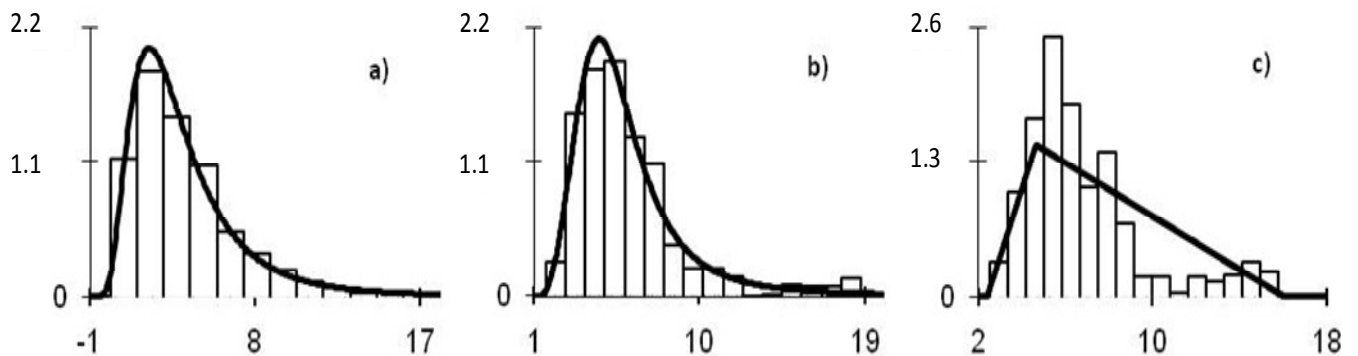
Finally, we find that for time horizons,  $n \geq 32$ , the conditional probability of realized volatilities is the Triangular distribution for breakfast (Figure 9(c)), lunch (Figure 10(c)), dinner (Figure 11(c)), and total sales



**Figure 9.** Conditional probability distributions of breakfast sales volatilities for horizons: (a)  $n = 3$  (bins: experimental data, solid lines fitting by the Beta General distribution); (b)  $n = 14$  (bins: experimental data, solid lines fitting by the Log-logistic distribution); and (c)  $n = 32$  (bins: experimental data, solid lines fitting by the Triangular distribution).



**Figure 10.** Conditional probability distributions of lunch sales volatilities for horizons: (a)  $n = 3$  (bins: experimental data, solid lines fitting by the Pearson distribution); (b)  $n = 14$  (bins: experimental data, solid lines fitting by the Log-logistic distribution); and (c)  $n = 32$  (bins: experimental data, solid lines fitting by the Triangular distribution).



**Figure 11.** Conditional probability distributions of dinner sales volatilities for horizons: (a)  $n = 3$  (bins: experimental data, solid lines fitting by the Pearson distribution); (b)  $n = 14$  (bins: experimental data, solid lines fitting by the Log-logistic distribution); and (c)  $n = 32$  (bins: experimental data, solid lines fitting by the Triangular distribution).

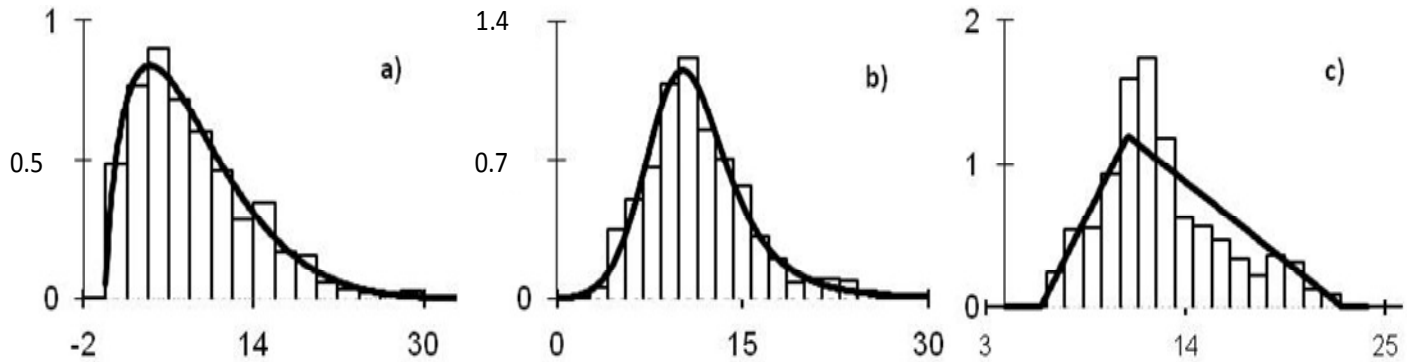
(Figure 12(c)).

**Conclusions**

In this work, we show that the time series of sales volatility of a restaurant (a tourism company seen as complex

system) can be characterized within a fractal geometry framework (a non-linear method for analysis). In this way we find a transition of the breakfast, lunch, dinner, and total sales volatilities from anti-persistent (negative correlations) to persistent (positive correlations) behavior in the horizon scale. This transition or crossover is accompanied by the changes in the type of volatility





**Figure 12.** Conditional probability distributions of total sales volatilities for horizons: (a)  $n = 3$  (bins: experimental data, solid lines fitting by the Beta General distribution); (b)  $n = 14$  (bins: experimental data, solid lines fitting by the Log-logistic distribution); and (c)  $n = 32$  (bins: experimental data, solid lines fitting by the triangular distribution).

distributions, which are light-tailed for short horizons and there are heavy-tailed for long-horizons (explained by power laws). The crossover from anti-persistent to persistent behavior has been observed in a wide variety of systems displaying generalized scaling dynamics with continuously varying exponent the existence of a “universal” mechanism which gives rise to crossover from antipersistent to persistent behavior in system of different nature could provide a new insight to the physics of complex system.

As we have seen, the complex system under study is characterized by multifractal (several Hurst exponent) dynamics. This finding will help in obtaining predictions on the future behavior of the system and on this reaction to external perturbation changes in the boundary conditions. One can test and interactively improve models of the system until they reproduce the observed scaling behavior. One example for such an approach is climate modeling, where the models were shown to need inputs from volcanoes and solar radiation in order to reproduce the long-term correlated (fractal) scaling behavior (Vyushin et al., 2004) previously found in observational temperature data (Koscielny-Bunde et al., 1988).

Our findings have potential implications in the construction of a branch of tools to control in-puts and forecasting the meals demand in order to improve the customer-service as well as reduce operative costs in restaurants that belong to Great Tourism Category hotels, because we have demonstrated that the sales volatilities analyzed get a critical point where positive correlations are displayed ( $n \geq 14$ ) and fitted by the heavy-tailed Log-logistic distribution ( $14 \leq n < 32$ ) expressed by power-laws.

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