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Full Length Research Paper

Nuclear symmetry energy and incompressibility of hot asymmetric nuclear matter in the Thomas-Fermi model

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The density dependence of nuclear symmetry energy and incompressibility of asymmetric nuclear matter have been investigated in the framework of the Thomas-Fermi approximation using the effective nucleon - nucleon interaction of Myers and Swiatecki in the new approach. The symmetry energy of about $28.25\ MeV$ is obtained at the saturation density. The symmetry energy and incompressibility of asymmetric nuclear matter are all in excellent agreement with the other many-body calculations.

Key words: Nuclear matter – symmetry energy-incompressibility-nucleon–nucleon interactions.

INTRODUCTION

The study of the nuclear matter symmetry energy $E_{\rm sym}(\rho_0)$ MeV, which essentially characterizes the isospin dependent part of the equation of state (EOS) of asymmetric nuclear matter, is currently an exciting topic of research in nuclear physics. Knowledge about the symmetry energy is essential in understanding many aspects of nuclear physics and astrophysics (Danielewicz et al., 2002; Lattimer and Prakash, 2004, 2007; Steiner et al., 2005; Baran et al., 2005; Li et al., 2008) as well as some interesting issues regarding possible new physics beyond the standard model (Sil et al., 2005; Krastev and Li, 2007; Wen et al., 2009). In recent years, significant progress has been made in determining the density dependence of $E_{\rm sym}(\rho_0)$ MeV (Baran et al., 2005; Li et al., 2008). The EOS of asymmetric nuclear matter and the density dependence of the nuclear symmetry energy are largely unknown except at ρ_0 where the nuclear symmetry energy has been determined to be around $30\ MeV$ from the empirical liquid-drop mass formula (Myers and Swiatecki, 1966; Pomorski and Dudek, 2003). Theoretical studies (Dieperink et al., 2003; Liu et al., 2002; Kaiser et al., 2002) based on microscopic many-body calculations and phenomenological approaches predict various different forms of the density dependence of the symmetry energy. Determining the exact form of the density dependence of the symmetry energy is important for studying the structure of neutron-rich nuclei (Brown, 2000; Horowitz and Piekarewicz, 2001; Furnstahl, 2002) and for studying the astrophysical origin, such as the structure of neutron stars and the dynamics of supernova collapse (Lattimer and Prakash, 2001, 2000; Hix et al., 2003). The saturation and bulk properties of symmetric nuclear matter have been studied successfully (Vidana and Polls, 2008; Van Dalen et al., 2004) but asymmetrical nuclear matter has not been studied very extensively.

The disadvantage of the microscopic treatment is the numerical complexity of the method. Because of this it is very tempting to use simpler models which are easier to deal with and make comparisons with respect to the properties of finite nuclei, the parameters of the mass formula and the properties of neutron stars. This is why we selected the new Thomas-Fermi approach of Myers and Swiatecki (Myers and Swiatecki, 1994, 1998).

Thomas-Fermi model

We shall not go in to details of this model, since they are described in greater detail in numerous investigations prior to this one (Myers and Swiatecki, 1994, 1998; Mohammadpour and Shamshirband, 2011). The calculation of the Thomas-Fermi model at finite temperature for asymmetric nuclear matter follows exactly the calculation at zero temperature except that we use the Fermi-Dirac distribution function,

$$f_\tau(p_1,T,\rho) = \left(1+\exp\left(\frac{1}{T}\left(\varepsilon_\tau(p_1,T,\rho)-\mu_\tau(T,\rho)\right)\right)\right)^{-1} \quad (1)$$

Instead of the step function $\Theta(p_F-|p|)$ at zero temperature. In this equation, $\varepsilon_\tau(p_1,T,\rho)$ is the single-particle energies and

$\mu_\tau(T,\rho)$ is the chemical potential at a given temperature and density for a non-interacting system. Because of momentum dependence of the single particle potential and use of Fermi-Dirac distribution function in excited state of Fermi liquid, we introduce an effective mass in single particle energy and simply use:

$$\varepsilon_\tau(p_1, T, \rho) = \frac{p_1^2}{2m^*(\rho, T)} + V_\tau(p_1) \qquad (2)$$

Where $m^*(\rho, T)$ is the effective mass and it is taken as a variational parameter, chemical potential will be obtained by choosing density as follows:

$$\rho = \frac{2}{h^3} \sum_\tau \int_{-\infty}^{+\infty} d^3p_1\, f_\tau(p_1) \qquad (3)$$

One then obtains for the energy per nucleon for the symmetric nuclear matter:

$$u = \frac{E}{A} = \frac{2}{\rho\, h^3} \sum_\tau \int_{-\infty}^{+\infty} d^3p_1 \left(\frac{p_1^2}{2m_\tau} + \frac{1}{2} V_\tau(p_1)\right) f_\tau(p_1) \qquad (4)$$

with the temperature dependent single-particle potential:

$$V_\tau(p_1) = -\frac{2}{h^3}\frac{2}{\rho_0} \times [T_{F,\tau} \int_\tau d^3p_2 \left(\alpha_l - \beta_l \left(\frac{p_{12}}{p_F}\right)^2 + \gamma_l \frac{p_F}{|p_{12}|} - \sigma_l \left(\frac{2\bar{\rho}}{\rho_0}\right)^{\frac{2}{3}}\right) f_\tau(p_2) + T_F \int_{-\tau} d^3p_2 \left(\alpha_u - \beta_u \left(\frac{p_{12}}{p_F}\right)^2 + \gamma_u \frac{p_F}{|p_{12}|} - \sigma_u \left(\frac{2\bar{\rho}}{\rho_0}\right)^{\frac{2}{3}}\right) f_{-\tau}(p_2)] \qquad (5)$$

At finite temperature, we must calculate the Helmholtz free energy per particle, that is:

$$F = u - Ts \qquad (6)$$

And the entropy per S particle is written as (Fetter and Walecka, 1971):

$$s = -\frac{2}{\rho\, h^3} \sum_\tau \int_{-\infty}^{+\infty} d^3p_1 \left[f_\tau(p_1) \ln f_\tau(p_1) + \left(1 - f_\tau(p_1)\right) \ln(1 - f_\tau(p_1))\right] \qquad (7)$$

Now, by minimizing the above free energy with respect to the effective mass we can find the appropriate effective mass and consequently free energy. Now $F(\rho, T, m^*)$ can be differentiated to yield various quantities of interest such as pressure $p(\rho, T, m^*)$, etc. The incompressibility is an essential quantity for nuclear matter and conventionally it is defined at zero temperature and at the saturation density ρ_0 for asymmetric nuclear matter where we have $p(\rho, \delta) = 0$ and thus it can be expressed as:

$$K_{sat}(\delta) = 9\, \rho_0^2 \left. \frac{\partial^2 (\frac{E(\rho,\delta)}{A})}{d\rho^2} \right|_{\rho=\rho_0} \qquad (8)$$

For asymmetric nuclear matter the isobaric incompressibility coefficient $K_{sat}(\delta)$ can be expressed up to the 2nd –order in δ as (Chen et al., 2009), $K_{sat}(\delta) = K_0 + K_{sat,2}\,\delta^2 + \square\,(\delta^4)$, the magnitude of 4th –order $K_{sat}(\delta)$ parameter is generally small. The 2nd –order $K_{sat}(\delta)$ parameter thus essentially characterizes the isospin dependence of the incompressibility of asymmetric nuclear matter at saturation density.

RESULTS AND DISCUSSION

One of important quantity that has a critical effect on the EOS of asymmetrical nuclear matter is the symmetry energy and its dependence on the density and temperature as well as asymmetry parameter δ which is defined as $\delta = \frac{\rho_n - \rho_p}{\rho}$ (δ=1, for pure neutron matter and, δ=0 for symmetric nuclear matter). In general, the symmetry energy is defined as:

$$E_{sym(\rho,T)} = \frac{1}{2} \left. \frac{\partial^2 E(\rho,\delta,T)}{\partial \delta^2} \right|_{\delta=0} \qquad (9)$$

But it is also possible to make an approximation (according to familiar semi-empirical mass formula) such as:

$$E_{app}(\rho, T) = E(\rho, \delta = 0, T) + E_{sym}(\rho, T)\delta^2 \qquad (10)$$

The density and temperature dependence of symmetry energy can be estimated from

$$E_{sym}(\rho, T) \cong E(\rho, \delta = 1, T) - E(\rho, \delta = 0, T) \qquad (11)$$

This implies that $E_{sym}(\rho, T)$ is the energy required to convert all the protons in the symmetric matter to neutrons. The validity of linear dependence of symmetry energy on δ^2 at three temperatures 5, 10 and 15 MeV and three different densities $\boldsymbol{\rho}$=0.1, 0.15 and 0.2 $fm^{-3}$are presented in Figure 1.

The symmetry energy in this model (28.25 MeV) at nuclear matter saturation density (0.161 fm−3), Figure 2, is comparable with those of Lee, Kuo, Li and Brown (LKLB) (Lee et al.,1998) (31.27 MeV) with Bonn one boson exchange potential (OBEP) in DBHF approach, Xu, Chen, Li and Ma (XCLM) (Xu et al., 2008) (31.6 MeV) with self consistent thermal model, Ducoin, Margueron and Chomaz (DMC) (Ducoin et al., 2008) (31.98 MeV), (26.83 MeV) and (29.26 MeV) with three Skyrme forces SLy230a, SGII and RATP, respectively, Santos and Providencia (SP) (2009) (33.7 MeV) with quark-meson coupling (QMC) model, Alonso and Sammarruca (AS) (2003) with OBEP in BHF (28.3 MeV) and DBHF (28.1 MeV) approach and Zuo, Lejenue, Lombardo and Mathiot (ZLLM) (Zuo et al., 2002) (30.5 MeV) with AV18 plus three-body force by using BHF calculation at nuclear matter saturation density, Vidana and Polls (VP) (2008) with AV_{18} in BHF $(33.2\ MeV)$, Zaryoni and Moshfegh (ZM) (2010) with AV_{14} in $LOCV$ (39.08 MeV), Basu, Chowdhury and Samanta (BCS) (Basu et al., 2009) with $DDM3Y$ in$RMF\,(30.71\,MeV)$. Although the nuclear symmetry energy at normal nuclear matter density for

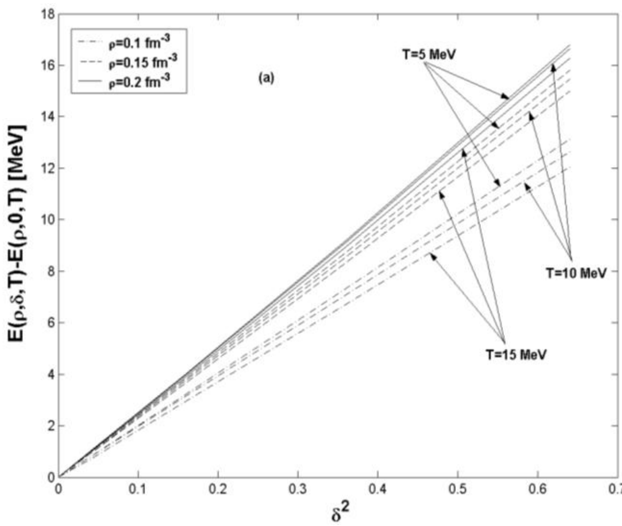

Figure 1. Thomas-Fermi calculation of quadratic dependence of symmetry energy in various density using the effective nucleon - nucleon interaction of Myers and Swiatecki.

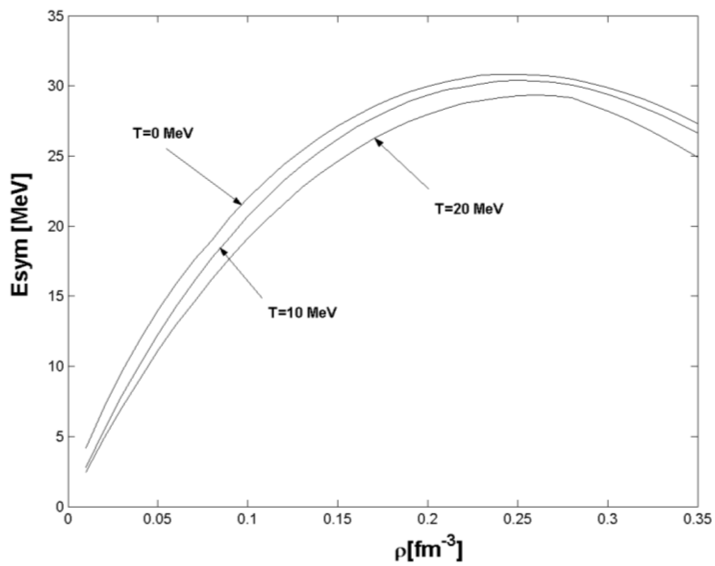

Figure 2. Density dependence of symmetry energy using an isospin and momentum dependent interaction of Myers and Swiatecki.

cold asymmetric nuclear matter is known to be around 30 MeV from the empirical liquid-drop mass formula (Steiner et al., 2005; Pomorski and Dudek, 2003; Danielewicz, 2003).

The existence of critical isospin asymmetry parameter δ_c at a given temperature indicates that for $\delta \succ \delta_c$ the system will not change completely into the liquid phase. Conversely, this suggests that at a fixed δ there exists a critical temperature T_c beyond which the system can only be in the gas phase at all pressures. In Figure 5 we present T_c as a function of δ in the Thomas-Fermi model. For symmetric nuclear matter δ=0 the critical temperature for liquid- gas phase transition in this model $T_c = 17.4\,MeV$. As asymmetry δ increases up to $\delta \geqslant 0.6$, T_c decreases rapidly. The Thomas- Fermi result for T_c agrees more or less with the results of Huber et al. (1998) and Moshfegh and Ghazanfari (2011). The critical temperatures for different asymmetries are given in Figure 3. The critical densities are shown in Figure 4.

The incompressibility of asymmetric nuclear matter at its saturation density is a basic quantity to characterize its EOS. In principle, this information can be extracted

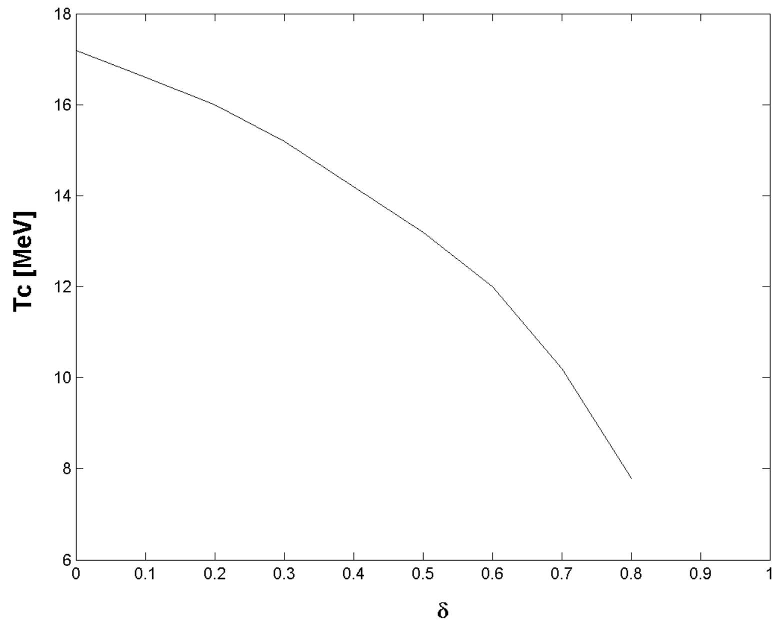

Figure 3. Critical temperature as functions of the asymmetry parameter δ.

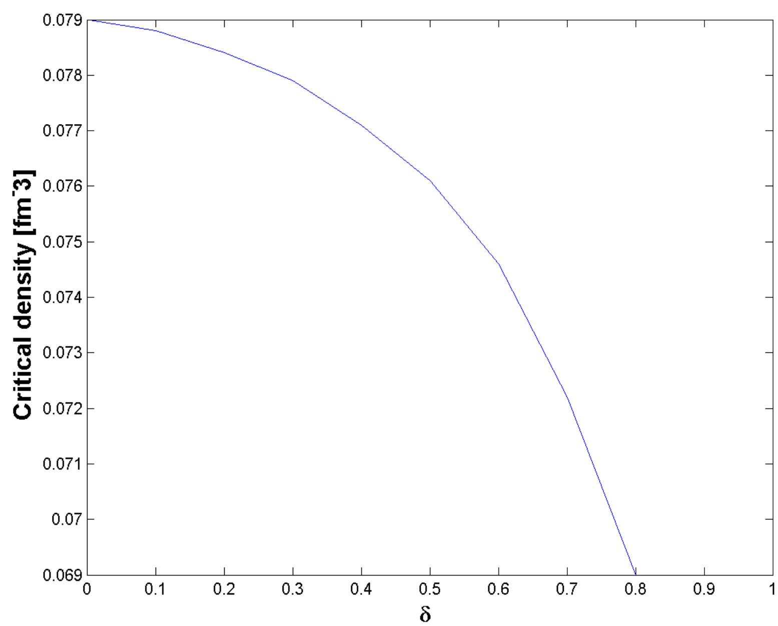

Figure 4. Critical density as a function of the asymmetry δ.

experimentally by measuring the GMR in neutron-rich nuclei (Blaizot, 1980, Blaizot and Grammaticos, 1981). The incompressibility at saturation density $K_{sat}(\delta)$ as a function of δ^2 in the Thomas-Fermi model is shown in Figure 5. It is seen that $K_{sat}(\delta)$ generally decreases with increasing isospin asymmetry. The isospin dependence of binding energy per nucleon E_{sat} of asymmetric nuclear matter at saturation density is shown in Figure 6 as function of δ^2 in this model. It is seen that E_{sat} increases with increasing isospin asymmetry and it's seen that E_{sat} display a linear dependence on δ^2. These results are good agreement with the very recent study (Chen et al., 2009; Piekarewicz and Centelles, 2009; Moshfegh and Ghazanfari, 2011).

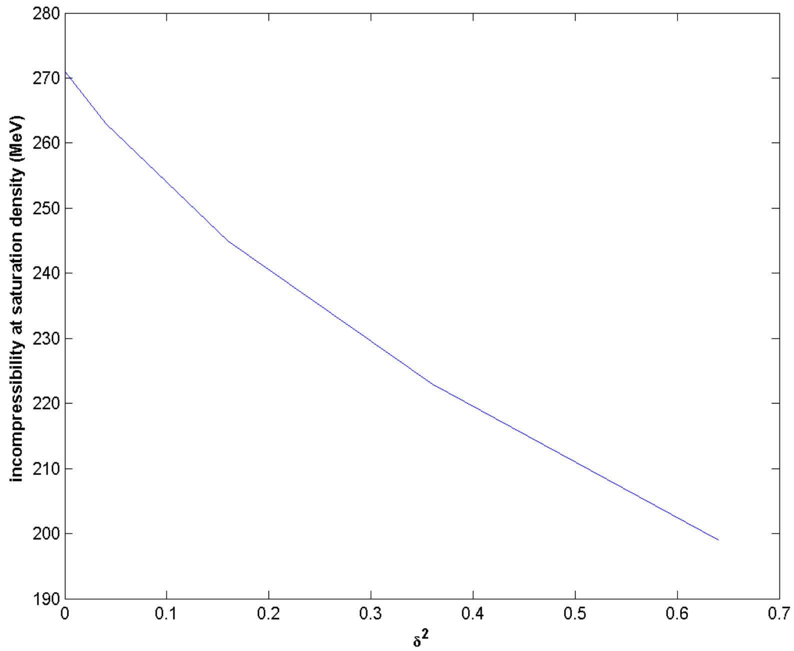

Figure 5. The incompressibility at saturation density as function of δ^2.

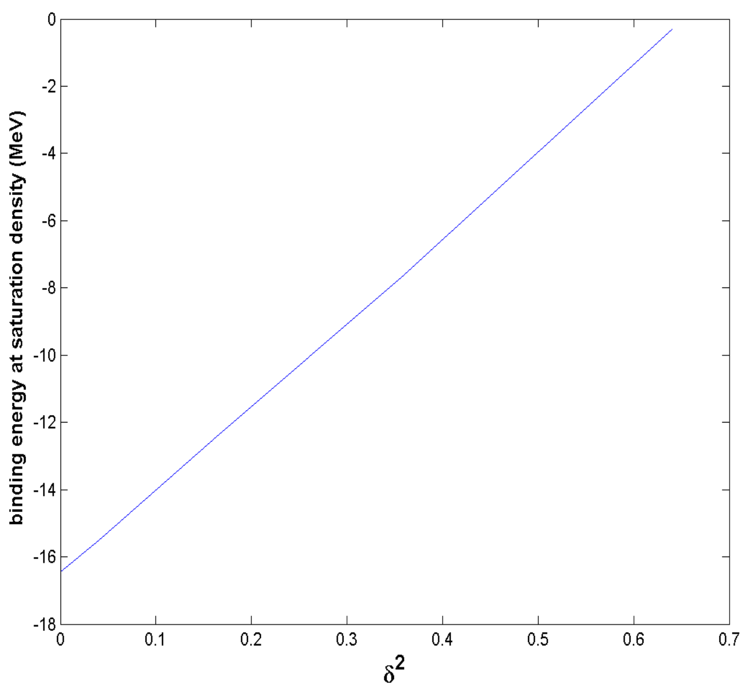

Figure 6. The binding energy at saturation density as function of δ^2.

Conclusion

We presented a macroscopic calculation of asymmetric nuclear matter in the frame of the Thomas-Fermi approximation using a recent modern parameterization of the effective nucleon -nucleon interaction of Myers and Swiateski. The results confirm the validity of the δ^2 law for the energy per nucleon in the wide range of density and asymmetric parameter. The symmetry energy coefficient at the saturation density obtained in this work is about 28.25 MeV. The empirical value of the symmetry is about 30 ± 4 MeV. We have also studied the incompressibility of an asymmetric nuclear matter at its saturation density. Our EOS for the asymmetrical nuclear matter and symmetry energy is compatible with the other many-body calculations. Finally, we hope we could develop our calculation for calculating β-stable matter using this new approach.

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