

Full Length Research Paper

Use of the simplex method to optimize analytical condition in structural analysis

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The nonlinear behavior is approximated as a sequence of successively changing linear systems over a short time interval. Solution of nonlinear equations needs to iterative method such as iterative algorithm. Iterative method attempts to solve a problem by finding successive approximations. The present study is concerned with methods of nonlinear static and dynamic analysis of structures, particularly for application to geometrically nonlinear space structures. The basis of the proposed method is a step-by-step procedure of parametric solution continuation using a predictor–corrector scheme. The prediction is made with help of some interpolation procedure. In this paper a simplex method of nonlinear dynamics response analysis is developed.

Key words: Analytical method, optimization of energy, iterative algorithm, finite element method, analyzing modal, nonlinear dynamic response.

INTRODUCTION

The Analytical method is amongst the most efficient methods known for finding null spaces, which is the final stage in integer factorization algorithms such as the quadratic sieve and number field sieve, and its development has been entirely driven by this application. The first iterative method for solving a linear system appeared in a letter of Gauss. The Analytical method is an iterative algorithm that is an adaptation of power methods to find Eigen values and eigenvectors of a square matrix or the singular value decomposition of a rectangular matrix (Brezinski and Sadok, 2002; Ghafari, 2008). Minimum potential dynamic energy asserts that a structure or body shall deform or displace to a position that minimizes the total potential energy, with the lost potential energy being dissipated as heat. The tendency to minimum total potential energy is due to the second law of thermodynamics, which states that the entropy of a system will maximize at equilibrium. The design process for nonlinear structure is a relatively complex problem. Indeed the equilibrium configuration is an unknown in the analysis of this kind of structures. The nonlinear structure

may be analyzed as discrete system or continuous membrane (Silva and Vellasco, 2008).

PARTIAL DIFFERENTIAL EQUATION (PDE)

Nonlinear structural analysis uses analytical method under a partial differential equation (PDE) that it is a differential equation in which the unknown function is a function of multiple independent variables and the equation involves its partial derivatives. The partial differential equations are broadly classified as linear and nonlinear (George, 2009). The coefficients of the unknown function and its derivatives in a linear differential equation are allowed to be (known) functions of the independent variable or variables. Nonlinear differential equations can exhibit very complicated behavior over extended time intervals [Huu-Tai, 2010]. The procedures to solving nonlinear differential equation will be rapidly by minimizing of potential dynamic method.

EQUATION OF MOTION FOR MULTI DEGREE (MDOF)

Equations of motion are equations that describe the

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behavior of a system. The equation of motion for a multi degree of freedom (MDOF) system can be written as (Liquis, 2003):

$$M \ddot{x} + C(t) \dot{x} + K(t) x = P(t) \tag{1}$$

Where; M= mass matrix; C (t) = damping matrix; K (t) = stiffness matrix; x = displacement vector; \dot{x} = velocity vector; \ddot{x} = acceleration vector; P (t) = Load vector

The assumption of a constant mass in the case of MDOF systems is arbitrary as it could be represented as a time varying quantity (Sundar, 2000).

Since m is a non-zero constant value, both sides of Equation (1) can be divided by m, as follows;

$$P = \frac{c(t)}{m}$$

$$Q = \frac{K(t)}{m}$$

$$F = \frac{p(t)}{m}$$

Eq. (1) can be written as:

$$\ddot{x} + P \dot{x} + Qx = F \tag{2}$$

The mathematical solution of Equation (2) depends on the values of P, Q and F. Equation (2) is a linear differential equation if P and Q are independent of x and remains so even if P and Q are functions of t (Jian-Bing, 2007).

THE EXPRESSION FOR THE TOTAL POTENTIAL ENERGY

The total potential energy is written as:

$$W=U+V \tag{3}$$

Where;

W= the total potential energy
 U= the strain energy of the system, and
 V= the potential energy of the loading.

Taking the unloaded position of the assembly as datum,

$$W = \sum_{n=1}^m U_n + \sum_{j=1}^j \sum_{i=1}^3 F_{ji} X_{ji} \tag{4}$$

Where;

M= total number of members,
 J = total number of cable joints,
 F_{ji} = external applied load on joint j in direction i, and
 X_{ji} = displacement of joint j in direction i.

The condition for structural equilibrium is that the total

potential energy of the system is a minimum, that is to say (Laier, 2010).

$$\partial W / \partial X_{ji} = 0 \quad (j = 1, 2, \dots, j) \ \& \ (i = 1, 2, 3) \tag{5}$$

Thus at the solution the gradient vector of the total potential energy function is zero (Hashamdar et al., 2011).

The gradient of the total potential energy

Differentiating Equation (12) with respect to X_{ji} gives the g_{ji} element of the gradient vector g as;

$$g_{ji} = \partial W / \partial X_{ji} = \sum_{n=1}^q \partial U_n / \partial X_{ji} - F_{ji} \tag{6}$$

Let

T_{jn} = the initial tension in member jn,
 T_{jn} = the instantaneous tension in member jn,
 e_{jn} = elastic elongation of member jn,
 E = young Modulus of Elasticity,
 A = cross-sectional area of cable,
 L_{jn} = length of member jn, and
 Q = number of member meeting at joint j

The expression for g_{ji} can then be written as:

$$g_{ji} = \sum_{n=1}^q \frac{\partial U_n}{\partial e_{jn}} \cdot \frac{\partial e_{jn}}{\partial X_{ji}} - F_{ji} \tag{7}$$

The strain energy of member jn is given as:

$$U_{jn} = T_{jn} e_{jn} + \frac{EA}{2L_{jn}} e_{jn}^2 \tag{8}$$

Differentiating U_{jn} with respect to e_{jn} yields

$$\partial U_{jn} / \partial e_{jn} = T_{jn} + \frac{EA}{L_{jn}} e_{jn} = T_{jn} \tag{9}$$

The initial and elongated length of member jn may be expressed as:

$$L_{jn}^2 = \sum_{i=1}^3 (X_{ni} - X_{ji})^2 \tag{10}$$

$$(L_{jn} + e_{jn})^2 = \sum_{i=1}^3 (X_{ni} - X_{ji} + x_{ni} - x_{ji})^2 \quad (11)$$

Where X_{ji} is the coordinate of joint j in direction i. Simplifying Equation (19) and substituting for L_{jn} from Equation (18) yields the following expression for e_{jn} [Man-Chung, 2005]:

$$e_{jn} = \frac{1}{2L_{jn} + e_{jn}} \sum_{i=1}^3 \{ (X_{ni} - X_{ji}) (2X_{ni} - 2X_{ji} + X_{ni} - X_{ji}) \} \quad (12)$$

Differentiating Equation (12) with respect to X_{ji} yields

$$\partial e_{jn} / \partial X_{ji} = \frac{-1}{L_{jn} + e_{jn}} (X_{ni} - X_{ji} + X_{ni} - X_{ji}) \quad (13)$$

Substituting Equations (12) and (11) into Equation (13) yields the expression for the gradient as [Peterson, 2008]:

$$g_{ji} = - \sum_{n=1}^q t_{jn} (X_{ni} - X_{ji} + X_{ni} - X_{ji}) - F_{ji} \quad (14)$$

Where; $t_{jn} = T_{jn} / (L_{jn} + e_{jn})$ is the tension coefficient of member j_n .

Minimum total potential energy

The correct value of X for which W is a minimum, that is, $g = 0$ can now be found by the iterative process.

$$X_{ji(k+1)} = X_{ji(k)} + S_{(k)} V_{ji(k)} \quad (15)$$

Where the suffices (k) and (k+1) denote the (k)th and (k+1)th iterate respectively and where (Benner and Martin, 2010).

V_{ji} = the element of the direction vector, and
 $S_{(k)}$ = the step length which defines the position along $V_{ji(k)}$ where the total potential energy is a minimum.

The expression for V_{ji} is used, given by:

$$V_{ji(k)} = -g_{ji(k)} + \frac{\sum_{j=1}^J \sum_{i=1}^3 g_{ji(k)} g_{ji(k)}}{\sum_{j=1}^J \sum_{i=1}^3 g_{ji(k-1)} g_{ji(k-1)}} V_{ji(k-1)} \quad (16)$$

The stationary point in the direction of descent can be found by expressing the total potential energy as a function of the step length along V_{ji} . Thus the required value of $S_{(k)}$ can be determined by the condition.

$$\partial W_{(k)} / \partial S_{(k)} = 0 \quad (17)$$

Calculation of the step length

The required polynomial for step length is found by substituting the expression for $X_{ji(k+1)}$ given by Equation (23) into a suitable expression for the total potential energy w.

Writing the strain energy term in Equation (12) as a function of the elongation, Equation (16), and at the same time substituting for X_{ji} using Equation (17) lead to; first the expression for the elongation as a function of S

$$e_{jn} = \frac{1}{2L_{jn} + e_{jn}} (a_1 + a_2 S + a_3 S^2) \quad (18)$$

Where;

$$a_1 = \sum_{i=1}^3 (2(x_{ni} - x_{ji})(x_{ni} - x_{ji}) + (x_{ni} - x_{ji})(x_{ni} - x_{ji})) \quad (19)$$

$$a_2 = \sum_{i=1}^3 2((x_{ni} - x_{ji} + x_{ni} - x_{ji})(v_{ni} - v_{ji})) \quad (20)$$

$$a_3 = \sum_{i=1}^3 (v_{ni} - v_{ji})^2 \quad (21)$$

And secondly to the expression for W in terms of the step length S and its derivative with respect to S as given below:

$$W = C_1 S^4 + C_2 S^3 + C_3 S^2 + C_4 S + C_5 \quad (22)$$

$$\partial W/\partial S = 4C_1 S^3 + 3C_2 S^2 + 2C_3 S + C_4 \quad (23)$$

Where;

$$C_1 = \sum_{n=1}^m \left(\frac{EA}{2L(2L+e)^2} a_3^2 \right)_n \quad (23 a)$$

$$C_2 = \sum_{n=1}^m \left(\frac{EA}{L(2L+e)^2} a_2 a_3 \right)_n \quad (23 b)$$

$$C_3 = \sum_{i=1}^m \left(\frac{T_o}{2L+e} a_3 + \frac{EA}{2L(2L+e)^2} (a_2^2 + 2a_1 a_3) \right)_n \quad (23 c)$$

$$C_4 = \sum_{n=1}^m \left(\frac{T_o}{2L+e} a_2 + \frac{EA}{L(2L+e)^2} a_1 a_2 \right)_n - \sum_{j=1}^J \sum_{i=1}^3 F_{ji} V_{ji} \quad (23 d)$$

$$C_5 = \sum_{n=1}^m \left(\frac{T_o}{2L+e} a_1 + \frac{EA}{2L(2L+e)^2} a_1^2 \right)_n - \sum_{j=1}^J \sum_{i=1}^3 F_{ji} X_{ji} \quad (23 e)$$

The diagram of frame made and members at joint are shown in Figures 1 and 4. The graph about vector decent is shown in Figure 2, and Figure 3 and it shows how vectors decent to reach the minimum of energy are shown.

ANALYTICAL TEST

The development of a mathematical control to ensure stability when using larger time steps is desirable. The mathematical model chosen is a circle flat net with 39 degrees of freedom. The circle flat net was also built as a finite element model and tested in order to verify the optimizing Analytical method theory given in this paper. The mass density influence the stability limit, under some circumstance scaling the mass density can potentially increase the efficiency of an analysis and the explicit dynamic uses a central difference rule to integrate the equation of motion explicitly through time.

From Table 1, comparisons are made by deflections due to concentrated load at node 4 and the results are similar to other of finite element modeling. A mode shape describes the expected curvature of a surface vibrating at a particular mode.

The damping matrix used an orthogonal damping matrix in which the damping ratio could be varied in the different mode. Figure 5 shows the displacement of node increase with distance to center of circle flat net.

The maximum amplitudes of response of the whole node are in the Z-direction on node 4. This size of time step coming from Analytical method was proved to be adequate. When the solution becomes unstable, the time

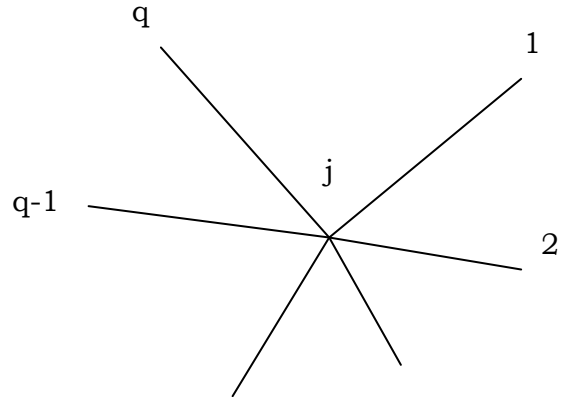


Figure 1. Number of member meeting at joint j.

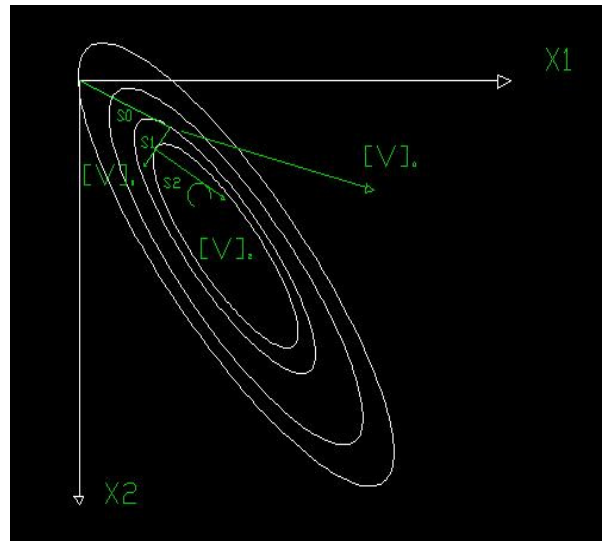


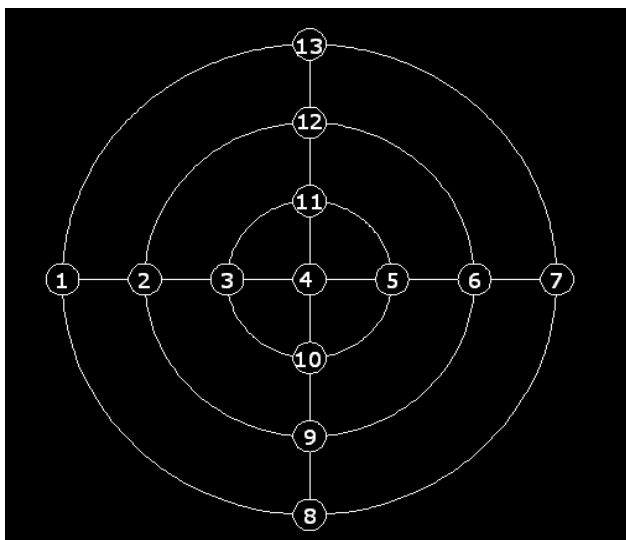
Figure 2. Geometric representation of decent vector.



Figure 3. Geometric representation of potential dynamic work for a function of two variables.

Table1. Deflections due to concentrated load at node 4.

LOAD(N) = 1000	Opt. Lanczos (T)	Finite element (E)	(T – E) / T*100
Z AXIS DEFLECTIONS(m) NODE 2 (LVDT)	178.6E-03	177.6E-03	0.56
Z AXIS DEFLECTIONS(m) NODE 3 (LVDT)	129.3E-03	127.9E-03	1.08
Z AXIS DEFLECTIONS(m) NODE 4 (LVDT)	50.75E-03	50.11E-03	1.26
Z AXIS DEFLECTIONS(m) NODE 5 (LVDT)	127.9E-03	127.15E-03	0.59
Z AXIS DEFLECTIONS(m) NODE 6 (LVDT)	50.75E-03	50.15E-03	1.18
Z AXIS DEFLECTIONS(m) NODE 9 (LVDT)	25.83E-03	24.33E-03	5.81
Z AXIS DEFLECTIONS(m) NODE 10 (LVDT)	74.46E-03	72.56E-03	2.55
Z AXIS DEFLECTIONS(m) NODE 11 (LVDT)	135.7E-03	133.25E-03	1.81
Z AXIS DEFLECTIONS(m) NODE 12 (LVDT)	135.7E-03	134.99E-03	0.52

**Figure 4.** Diagram of frame made.

history response of solution variable such as displacements will usually oscillate with increasing amplitude and

also the mass scaling was controlled by time incrementation.

In order to results, the proposed method for the non-linear dynamic response analysis of structures is based upon the minimization process. The size of the time step used for the dynamic response analysis was in all cases equal to half the smallest periodic time of the net concerned. The Figure 6 shows the 95% of mass attend to analyzing in preliminary 5 modes.

The differential percentage results are given in Table 2. Participation of frequency is given in Figure 7 for the whole of the mode in structures. Figure 7 shows the amount of frequency dramatically increases from mode 1 until mode 30.

In Figure 7 shows the frequencies appeared to level off between mode 1 and mode 2. From this mode onwards, the deflection dramatically climbed to reach mode 5.

CONCLUSIONS

Advantages of simplex method are such as, it solves the complex non linear Eigen value problem and it is able to account for a strong variation of the modulus with

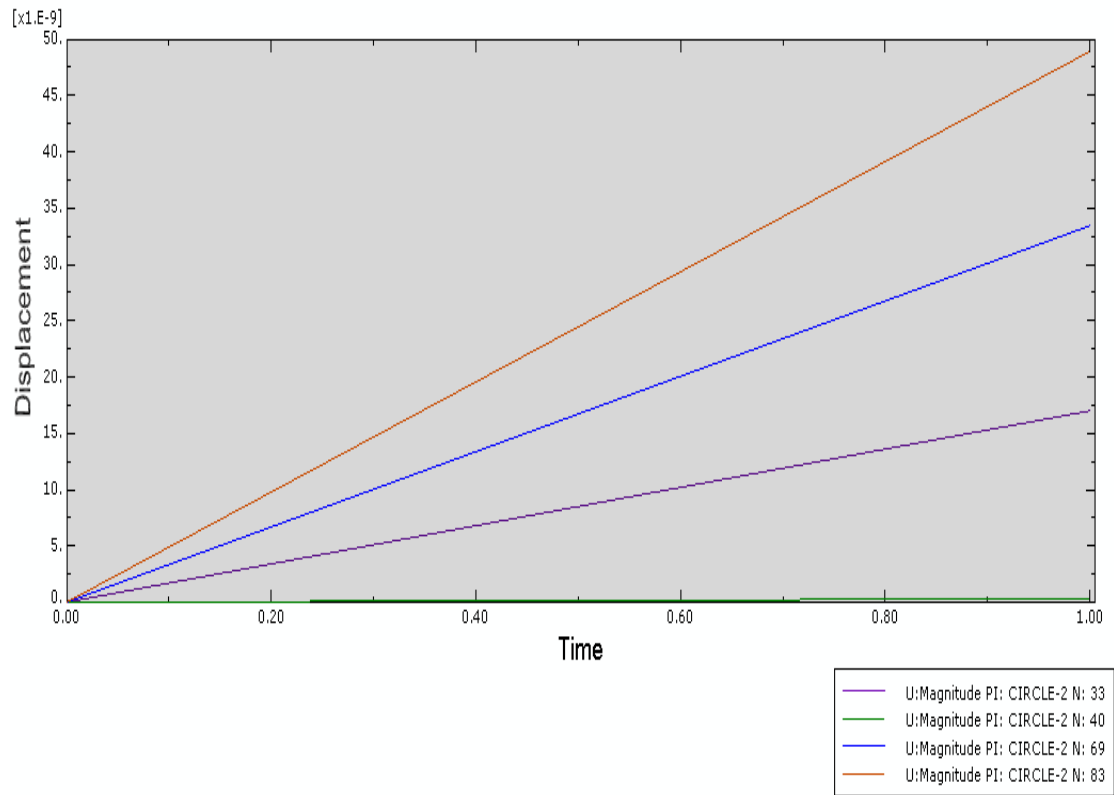


Figure 5. The displacements according to nodes.

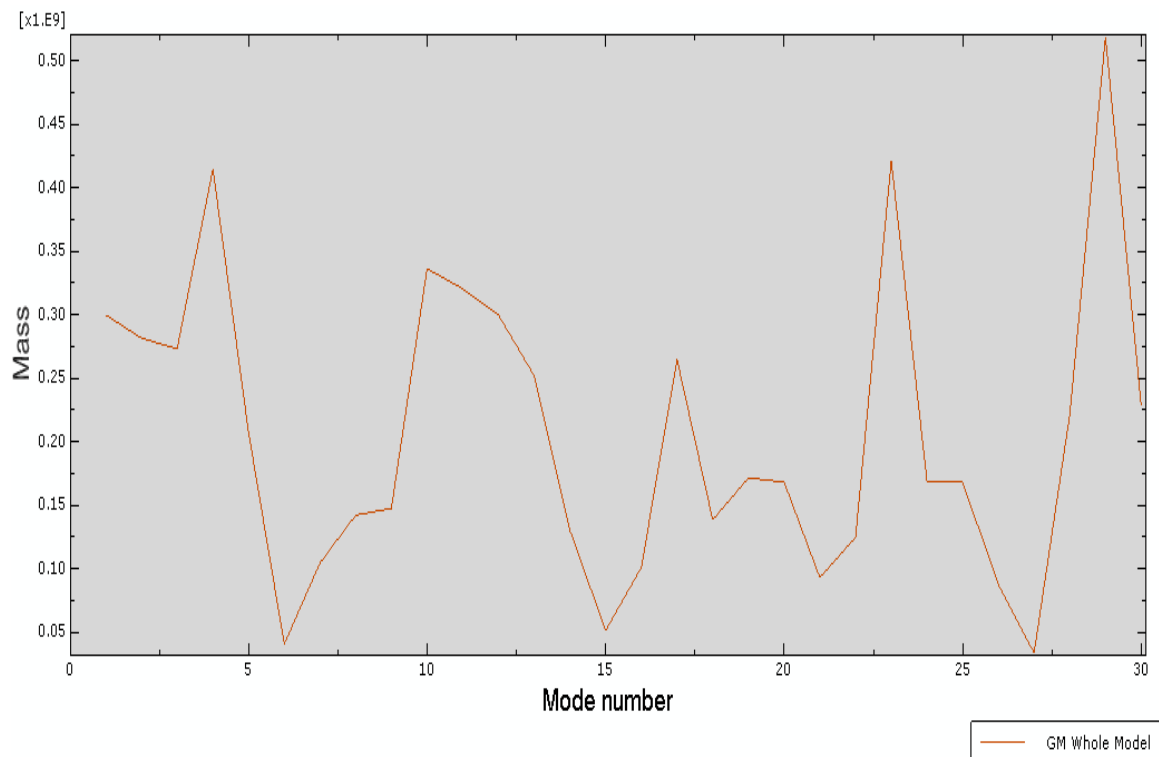
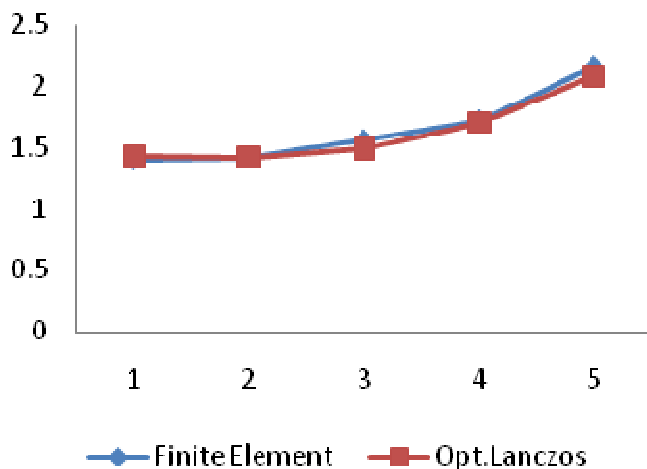


Figure 6. Distribution of general mass over of modes.

Table 2. Differences frequency results in 5 mode shapes.

Mode	Frequency renovation explicit	Frequency (Hz) finite element	Differentials percentage
1	1.4083	1.4321	1.69
2	1.42645	1.4151	0.8
3	1.5645	1.4945	4.47
4	1.7224	1.7041	1.06
5	2.1568	2.0823	3.45

**Figure 7.** comparison graph for finite element and optimization of energy.

frequency. The object of this work was principally to develop an Analytical method analysis theory and verify the theory by numerical and finite element testing. The propose method was found to be stable for time steps equal to less or less than half the smallest periodic time of the system. Comparison of finite element and theoretically predicted values showed that the deflection calculated by the proposed theory gives reasonably accurate results.

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