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# A study of electricity market volatility using long memory heteroscedastic model

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An accurate wholesale electricity market forecast has become an essential tool in bidding and hedging strategies in competitive electricity markets. This paper provides a dynamic asymmetric long memory heteroscedastic model to account the high volatile daily wholesale electricity markets in New England and Louisiana. This model implemented power Cox-Box transformation (Tse, 1998) under the Chung's (1999) model specification to the time-varying volatility. The model is able to capture various empirical stylized facts that commonly observed in electricity markets including clustering volatility, news impact, heavy-tailed and long memory volatility. Under the forecast evaluations, the long memory model outperformed the traditional model in all the forecast time-horizons. Finally, the outcome of the analysis is further applied in quantifying the market risk in term of value-at-risk.

**Key words:** Electricity markets, long memory generalized autoregressive conditional heteroskedasticity (GARCH), value-at-risk, time series analysis.

## INTRODUCTION

Price forecasting is an essential tool in pool bidding and bilateral contract systems of electricity-market deregulation. The deregulated market has created competition among the electricity producers and also great opportunities to increase their sales and profits. In bidding system, an accurate poll price forecast is important for the producers or retailers to optimally arrange their production schedule and costs. The future pool price provides valuable information for producers to hedge against the market volatility through bilateral contracts. Besides price level, the underlying data generating processes of price changes and volatility are also important in forecast evaluation of electricity markets.

In recent years, time-varying volatility forecasting in deregulated electricity markets has received considerable

attentions from researchers and practitioners. Autoregressive conditional heteroscedastic (ARCH) modelling (Engle, 1982) is among the famous method that has been widely used to capture the clustering volatility in energy markets. Carcia et al. (2005) has proposed a generalized ARCH to forecast hourly prices in the deregulated electricity markets of Spain and California. They reported that, the average forecast errors for both the markets are around 9%, depending on the studied month. Leme et al. (2008) using a similar generalized ARCH model to study the weekly Brazilian energy market. They found that four of the studied market regions have heteroscedastic effect in their volatility. Li et (2008) proposed a nonparametric generalized al. autoregressive conditional heteroskedasticity (GARCH) model and applied it on the power price in California market. Nunes et al. (2008) focused on the daily Spanish seasonal electricity market using autoregressive integrated moving average (ARIMA)-GARCH. They reported this model is capable to evaluate and predict the prices from January 1998 to August 2005. Besides ARCH

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model, other approaches such as artificial intelligent (Amjady, 2006; Gao et al., 2000; Xu et al., 2004), and wavelet analysis (Conejo et al., 2005; Pindoriya et al., 2008) have also addressed the same problem.

In order to obtain accurate forecasts, the statistical model should able to capture numerous financial market stylized facts (Cont, 2001). One of the interesting stylized facts is the long memory behavior which has been observed in most of the worldwide financial markets. Long memory processes are often observed in the field of hydrology, internet traffic analysis, finances and cardiology. One of the earliest studies is investigated by Hurst (1951) who observed the long-term storage capacity of Nile's river reservoirs and discovered the presence of long memory in hydrology. Mandelbrot (1967) on the other hand explained on how the length measurements of the United Kingdom coastlines are dependent on the scale measurements in term of long memory structure. Granger and Joyeux (1980) and Hosking (1980) reported the data generating processes that underlied the financial time series are long memory using the fractional differencing parameter estimation. In the world-wide-web traffic analysis, Leland et al. (1994) demonstrated that the Ethernet local area network (LAN) traffic is statistically long memory with millions of high quality Ethernet traffic measurements collected between year 1989 and 1992. Besides this, long memory processes are also found in the DNA nucleotide sequences (Lopes and Nune, 2006). The presence of long memory phenomenon in the electricity market can be explained using the heterogeneous market (Cheong, 2010; Docorogna et al., 2001) concept where the retailers and consumers are different from geographical location, preference to the type of energy, degree of energy information, wealth constraint, nature of consumption (commercial or home) and awareness of energy crisis. The combinations of these dissimilar time-response volatilities among the heterogeneous market participants are believed to produce hyperbolic autocorrelation decays or long memory in the return volatility. Besides this, the implemented model also should able to account the possible leverage effect (volatility response to upward and downward prices), dynamic power of volatility representation (common specification is in conditional variance (Bollerslev, 1986) or conditional standard deviation (Taylor, 1986) and skewed-heavy tail distribution assumption.

This study aims to evaluate one day-ahead forecast of price changes with long memory volatility using a dynamic asymmetric long memory ARCH model. We adopt the fractionally integrated asymmetric power autoregressive conditional heteroscedastic (FIAPARCH) introduced by Tse (1998). However, due to the drawback of model parametrization that will cause difficulty in both estimation and the interpretation of estimated results, the Chung's (1999) model specification has been selected. In order to capture possible heavy-tailed and skewness behaviors, a skewed student-t distribution is used in the studied electricity markets. It is worth noted that most of the ARCH applications in electricity markets (Carcia et al., 2005; Leme et al., 2008; Nune et al., 2008; Xiong et al., 2008) have considered conditional mean equation as logarithm prices ( $p_t$ ). As a result, the  $p_t$  suffers from long order of autoregressive (AR) lags which against the Box-Jenkins parsimonious principle, for example Garcia et al. (2005) reported 12 lags for Spanish and California markets. An alternative to lessen this shortcoming is by redefining the log prices as log price changes or more commonly named as continuous compounded return. This definition is a scale-free summary of possible profit opportunity and provides the stationarity condition of time series.

For empirical study, the daily wholesale data NEPOOL mass and Entergy hubs for the New England and Louisiana are selected respectively for discussion and analysis. The forecast evaluations are compared between the long memory ARCH model and traditional ARCH models. Finally the forecast results are implemented in the risk quantification in term of value-at-risk (Jorion, 1997).

## METHODOLOGY

## Fractionally integrated asymmetric power autoregressive conditional heteroscedastic (ARCH)

The joint-estimation of ARMA-ARCH model consisted of two components, the conditional mean and time-varying volatility equations. Let  $r_{\rm t}$  be a general percentage compounded return stochastic process with  $100 \times \log(\frac{P_{\rm r}}{P_{\rm r-1}})$ . This series is usually

serially uncorrelated, but not independent in higher moment such as variance. For a given historical data  $I_{t-1}$  available at time t–1, the price changes  $r_t$  (conditional mean equation) is defined as

$$r_t = \mu_t - a_t a_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \stackrel{i.i.d}{\sim} f(0,1); \tag{1}$$

where  $f(\cdot)$  is the density function of  $\varepsilon_t$ . The conditional mean  $E(r_t | I_{t-1}) = E_{t-1}(r_t) = \mu_t$  often follows a stationary ARMA (m,n) model specified as

$$\mu_{t} = \theta_{0} + \sum_{i=1}^{m} \theta_{i} r_{t-i} - \sum_{j=1}^{n} \vartheta_{j} a_{t-j} .$$
<sup>(2)</sup>

To accommodate the possibility of long memory volatility, the fractionally integrated (FI) ARCH is introduced by Baillie et al. (1996), Engle and Lee (1999) and Tse (1998). Long memory processes often relate to their autocorrelation functions. For example, a fractionally integrated ARMA is said to be long memory when its autocorrelation functions are not summable

0.5 (Cheong, 2010). Under the Robinson and Henry (1999) specification, the squares of shock  $a_t^2$  is rewritten as:

$$a_t^2 = \omega + \Omega(B)\eta_t \tag{3}$$

With

$$\eta_t = a_t^2 - \sigma_t^2, \Omega(B) = \omega_0 + \omega_1 B + \omega_2 B^2 + \dots = \sum_{i=0}^{\infty} \omega_i B^i$$
  
and  $0 \neq \sum_{i=0}^{\infty} \omega_i^2$  to  $\sum_{i=0}^{\infty} (a_i^2) = b \ge 1$  is long memory

and  $0 < \sum_{i=0} \omega_i^2 < \infty$  . The  $\{ {m 
ho}_h(a_i^2), h \ge 1 \}$  is long memory

under the finite parametrization of  $\Omega(B)$  :

$$\Omega(B) = \frac{\left(1 - \beta(B)\right)}{\Psi(B)\left(1 - B\right)^d} \eta_t, \qquad (4)$$

for  $d \in (0,0.5)$  with the fractional-differencing operator  $(1-B)^d$ . More precisely, the autocorrelations  $\{\rho_h(a_t^2), h \ge 1\}$  decays at a slow hyperbolic rate that satisfies

$$\lim_{h \to \infty} \rho_h(a_t^2) = \frac{\sum_{i=0}^{\infty} \omega_i \omega_{i+h}}{\sum_{i=0}^{\infty} \omega_i^2} \sim Ch^{2d-1}.$$
 (5)

For long memory electricity time series, the FIAPGARCH model (Tse, 1998) is used to accommodate the various statistical properties of price changes and time-varying volatility in the electricity markets. The aforementioned model is a combination of Ding et al. (1993) asymmetry power ARCH and Granger and Joyeux (1980) fractionally integrated filter with the following specification:

$$\sigma_t^{\delta} = \frac{\alpha_0}{1 - \beta(B)} + \left\{ 1 - \frac{\varphi(B)(1 - B)^d}{1 - \beta(B)} \right\} \left( \left| a_t \right| - \gamma a_t \right)^{\delta}, \quad (6)$$

where

$$\alpha(B) = \left(\alpha_1 B + \dots + \alpha_q B^q\right), \ \beta(B) = \left(\beta_1 B + \dots + \beta_p B^p\right)$$

and 
$$\varphi(B) = \frac{1 - \alpha(B) - \beta(B)}{1 - B}$$
 represent the lag polynomials

with  $\alpha_0$  is the unconditional variance of  $a_t$ ;

$$(1-B)^{d} = \sum_{n=0}^{\infty} (-1)^{n} \frac{d(d-1)...(d-k+1)}{k!} \quad B^{d} \text{ with } d \in [0,1],$$

denotes the fractional integrated operator;  $\gamma = \log(\phi) \in (-1, 1)$  denotes the asymmetric effect, for example, a negative  $\gamma$  indicates negative shocks give rise to stronger volatility than positive shocks at a same magnitude or sometimes refers to leverage effect in finance perspective;  $\delta$  indicates the Box-Cox transformation for the most appropriate volatility representation. The term 'dynamic' refers to the any possible real value of  $\delta$ .

This specification covers a series of ARCH model under specific conditions, for example the GARCH (Bollerslev, 1986) model with  $\gamma$ =0,  $\delta$ =2 and d=0; the Ding et al. (1993) asymmetric power (AP) GARCH with  $\gamma$ ,  $\delta$  and d=0; the BBM (1996) FIGARCH with  $\gamma$ =0,  $\delta$ =2 and FIAPGARCH (Tse, 1998) with  $\gamma$ ,  $\delta$  and d. It is worth noting that the Baillie, Bollerslev and Mikkelsen (BBM) type FIGARCH (including FIAPGARCH by Tse, 1998) suffers from the model parametrization (Chung, 1999) which causes difficulty in both estimation and the interpretation of estimated results. The FI model is structurally different from the original ARFIMA (Granger and Joyeux, 1980). However, according to Chung (1999), a simple rearrangement of the long memory volatility can avoid such discrepancy in the FIGARCH parameterization. Similar approach can be implemented to FIAPGARCH model based on Equation 6 as follow:

$$\begin{bmatrix} 1 - \beta(B) \end{bmatrix} \sigma_t^{\delta} = -\beta(B)(|a_t| - \gamma a_t)^{\delta} + (|a_t| - \gamma a_t)^{\delta} - \varphi(B)(1 - B)^d [(|a_t| - \gamma a_t)^{\delta} - \alpha_0]$$
(7)

Although this specification is quite similar to the ARFIMA, it does not fully replicate the exact structural form of ARFIMA (for example, a stationary and invertible ARFIMA, its *d* lies between -0.5 and +0.5 while for FIAPGARCH, the *d* is permitted to exceed 0.5 however less than unity).

#### The flow of analysis

The procedures of preliminary analysis, model identification, estimation, diagnostic and forecasting follows the standard Box-Jenkins framework as follow:

Step 1: Preliminary analysis focuses on the graphical illustration, descriptive statistics and normality tests;

Step 2: The subset models for conditional mean is identified using autocorrelation function (ACF) and partial ACF whereas the timevarying volatility specification is detected by the Engle Lagrange multiplier and Ljung-Box statistics. Next, the presence of long memory processes is identified using ACF, variance-time plot and rescaled-range analysis of volatility proxies.

Step 3: After the functional form models have been initiated, the joint estimations (vector parameter,  $\psi$ ) involves the conditional mean parameters  $\omega' = (a_0, a_1, ..., b_1, b_2, ...)$  and the density function parameters g', as well as the conditional variance parameters,  $\theta' = (\alpha_0, \alpha_1, ..., \gamma, \delta, d, \beta_0, \beta_1, ...)$ , all set at time t and  $c = \frac{(r_t - \mu_t)}{c}$  are estimated using the maximum likelihood

$$\mathcal{E}_t = \sigma_t / \sigma_t$$
 are estimated using the maximum likelihood

method. For long memory model, the truncation lag of the slow decaying fractional difference operator  $\left(1\text{-B}\right)^d$  is set to 1000

$$(1-B)^{d} = \sum_{n=0}^{1000} (-1)^{n} \frac{d(d-1)...(d-k+1)}{k!} B^{d}$$

which is in line with the common BBM (Baillie et al.,1996) setting. Due to the nonlinearity condition, the iterative optimization algorithm is used instead of analytical derivative approach with the log-likelihood function  $L_N$  as follows:

$$\frac{\partial L_N}{\partial \psi} \approx \frac{\partial L_N}{\partial \psi^{(0)}} + \left(\psi - \psi^{(0)}\right) \frac{\partial^2 L_N}{\partial \psi^{(0)} \partial \psi^{(0)}},\tag{8}$$

where  $\psi^{(0)}$  denotes the trial values of the estimates and  $\partial^2 I$ 

 $rac{\partial^2 L_N}{\partial \psi^{(0)} \partial \psi^{(0)}}$  represented the Hessian matrix. Rearranging the

terms in the form Newton-Raphson algorithm, the  $\left(k+1\right)^{\text{th}}$  vector set of parameters values is defined as

$$\boldsymbol{\psi}^{(k+1)} = \boldsymbol{\psi}^{(k)} - \left(\frac{\partial^2 L_N}{\partial \boldsymbol{\psi}^{(0)} \partial \boldsymbol{\psi}^{(0)}}\right)^{-1} \frac{\partial L_N^{(k)}}{\partial \boldsymbol{\psi}}.$$
 (9)

For asymmetric (non-zero skewness) and heavy-tailed (kurtosis exceeded 3) price changes shock, a standardized skewed student-t (Lambert and Laurent, 2001) log-likelihood is used as follows:

$$L_{N} = N \begin{cases} \ln \Gamma \left\lfloor \frac{v+1}{2} \right\rfloor - \ln \Gamma \left\lfloor \frac{v}{2} \right\rfloor - \frac{1}{2} \ln(\pi(v-2)) \\ + \ln \left( \frac{2}{skew + \frac{1}{skew}} \right) + \ln(s) \end{cases}$$
$$-\frac{1}{2} \sum_{t=1}^{N} \left( \ln(\sigma_{t}^{2}) + (1+v) \ln \left( 1 + \frac{(s\varepsilon_{t} + m)^{2}}{v-2} skew^{-2I_{t}} \right) \right)$$
$$I_{t} = \begin{cases} 1 & \text{for } \varepsilon_{t} \ge -\frac{m}{s} \\ -1 & \text{for } \varepsilon_{t} < -\frac{m}{s} \end{cases}$$

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Where

$$mean, m = \frac{\Gamma\left[\frac{v+1}{2}\right]\sqrt{v-2}}{\sqrt{\pi}\Gamma\left[\frac{v}{2}\right]} \left(skew - \frac{1}{skew}\right) \text{ and}$$
$$stdev, s = \sqrt{skew^2 + \frac{1}{skew^2} - m^2}$$

*skew* and v denote the skewness parameter and degree of freedom from the extended student-t distribution. It is obvious that, when *skew* equivalent to 1, the distribution reduces to symmetric standardized student-t distribution.

Step 4: All the models are diagnosed using the Ljung-Box statistics for both standardized and squared residuals. The acceptance of the test statistics indicated no significant autocorrelation in the conditional mean and variance equations. For heteroskedastic effect, the Engle LM ARCH test is used upon the squared standardized residuals. The adequacy test is further examined by the Engle and Ng (1993) test to check the asymmetry volatility impact response to good/bad news. The negative/positive-size-bias tests examined whether the squared standardized residuals are indeed i.i.d. If the information  $a_{L^{+}}$  provides predictive component to residuals, then the conditional variance are miss-

specified. Let  $S_t^-$  be a dummy variable that takes value of 1 if  $a_{t-1}$  is negative and zero otherwise. Also, let  $S_t^+ = 1 - S_t^-$ . The negative and positive size-bias test statistics are defined as the *t-ratio* for the coefficients  $\gamma_{t2}$  and  $\gamma_{22}$  in the regression models:

$$\tilde{a}_{t}^{2} = \gamma_{11} + \gamma_{12} S_{t}^{-} a_{t-1} + \gamma_{13} z_{t-1} + \varepsilon_{t}$$
(11)

$$\widetilde{a}_{t}^{2} = \gamma_{21} + \gamma_{22} S_{t}^{+} a_{t-1} + \gamma_{23} z_{t-1} + \varepsilon_{t}$$
(12)

where the  $\varepsilon_t$  is a white noise error. Step 5: Each volatility model is estimated *H* times bases on fix period of  $T_1$  observations in the selected time period. A rolling parameter estimations are implemented, for example, a first one-day ahead forecast at  $t = T_1+1$ is using the estimation t=1 to  $T_1$  while the estimation from t=2 to  $T_1+1$  is used to forecast the volatility at  $t = T_1+2$ . Therefore, *H* oneday ahead volatility forecasts can be obtained by using the rolling estimations procedures for  $\hat{\sigma}_{(h),t}^2$ , where h=1,..., H. For forecast evaluation two loss functions are used to evaluate the predictive accuracy of a volatility model:

L<sub>1</sub>: MSE =  

$$\frac{1}{H} \sum_{h=t+1}^{t+H} |actual_{h} - forecast_{h}|^{2}; \qquad (13)$$

L<sub>2</sub>: MAE =  

$$\frac{1}{H} \sum_{h=t+1}^{t+H} |actual_{h} - forecast_{h}|; \qquad (14)$$

#### CASE STUDIES, ANALYSIS RESULTS

## **Applied data**

In the empirical study, the Louisiana and New England daily wholesale electricity data are selected in the long memory ARCH implementation. Figure 1 illustrates the price level and return series for both the markets. The wholesale pricing is once belongs to exclusive domain for large retail supplier, however due to market liberalization it is now open up to end-users such as New England. Both data sets commence from January year 2002 until the end of December year 2008 with a total of 1762 and 1601 observations for Louisiana and New England respectively. The aforementioned data are used for estimations and the observations in year 2009 are reserved for forecast and forecast evaluations. The daily wholesale data can be obtained from the Intercontinental Exchange (ICE) which updated biweekly by the Energy Information Administration (EIA). The ICE (https://www.theice.com) is a major execution venue for over-the-counter (OTC) trading. These power indices are taken directly from transactions executed on the ICE platform representing approximately 70% of next day trading activity.



Figure 1. Price level and return series from January 2002 to December 2008.

Table 1 presents the descriptive statistics report for mean, standard deviation, skewness and kurtosis of the unconditional returns. All the returns are deviated from normal distribution with non-zero skewness and excess kurtosis. Normality test using Jarque-Bera statistic bases on these two measurements show that all the return series reject the normality condition with zero skewness and kurtosis exceed three.

In addition, a series of quantile-quantile plots with normality against all the series do not follow a straight line especially at both tails. However, better results are observed when the normal distribution is replaced by student-t distribution. Figure 2 illustrates the aforementioned arguments. As a result, the normality assumption for return series may underestimate the empirical returns series with significant number of extreme values. Thus, a heavy-tailed distribution is more preferable in the model specification.

## Identification result

For the subset model of mean equation, the ACF and PACF suggest that both the return series follow ARMA

(2,1) with the specification

$$\mu_t = \theta_0 + \sum_{i=1}^2 \theta_i r_{t-i} - \sum_{j=1}^1 \vartheta_j a_{t-j}$$

The Ljung Box statistic is based on the ACF of  $a_t^2$ :  $Q(k) = N(N+2)\sum_{i}^{k} \frac{\hat{\rho}_i^2}{N-i}$  with pull hungthesis

 $\overline{t} = N - t$  with null hypothesis  $H_0: \rho_1 = ... = \rho_k = 0$ . Another heteroscedastic test is Engle statistic which is based on linear regression  $a_t^2 = \lambda_0 + \sum_{i=1}^k \lambda_i a_{t-i}^2 + z_t$  where  $z_t$  is error term with null hypothesis  $H_0: \lambda_1 = ... = \lambda_k = 0$ .

The time varying volatility is studied by examining the possible serial correlation and dependence analysis in the volatility proxy. Table 2 reports that all the series reject both the null hypotheses with serial correlation and dependence in the first six lag. In other words, the

Statistics	Louisiana	New England
Mean	0.026037	0.035846
Std. Dev.	8.936963	12.19540
Skewness	0.244830	0.364211
Kurtosis	9.014292	14.40199
Jarque-Bera	2673.209	8707.843
P-value	0.000000	0.000000
Observations	1762	1601

 Table 1. Descriptive statistics for return series.



Figure 2. Quantile-quantile plots for Louisiana market and New England market.

heteroscedastic effects are evidenced in both the electricity markets. Variance-time (VT) and rescaled-range (R/S) plots are based on the regression as follows:

VT: log(V[x(n)]) = log(V[x]) - (2H-2) log(n),

$$x_{k}^{(n)} = \frac{1}{n} \sum_{i=kn-(n-1)}^{kn} x_{i}$$

 Table 2. Heteroscedastic test.

Market	Engle LM statistic, LM(6)	Ljung-Box statistic, Q(6)	
Louisiana	48.94964(0.000)	402.07(0.000)	
New England	30.40733(0.000)	247.11(0.000)	

Table 3. Long memory hurst's parameter.

Proxy	Absolute return				
Hurst parameter	Variance-time plot	R <sup>2</sup>	R/S	R <sup>2</sup>	
Louisiana	0.659	0.9863	0.686	0.9921	
New England	0.652	0.9585	0.683	0.9938	
Proxy		Square retu	ırn		
110/1		Oquarerole			
Hurst value	Variance-time plot	R <sup>2</sup>	R/S	R <sup>2</sup>	
Hurst value	Variance-time plot 0.643	<b>R</b> <sup>2</sup> 0.9921	<b>R/S</b> 0.674	<b>R<sup>2</sup></b> 0.9975	
Hurst value Louisiana New England	Variance-time plot 0.643 0.635	<b>R</b> <sup>2</sup> 0.9921 0.9771	<b>R/S</b> 0.674 0.673	<b>R<sup>2</sup></b> 0.9975 0.9964	



Figure 3. R/S and VT plot for square-return.



$$\frac{R}{S} = \frac{\max_{1 \le j \le N} \left[ \sum_{k=1}^{j} (X_{k} - M(L)) \right] - \min_{1 \le j \le N} \left[ \sum_{k=1}^{j} (X_{k} - M(L)) \right]}{\sqrt{\frac{1}{N} \left[ \sum_{k=1}^{N} (X_{k} - M(L))^{2} \right]}}$$

Next, the existence of long memory processes in volatility (proxies by absolute and square return) is examined using VT and R/S analysis. Table 3 presents the estimated Hurst's parameters are approximately equivalent to 0.6500 under the regression analysis. Figure 3 illustrates the regression plots for both proxies, squares and absolute return. Overall, the R/S provides more accurate estimations compares to VT method under the coefficient of determination measurements. Both the long memory volatilities are evidence with Hurst values between 0.5000 to 1.0000 in the series. Due to this, it is more suitable to include long memory property in the GARCH model specification.

## Estimation result and diagnostic checking

Table 4 presents the results of maximum likelihood for joint-estimation ARMA-GARCH based on Equation 6 to 10. The result and implication for each coefficient is explained as follows:

Estimation	Louisiene	New England	
Mean	Louisiana	New England	
Constant, $\theta_0$	0.100774 (0.9971)	0.157135 <sup>*</sup> (1.371)	
AR(1), θ <sub>1</sub>	0.829295 *(14.24)	0.624152 <sup>*</sup> (11.68)	
AR(2), θ <sub>2</sub>	-0.178100 *(-6.946)	-0.111695 <sup>*</sup> (-4.212)	
MA(1), ϑ <sub>1</sub>	-0.789449 *(-14.16)	-0.701472 <sup>*</sup> (-13.73)	
Variance			
Constant, $\alpha_0$	132.639274 (1.905)	48.69815 <sup>*</sup> (2.556)	
ARCH, α1	0.206930(1.381)	0.108793 (0.5970)	
GARCH, β₁	0.436339 *(2.361)	0.268781 <sup>*</sup> (1.357)	
Long memory, d	0.430906 <sup>*</sup> (5.268)	0.377616 <sup>*</sup> (8.118)	
News impact, $\gamma$	-0.495339 (0.7108)	-0.613679 <sup>*</sup> (-4.680)	
Power, $\delta$	1.880342 <sup>*</sup> (10.64)	1.487252 <sup>*</sup> (13.33)	
Tail property			
Tail index, <i>v</i>	4.861406*(9.198)	4.385438 <sup>*</sup> (9.930)	
Skewness, skew	0.083231*(2.672)	0.263467 <sup>*</sup> (7.653)	

Table 4. Maximum likelihood estimation.

The value in the parenthesis denotes the t-value.

The return series for California market is fitted by an ARMA(2,1) for both the Louisiana and New England markets. This implies that, the historical price changes for both the markets have impacts to the current electricity price changes.

Instead of prefixing the power transformation ( $\delta$ ) of volatility, the dynamic  $\delta$  is selected based on the optimal log-likelihood function in the iterative searching algorithm. The Louisiana market indicates  $\delta$  equivalent to 1.882137 which is in favor of the Bollerslev's volatility specification with conditional variance. On the hand, the New England market is indifferent in either Taylor's or Bolleslev's specification with  $\delta$  equivalent to 1.487252.

The tail-index *vs* for both markets are 4.861406 and 4.385438 with the fatter tail for New England market than Louisiana market. Besides this, the error terms for the density function for all the markets are positively skewed which implies that most the observations are concentrated at the negative side of the distribution.

The long memory parameter *d* documents 0.430906 and 0.377616 for Louisiana and New England markets. In short, the New England market indicates higher long memory intensity over New England markets. The presence of long memory phenomenon in the electricity market can be explained using the heterogeneous market (Cheong, 2010; Dacorogna et al., 2001) concept where the retailers and consumers are different from geographical location, preference to the type of energy, degree of energy information, wealth constraint, nature of consumption (commercial or home) and awareness of energy crisis. The combinations of these dissimilar timeresponse volatilities among the heterogeneous market participants are believed to produce hyperbolic autocorrelation decays or long memory in the return volatility.

Only the New England's volatility asymmetry parameter  $\phi$  indicates the news impact is asymmetric. In short, the New England market implies that the upward movement (rise in price) has a stronger impact on the next day volatility than a plunge of the same magnitude. However, this phenomenon is not observed in the Louisiana markets.

For the diagnostic checking, Table 5 indicates that all the models are adequate with no serial correlation, ARCH effect and asymmetric impact to the news at 1% significance level.

#### Forecast and forecast evaluation results

The one-day ahead return and volatility forecasts are conducted for Louisiana and New England electricity markets. The rolling forecasting is implemented in four different time horizons with 5, 20, 60 and 150 one-day ahead forecasts. A static forecasting is conducted by rolling re-estimation once a day from a fixed size of 1762 and 1601 for Louisiana and New England respectively. Table 6 reports that, the return forecast evaluations with the measurement of mean absolute error (MAE) and mean squared error (MSE). Since the MSE is relatively more sensitive to extreme value in the series, it is observed that MSE are higher than MAE in all the time horizons forecast evaluations. Figures 4 and 5 illustrate the outcomes of the forecasts for both the markets using ARMA(2,1) models. It is worth to note that, the return

Table 5. Diagnostic checking.

Diagnose	Louisiana	New England
ã, , <b>Q(6)</b>	7.24754 [0.0644127]	9.21628 <sup>*</sup> [0.0265493]
$\tilde{a}_{\iota}^{2}$ Q(6)	3.83943 [0.4281721]	1.12627 [0.8900827]
ARCH(6)	0.68998 [0.6578]	0.18897 [0.9800]
Negative bias	1.06996 0.28464	0.77987 0.43547
Positive bias	0.21111 0.83280	0.00039 0.99969

Notes:

1.  $\tilde{a}_{t}$  represents the standardized residual. Ljung box serial correlation test (Q-statistics) on  $\tilde{a}_{t}$  and  $\tilde{a}_{t}^{2}$ : null hypothesis – no serial correlation; LM ARCH test: null hypothesis - no ARCH effect; 2. The parentheses values represent the standard error and p-value for model estimation and diagnostic analysis respectively; . (\*) denotes 5% level of significance.

Table 6.	Return	forecast	evaluations.
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	Louisiana		New England	
Forecast	MAE	MSE	MAE	MSE
5 day-ahead	4.875109	45.7746	4.875109	45.7746
20 day-ahead	6.493351	69.69166	14.96263	359.3206
60 day-ahead	6.715448	69.57441	10.89982	228.9269
150 day-ahead	6.394046	63.11644	6.953547	108.6783



Figure 4. Return forecasts for Louisiana and New England market.

forecast can be improved if other influential aspects (Torre et al., 2003) such as transmission line congestion or other relevant aspects. Tables 7 and 8 present the volatility forecast evaluations for long memory GARCH and inclusion of standard GARCH as comparison purposes. It is found that overall the forecast evaluations for long memory GARCH is better than the standard GARCH in the entire time horizon. Similar volatility forecast evaluation results are observed in where MSE is relatively more sensitive to extreme volatility than MAE. Thus, the error forecasts using the MAE is smaller. From Figure 5, the forecast volatility series are responsive to the actual volatility movements. As a conclusion, it appears that long memory model is out-performed the standard ARCH model for all the time horizon forecasts with smaller loss functions.



Figure 5. Volatility forecasts for Louisiana market and New England market.

 Table 7. Volatility forecast evaluations for Louisiana market.

Medel	Long memory GARCH		Standard GARCH	
wodei	MAE	MSE	MAE	MSE
5 day-ahead	146.958	22169.37	158.798	26043.9
20 day-ahead	112.6326	23737.14	116.9692	24192.8
60 day-ahead	84.1231	14320.88	85.66874	14491.07
150 day-ahead	70.65394	9877.373	72.27973	9969.246

Table 8. Volatility forecast evaluations for New England market.

Marial	Long mem	Long memory GARCH		GARCH
Model	MAE	MSE	MAE	MSE
5 day-ahead	93.85406	10492.99	124.6625	17018.3
20 day-ahead	392.1333	372194.3	403.9945	330081.7
60 day-ahead	270.4836	251197.2	301.2774	245464.7
150 day-ahead	131.2295	102442.7	145.2909	100518.2

### Application in market risk

The estimated results have immediate application in market risk of electricity markets. One of the possible scenario is says a retailer earns a gross profit of \$1million (by selling a certain volume of electricity) for a particular day. If the retailer would like to know the worst loss in the next five days under normal market condition, this market risk can be quantified using the definition of value-at-risk (Jorion, 1997). The VaR is normally define as the worst loss for a given confidence level (for instance 95%) which means one is 95% certain that at the end of a chosen risk horizon (for example, five day ahead for this specific study) there will be no greater loss, that is the VaR under normal market conditions. In portfolio analysis, the VaR

often acts as a tool to alert investors for their possible expose risks under a particular portfolio. Consider the fitted model for Louisiana market, the estimated values at t = 1762 are  $r_{1762}$  = -33.4513 and  $\sigma^2_{1762}$  = 110.6999. Thus, the five days ahead forecasts are  $r_{1762}(5)$  = 0.242063 and  $\sigma^2_{1762}(5)$  = 110.7254. For lower tail 5% quantile, the value is  $r_t + t_{\alpha=0.05,(v=4.861406, skew=0.083231)}\sigma$  = 0.242063 – 1.4954 × 110.7254 = -15.4935% (negative sign indicates the loss). The 95% VaR for a position of \$1 million is \$1millionx0.154935 = \$154935, in this condition the long memory model still hold. In other words, with 95% confidence the potential loss of holding this position in next five day is \$154935 or less. Similarly, the lower 1% quantile can be determined as  $r_t + t_{\alpha=0.01,(v=4.861406, skew=0.083231)}, \sigma$  = 0.242063 – 2.4531 × 110.7254 = -25.571%.

This time, with higher confidence level 99% the maximum potential loss increases to \$255710. It is worth to note that the VaR is directly influences by the parametric distribution assumption, for this specific study we used skewed student-t distribution. If other assumption such as normal distribution or generalized error distribution is used, the value of VaR will be varied based on the behaviour of the tail distribution.

## CONCLUSION

This paper investigated the time varying volatility of two electricity markets for New England and Louisiana in terms of their long memory, asymmetric news impact and skewed heavy-tailed behaviors. The empirical findings that may attract the interests of academicians and researchers are as follows:

First, the power transformation of time-varying volatility for Louisiana market is in favor of variance representation; whereas, it is indifferent for New England in either variance or standard deviation form. The type of volatility representation has important role in forecast and risk evaluations

Second, both the markets indicated positively skewed and heavy-tailed distribution which implied the concentration of more negative returns in the studied duration.

Third, only the New England market evidenced the presence of leverage effect.

This implied that the downward movements (shocks) in the New England market are follow by greater volatilities than upward movements of the same magnitude. Fourth, both the markets indicated long memory behavior in terms of their time-varying volatilities. This suggested that the fluctuation of prices has permanent effects on its volatility. The long-persistence volatility can be explained in the micro manner where the heterogeneous market participants interpreted the same information differently according to their trading opportunities. The electricity markets which composed by participants with different reaction times to news (new information) have created a volatility cascade ranging from low to high frequencies. These combinations of dissimilar volatilities are believed to produce the very slow-decaying volatility in markets. Therefore, investors or policy makers should take into account this behavioral economic in their future research or investment. Fifth, the out-sample forecast evaluations indicated superior performance for long memory model over the standard GARCH models. In other words, the richer specifications of the implemented long memory model gained additional accuracy over the traditional model. One of the direct applications of estimated timevarying volatility is the quantitative market risk measurement using value-at-risk. From the economic point of view, the electricity market risk is a vital issue for financial institutes (including private or government investments)

because a large amount of wealth can be lost due to failure of supervising and controlling the financial risks.

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