International Journal of the Physical Sciences Vol. 6(15), pp. 3645-3652, 4 August, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.437
ISSN 1992-1950 ©2011 Academic Journals

Full Length Research Paper

Dimension reduction based direction of arrival estimation with partially calibrated distributed electromagnetic component sensor arrays

Hui Chen*, Qun Wan and Yipeng Liu

Department of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China.

Accepted 27 May, 2011

In this paper, we proposed a dimension reduction, based on the direction of arrival and estimation approach, using an array of spatially distributed electric and magnetic component sensors, and the array comprises subarrays that are well calibrated individually but not with each other. To verify our proposed method, computational complexity of the proposed method was analyzed and compared with existing method, it was shown that the new dimension reduction based method reduced the computational loads significantly. Furthermore, we discussed DOA identifiability issues for such arrays and derived a relevant Cramer-Rao bound. Computer simulations were conducted to illustrate the effectivity of the proposed estimator.

Key words: Direction of arrival estimation, dimension reduction, distributed electromagnetic component sensor array.

INTRODUCTION

Most existing direction of arrival (DOA) estimation methods employ scalar sensor arrays in which the output of each sensor is a scalar corresponding to, for example, the pressure in the acoustic case or a scalar function of the electric filed in the electromagnetic case. Whereas it is well known that the spatial electromagnetic signal is a vector signal, complete information of electromagnetic field is a six-dimensional complex vector; and the main advantage of the vector sensor array is that it makes use of all available electromagnetic information and should outperform the scalar sensor array in accuracy of DOA estimation. So the problem of estimating electromagnetic wave parameters using antenna arrays with diversely polarized elements is important in many applications. In addition to DOA (Nehorai and Paldi, 1994), using the polarization information in waveforms about targets can improve the performance of active sensing systems such as radar (Sowelam and Tewfik, 2000; Hurtado et al., 2009) and increase the capacity of communication systems. In radar, polarimetric scattering information is useful for discriminating the targets' features such as geometrical structure, shape, orientation and so on. Besides, we know the problem of DOA estimation using partly calibrated sensor arrays is important in several practical applications, such as the situation where subarray-based sparse arrays are used (Zoltowski and Wong, 2000; Pesavento et al., 2001). In this condition, the aperture of the whole array is much larger than the one of each subarray. Therefore, each subarray can be assumed to be well calibrated but the calibration of the whole array may be poor due to completely unknown or imprecisely known inter-subarray displacements, imperfect time synchronization of different subarrays, unknown channel mismatches between some subarrays that are located far away from each other, or a combination of the above mentioned effects. The conventional subspace algorithm such as multiple signal classification (MUSIC) cannot be used in the above mentioned situations as they are very sensitive even to very small array manifold model errors (Friedlander, 1990). Generally, full offline calibration of the whole array is an existing solution to the aforementioned problem, which may be an extremely complicated task (Porat and Friedlander, 1997). Several self-calibration of solutions

*Corresponding author. E-mail: chenhui123@uestc.edu.cn.

(Rockah and Schultheiss, 1987; Friedlander and Weiss, 1991; Viberg and Swindlehurst, 1994; Ng and See, 1996; Flanagan and Bell, 2000) have also been proposed to enable joint calibration of the array and estimation of the source DOAs, but their computational complexity are very high, and performance will be severely degenerated in the case of large sensor position errors (Flanagan and Bell, 2000). Besides, these methods cannot be employed when the time synchronization of subarrays is imperfect or the channel mismatches are unknown.

A particular solution to the problem of DOA estimation in partly calibrated arrays has been recently proposed (Pesavento et al., 2001, 2002). This method is referred to as the rank reduction estimator (RARE) and developed solely for the specific case of unknown inter-subarray displacements. It enjoys simple implementation and desired performance which is close to the corresponding Cramer-Rao bound (CRB). Whereas practical applications of these approaches are restricted by the array geometry requirement that each subarray must belong to the particular class of identically oriented linear array with inter-element spacing that are integer multiples of a certain shortest baseline. To overcome these restrictions, the authors (See and Gershman, 2004) developed a general signal model that is applicable to subarray-based partly calibrated sensor arrays whose subarray geometry is arbitrary, and propose a subspace-based approach to DOA estimation in partially calibrated arrays, that can be viewed as a spectral search-based extension of the original root-RARE algorithm of Pesavento et al. (2001, 2002). However, this MUSIC-like algorithm called G-RARE only suits for scalar arrays. To achieve better resolution and accuracy performance by using large aperture, See and Nehorai (2003) extended the application of G-RARE from a scalar sensor array to distributed electromagnetic component sensor array (DEMCA), whereas it suffers the problem of high computation complexity.

Recently, some new position estimation methods (Chen et al., 2008; Chen et al., 2010) for wireless sensor network will be used to further improve the estimation performance in our future study. Here, we proposed a dimension reduction based subspace approach to achieve better estimation performance of multiple sources using partially calibrated DEMCAs. Spectral cost function was addressed using subspace orthogonal firstly. Then, we introduced a simplified 2-dimension spectral cost function by combining multidimensional parameter, which reduced the computational loads significantly without degenerating performance. Moreover, to verify the performance of our proposed method, CRB was computed and the existing method was compared with ours, simulation results illustrated the validity of the proposed method finally.

MEASUREMENT MODEL

The measurement model of a compact 6-dimension electromagnetic vector sensor (EMVS) is given by Nehorai and Paldi (1994),

$$\begin{bmatrix} \mathbf{y}_E(t) \\ \mathbf{y}_H(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_3 \\ \mathbf{U} \end{bmatrix} \mathbf{V}\,\mathbf{Q}\,\mathbf{w}\,s(t) + \begin{bmatrix} \mathbf{e}_E(t) \\ \mathbf{e}_H(t) \end{bmatrix}, \tag{1}$$

where $\boldsymbol{I}_3$ is the third order identity matrix, and

$$\mathbf{U} \triangleq \begin{bmatrix} 0 & -\mathrm{u}_3 & \mathrm{u}_2 \\ \mathrm{u}_3 & 0 & -\mathrm{u}_1 \\ -\mathrm{u}_2 & \mathrm{u}_1 & 0 \end{bmatrix}. \tag{2}$$

The vector $\mathbf{u} = [\mathrm{u}_1, \mathrm{u}_2, \mathrm{u}_3] = [\cos\theta\cos\phi,\ \sin\theta\cos\phi,\ \sin\phi]^T$ indicates the unit direction vector from sensor to source, the matrix $\mathbf{V}$ is given by

$$\mathbf{V} = \begin{bmatrix} -\sin\theta & -\cos\theta\sin\phi \\ \cos\theta & -\sin\theta\sin\phi \\ 0 & \cos\phi \end{bmatrix}, \tag{3}$$

the wave polarization is defined by

$$\mathbf{Q} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \tag{4}$$

And

$$\mathbf{w} = \begin{bmatrix} \cos\beta \\ j\sin\beta \end{bmatrix}. \tag{5}$$

Where θ, ϕ, α and β are the azimuth, elevation, polarized ellipse's orientation and eccentricity angles respectively, $\mathbf{e}_E(t)$ and $\mathbf{e}_H(t)$ are the noise component of electric and magnetic fields respectively.

Assuming that the signal sources are narrowband, the measurement model of a DEMCA in multiple sources environment becomes (See and Nehorai, 1999)

$$\begin{bmatrix} \mathbf{y}_E(t) \\ \mathbf{y}_H(t) \end{bmatrix} = \sum_{k=1}^{K} \mathbf{a}(\boldsymbol{\theta}^{(k)}) s_k(t) + \begin{bmatrix} \mathbf{e}_E(t) \\ \mathbf{e}_H(t) \end{bmatrix}, \tag{6}$$

$$\mathbf{a}(\boldsymbol{\theta}^{(k)}) = \boldsymbol{\Gamma}(\theta^k, \phi^k)\boldsymbol{\Omega}\begin{bmatrix} \boldsymbol{I}_3 \\ \mathbf{U}_k \end{bmatrix}\mathbf{V}_k\mathbf{Q}_k\mathbf{w}_k = \mathbf{b}(\theta^k, \phi^k)\mathbf{w}_k, \tag{7}$$

where $\boldsymbol{\theta}^{(k)} = (\theta^k, \phi^k, \alpha^{(k)}, \beta^{(k)})$ denotes the direction and polarization information of the k-th source signal. $\boldsymbol{\Gamma}(\theta, \phi)$ is an $N \times N$ (N is the number of component sensors) diagonal matrix whose n-th diagonal entry is given by $\left[\boldsymbol{\Gamma}(\theta, \phi)\right]_{nn} = e^{j2\pi \mathbf{q}_n^T \mathbf{u}/\lambda}$, for $n = 1, ..., N$. This matrix provides the phase shift between the

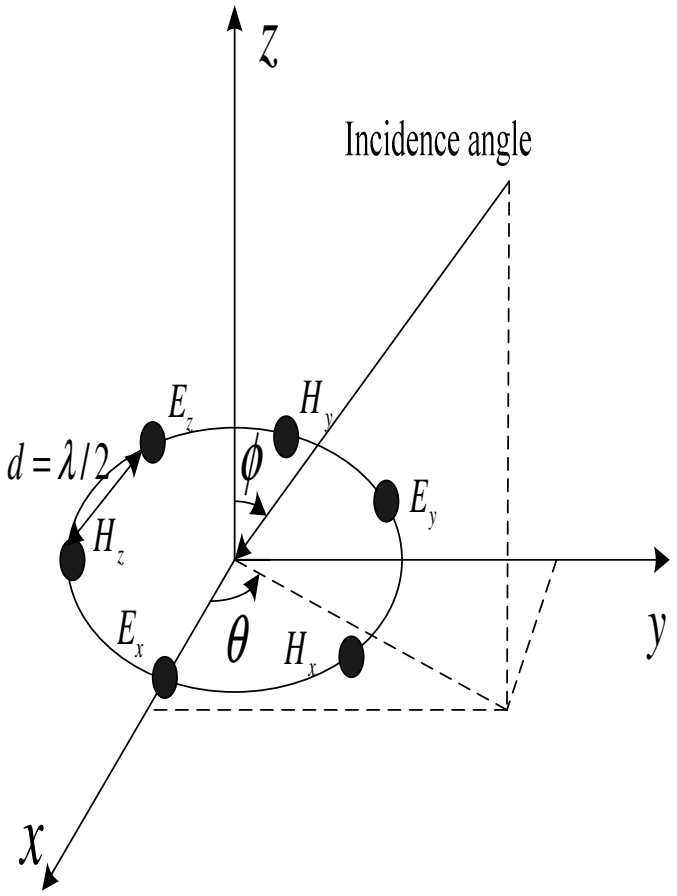

Figure 1. Distributed electromagnetic component sensor array.

vector sensor centre and position $\boldsymbol{q}_n$ of the n-th element of the vector sensor. $\boldsymbol{\Omega}$ is an $N\times 6$ selection matrix with elements of 1 or 0 to pick out a choice from 6 components of the electromagnetic vector sensor. For example, $\boldsymbol{\Omega}=\boldsymbol{I}_6$ when DEMCA is placed as shown in Figure 1.

Unlike the compact EMVS, DEMCA generalizes the vector sensor arrays and allows the differential delay measurements resulting from diverse placement of the component sensors and electromagnetic field measurements to be jointly exploited in estimating the source parameters. Given both the complete electromagnetic and spatial information, better parameter estimation with a smaller aperture array can be expected over a wide frequency range as compared to either a vector sensor or a scalar array. In the following, the operators $(\cdot)^H$, $(\cdot)^T$ and $\mathrm{E}\{\cdot\}$ denote the Hermitian transpose, transpose, and statistical expectation respectively. The symbol diag $\{z_1, z_2\}$ represents a diagonal matrix with diagonal entries z_1, z_2 and blkdiag $\{\mathbf{Z}_1, \mathbf{Z}_2\}$ represents a block diagonal matrix with diagonal entries $\mathbf{Z}_1$, $\mathbf{Z}_2$. Assume that an array is composed of L DEMCA subarrays, the l-th DEMCA subarray is denoted as

$$\mathbf{a}_l(\boldsymbol{\theta}) = P_l(\theta,\phi)\cdot\mathbf{a}(\boldsymbol{\theta}), \tag{8}$$

where $P_l(\theta,\phi)=e^{j2\pi \boldsymbol{p}_l^T \mathbf{u}/\lambda}$ represents the phase of the planewave arriving at the l-th DEMCA subarray at the position $\boldsymbol{p}_l$ $(l=1,...,L)$. Then, the steering vector of the array comprising L partially calibrated DEMCAs is given by

$$\mathbf{a}_F(\boldsymbol{\theta}^{(k)}) = \boldsymbol{D}(\boldsymbol{\theta}^k,\phi^k)\boldsymbol{F}(\mathbf{Q}_k\mathbf{w}_k)\mathbf{h}^{(k)}, \tag{9}$$

where

$$\boldsymbol{D}(\boldsymbol{\theta}^k,\phi^k) = \mathrm{blkdiag}(\mathbf{c}_1(\boldsymbol{\theta}^k,\phi^k),\mathbf{c}_2(\boldsymbol{\theta}^k,\phi^k),...,\mathbf{c}_L(\boldsymbol{\theta}^k,\phi^k)),$$

$$\mathbf{c}_i(\boldsymbol{\theta}^k,\phi^k) = P_i(\boldsymbol{\theta}^k,\phi^k)\boldsymbol{\Gamma}(\boldsymbol{\theta}^k,\phi^k)\boldsymbol{\Omega}\begin{bmatrix}\boldsymbol{I}_3\\ \mathbf{U}_k\end{bmatrix}\mathbf{V}_k,$$

$$\boldsymbol{F}(\mathbf{Q}_k\mathbf{w}_k) = \mathrm{blkdiag}\underbrace{(\mathbf{Q}_k\mathbf{w}_k,\mathbf{Q}_k\mathbf{w}_k,....,\mathbf{Q}_k\mathbf{w}_k)}_{L\text{ terms in all}},$$

$\mathbf{h}^{(k)}=[h_1^{(k)},h_2^{(k)},...,h_L^{(k)}]^T$, h_l denotes the calibration errors due to inter subarray displacement error, receiver channel mismatch and sampling offsets among the subarrays. Then, we can model array snapshot compactly in matrix form with $\mathbf{a}_F(\boldsymbol{\theta})$,

$$\mathbf{Y}(t)=\mathbf{A}(\boldsymbol{\Theta})\mathbf{S}(t)+\mathbf{e}(t),\quad t=1,...,T \tag{10}$$

where $\mathbf{A}(\Theta)=[\mathbf{a}_F(\boldsymbol{\theta}^{(1)}),\mathbf{a}_F(\boldsymbol{\theta}^{(2)}),...,\mathbf{a}_F(\boldsymbol{\theta}^{(K)})]$, $\boldsymbol{\Theta}=[\boldsymbol{\theta}^{(1)},\boldsymbol{\theta}^{(2)},...,\boldsymbol{\theta}^{(K)}]$, $\mathbf{S}(t)=[\boldsymbol{s}_1(t),\ \boldsymbol{s}_2(t),...,\ \boldsymbol{s}_K(t)]$. And we make the following commonly used assumptions on the model (10):

A: The source signal sequence $\{s(1),\ s(2),...,\ s(T)\}$ is a sample from a temporally uncorrelated stationary (complex) Gaussian process with zero mean and $\mathrm{E}\{s(m)s^H(n)\}=R_s\delta_{m,n}$, $\mathrm{E}\{s(m)s^T(n)\}=0$ (for all m and n), where $\delta_{m,n}$ is the Kronecker delta.

B: The noise $\mathbf{e}(t)$ is (complex) Gaussian distributed with zero mean and $\mathrm{E}\{\mathbf{e}(m)\mathbf{e}^H(n)\}=\sigma^2\boldsymbol{I}\delta_{m,n}$, (for all m and n).

It is also assumed that the entries of $\boldsymbol{s}(t)$ and $\mathbf{e}(t)$ are independent with each other. With $\mathbf{Y}(t)$, the sample estimate of the array covariance matrix

$$\boldsymbol{R}\triangleq\mathrm{E}\{\mathbf{Y}(t)\mathbf{Y}^H(t)\}=\mathbf{A}(\Theta)\boldsymbol{R}_s\mathbf{A}(\Theta)^H+\sigma^2\boldsymbol{I} \tag{11}$$

is given by

$$\hat{\boldsymbol{R}}=\frac{1}{T}\sum_{t=1}^{T}\mathbf{Y}(t)\mathbf{Y}^H(t) \tag{12}$$

Where $\boldsymbol{R}_s\triangleq\mathrm{E}\{\mathbf{S}(t)\mathbf{S}^H(t)\}$ is the source covariance matrix, σ^2 is the identical variance in each sensor. When $K<6L$, the eigen-decomposition of the matrices (11) and (12) can be expressed in the form

$$\boldsymbol{R}=\boldsymbol{E}_s\boldsymbol{\Lambda}_s\boldsymbol{E}_s^H+\boldsymbol{E}_N\boldsymbol{\Lambda}_N\boldsymbol{E}_N^H, \tag{13}$$

$$\hat{\boldsymbol{R}}=\hat{\boldsymbol{E}}_s\hat{\boldsymbol{\Lambda}}_s\hat{\boldsymbol{E}}_s^H+\hat{\boldsymbol{E}}_N\hat{\boldsymbol{\Lambda}}_N\hat{\boldsymbol{E}}_N^H. \tag{14}$$

where the $K\times K$ diagonal matrices $\boldsymbol{\Lambda}_s$ and $\hat{\boldsymbol{\Lambda}}_s$ contain the K signal subspace eigenvalues of $\boldsymbol{R}$ and $\hat{\boldsymbol{R}}$ respectively, and the $(6L-K)\times(6L-K)$ diagonal matrices $\boldsymbol{\Lambda}_N$ and $\hat{\boldsymbol{\Lambda}}_N$ contain the $6L-K$ noise subspace eigenvalues of $\boldsymbol{R}$ and $\hat{\boldsymbol{R}}$ respectively. In turn, $\boldsymbol{E}_s$ and $\hat{\boldsymbol{E}}_s$ are $6L\times K$ matrices whose columns are the signal subspace eigenvectors corresponding to the K largest eigenvalues of $\boldsymbol{R}$ and $\hat{\boldsymbol{R}}$ respectively, while $\boldsymbol{E}_N$ and $\hat{\boldsymbol{E}}_N$ are $6L\times(6L-K)$ matrices whose columns are the noise subspace eigenvectors corresponding to the $6L-K$ smallest eigenvalues of $\boldsymbol{R}$ and $\hat{\boldsymbol{R}}$ respectively.

DOA ESTIMATION

Now, we consider the well known spectral MUSIC algorithm which estimates the signal DOAs from the K highest peaks of the following function (Zoltowski and Wong, 2000).

$$f(\boldsymbol{\theta})=\frac{1}{\mathbf{a}_F^H(\boldsymbol{\theta})\boldsymbol{E}_N\boldsymbol{E}_N^H\mathbf{a}_F(\boldsymbol{\theta})}. \tag{15}$$

In the ideal case of exactly known $\boldsymbol{R}$, the DOAs can be found from the equation

$$\mathbf{a}_F^H(\boldsymbol{\theta})\boldsymbol{E}_N\boldsymbol{E}_N^H\mathbf{a}_F(\boldsymbol{\theta})=0. \tag{16}$$

To find the K highest peaks of (15), we have to use an exhaustive multidimensional search with respect to θ, ϕ, α and β, which becomes totally impractical. To overcome this problem, inserting (9) into (15), we can rewrite the equation (15) as

$$\begin{aligned}f(\boldsymbol{\theta})&=\frac{1}{\left\|\mathbf{a}_F^H(\boldsymbol{\theta})\boldsymbol{E}_N\right\|^2}=\frac{1}{\left\|\boldsymbol{E}_N^H\mathbf{a}_F(\boldsymbol{\theta})\right\|^2}=\frac{1}{\left\|\boldsymbol{E}_N^H\boldsymbol{D}(\theta,\phi)\boldsymbol{F}(\mathbf{Q}\mathbf{w})\mathbf{h}\right\|^2}\\&=\frac{1}{[\boldsymbol{F}(\mathbf{Q}\mathbf{w})\mathbf{h}]^H\boldsymbol{D}^H(\theta,\phi)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{D}(\theta,\phi)[\boldsymbol{F}(\mathbf{Q}\mathbf{w})\mathbf{h}]}\\&=\frac{1}{\boldsymbol{g}^H\boldsymbol{C}\,\boldsymbol{g}}.\end{aligned} \tag{17}$$

Where $\boldsymbol{g}=\boldsymbol{F}(\mathbf{Q}\mathbf{w})\mathbf{h}$ and $\boldsymbol{C}=\boldsymbol{D}^H(\theta,\phi)\boldsymbol{E}_N\boldsymbol{E}_N^H\boldsymbol{D}(\theta,\phi)$. From (17), it is obviously shown that the polarization information and error information are only contained in vector $\boldsymbol{g}$, so the matrix $\boldsymbol{C}$ is independent of polarization and error information. Besides, the sample matrix $\hat{\boldsymbol{C}}=\boldsymbol{D}^H(\theta,\phi)\hat{\boldsymbol{E}}_N\hat{\boldsymbol{E}}_N^H\boldsymbol{D}(\theta,\phi)$ is used instead of $\boldsymbol{C}$ in practice. Therefore, the signal DOAs can be found from the K highest peaks of the following spectral function:

$$f_1(\boldsymbol{\theta})=\frac{1}{\lambda_{\min}\{\hat{\boldsymbol{C}}\}}, \tag{18}$$

where $\lambda_{\min}\{\cdot\}$ is the operator that returns the smallest eigenvalue of a Hermitian matrix.

We can still analyze this problem in aspect of reduction of rank (See and Gershman, 2004), in general, matrix $\hat{\boldsymbol{C}}$ is full rank if

$$L\le 6L-K. \tag{19}$$

Because the column rank of $\hat{\boldsymbol{C}}$ is not less than L, Therefore, (16) can be true only if the matrix $\hat{\boldsymbol{C}}$ drops rank so that

$$\mathrm{rank}(\hat{\boldsymbol{C}})<L \tag{20}$$

So the determinant of $\hat{\boldsymbol{C}}$ as well as its smallest eigenvalue will tend to have a minimum when $\boldsymbol{\theta}$ coincides with one of the signal directions $\{\boldsymbol{\theta}_l\}_{k=1}^K$. Therefore, another alternative spectral function can be used

$$f_2(\boldsymbol{\theta})=\frac{1}{\det\{\hat{\boldsymbol{C}}\}}. \tag{21}$$

UNIQUENESS AND IDENTIFIABILITY

Here, we show the uniqueness of the signal DOA estimates obtained

from the rank reduction criterion (18) in the case $T \to \infty$ and the identifiability issues. Define a parameter γ_l first, if the manifold of the l -th subarray is unambiguous, $\gamma_l = 1$, otherwise, $\gamma_l = 0$. According to See and Gershman (2004), if the following condition is satisfied:

$$K < \sum_{l=1}^{L} \gamma_l (M_l - 1) \quad , \tag{22}$$

where M_l is the sensor number of the l -th subarray, and the true parameter $\mathbf{h}$ is fixed in (16). Then the signal DOAs $\{(\theta_k, \phi_k)\}_{k=1}^{K}$ is the whole set of possible solution to (16). Moreover, if $\mathrm{rank}(\hat{\boldsymbol{C}}) < L$, (16) holds true. Obviously our proposed method satisfies all of the conditions mentioned previously.

CRAMER RAO BOUND

Here, we derive the stochastic CRB of direction finding problem in subarray-based partly calibrated arrays. Besides, this bound regards the inter-subarray parameters to be unknown. To derive the CRB expressions conveniently, we rewrite the steering vector of the array comprising *L* partially calibrated DEMCAs as follows (See and Nehorai, 2003),

$$\mathbf{a}_F(\boldsymbol{\theta}^{(k)}) = \boldsymbol{G}(\theta^k, \phi^k, \mathbf{w}_k)\mathbf{h}^{(k)} \quad , \tag{23}$$

Where

$$\boldsymbol{G}(\theta^k, \phi^k, \mathbf{w}_k) = \begin{bmatrix} \mathbf{a}_1(\boldsymbol{\theta}^{(k)}) & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{a}_2(\boldsymbol{\theta}^{(k)}) & 0 & \cdots & 0 \\ 0 & 0 & \ddots & 0 & \vdots \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \mathbf{a}_L(\boldsymbol{\theta}^{(k)}) \end{bmatrix} .$$

Alternatively, the expression (23) can also be rewritten as (See and Nehorai, 2003)

$$\mathbf{a}_F(\boldsymbol{\theta}^{(k)}) = \boldsymbol{\Phi}(\theta^k, \phi^k, \mathbf{h}^{(k)})\mathbf{w}_k \quad , \tag{24}$$

where

$$\boldsymbol{\Phi}(\theta^k, \phi^k, \mathbf{h}^{(k)}) = \begin{bmatrix} P_1(\theta^k, \phi^k)\mathbf{b}_1(\theta^k, \phi^k)h_1^{(k)} \\ P_2(\theta^k, \phi^k)\mathbf{b}_2(\theta^k, \phi^k)h_2^{(k)} \\ \vdots \\ P_L(\theta^k, \phi^k)\mathbf{b}_L(\theta^k, \phi^k)h_L^{(k)} \end{bmatrix} .$$

Here, using (23), the snapshot model (10) can be rewritten as

$$\mathbf{Y}(t) = \sum_{k=1}^{K} \boldsymbol{G}(\theta^{(k)}, \phi^{(k)}, \mathbf{w}^{(k)})\mathbf{h}^{(k)} \boldsymbol{s}_k(t) + \mathbf{e}(t), \quad t = 1, ..., T \quad . \tag{25}$$

From (25), it is clear that each vector $\mathbf{h}^{(k)}$ is identifiable up to a scaling constant. To avoid scaling ambiguity in the computation of the CRB, we fix the first elements of the vectors $\mathbf{h}^{(k)}$ $(k = 1, \ldots, K)$, that is, assume that they are known. Let us introduce the following $2KL \times 1$ vector:

$$\boldsymbol{\eta} \triangleq [\boldsymbol{\theta}^T, \boldsymbol{\xi}_2^T, \ldots, \boldsymbol{\xi}_L^T, \boldsymbol{\varsigma}_2^T, \ldots, \boldsymbol{\varsigma}_L^T]^T \quad , \tag{26}$$

where

$$\boldsymbol{\xi}_l \triangleq [\mathrm{Re}\{h_{1,l}\}, \ldots, \mathrm{Re}\{h_{K,l}\}]^T , \boldsymbol{\varsigma}_l \triangleq [\mathrm{Im}\{h_{1,l}\}, \ldots, \mathrm{Im}\{h_{K,l}\}]^T .$$

Let the snapshots satisfy the following stochastic model

$$\boldsymbol{s}(t) \sim \mathrm{N}\{0, \boldsymbol{R}_s\} \quad , \tag{27}$$

The unknown parameters of the problem include the elements of the vector η , the noise variance σ^2 , and the parameters of the source covariance matrix $\boldsymbol{R}_s$. Concentrating the problem with respect to the parameters of the source covariance matrix and the noise variance, we have the following equation for the entries of the $2KL \times 2KL$ CRB matrix (Nehorai and Paldi, 1994),

$$[\mathrm{CRB}^{-1}(\boldsymbol{\eta})]_{kl} = \frac{2N}{\sigma^2} \mathrm{Re}\{\mathrm{tr}(\boldsymbol{\Sigma} \frac{\partial \mathbf{A}^H(\boldsymbol{\Theta})}{\partial \boldsymbol{\eta}_l} \Pi_C \frac{\partial \mathbf{A}(\boldsymbol{\Theta})}{\partial \boldsymbol{\eta}_k})\}$$
$$(k = 1, ..., K;\ l = 1, ..., L) \quad , \tag{28}$$

where $\boldsymbol{\Sigma} = \boldsymbol{R}_s(\mathbf{A}^H(\boldsymbol{\Theta})\mathbf{A}(\boldsymbol{\Theta})\boldsymbol{R}_s + \sigma^2 \boldsymbol{I})^{-1}\mathbf{A}^H(\boldsymbol{\Theta})\mathbf{A}(\boldsymbol{\Theta})\boldsymbol{R}_s$ is the $K \times K$ matrix, tr(·) denotes matrix trace operator, and $\Pi_C = \boldsymbol{I} - \mathbf{A}(\boldsymbol{\Theta})(\mathbf{A}^H(\boldsymbol{\Theta})\mathbf{A}(\boldsymbol{\Theta}))^{-1}\mathbf{A}^H(\boldsymbol{\Theta})$ is the $6L \times 6L$ orthogonal projection matrix. Here, for notational simplicity, the explicit dependence on $\boldsymbol{\Theta}$ will be occasionally omitted. Thus,

$$\frac{\partial \mathbf{A}}{\partial \theta^k} = \underbrace{\left[\mathbf{0} \cdots \frac{\partial (\boldsymbol{G}(\theta^k, \phi^k, \mathbf{w}_k)\mathbf{h}_k)}{\partial \theta^k} \cdots \mathbf{0}\right]}_{6L \times K}$$

$$\frac{\partial \mathbf{A}}{\partial \phi^k} = \underbrace{\left[\mathbf{0} \cdots \frac{\partial (\boldsymbol{G}(\theta^k, \phi^k, \mathbf{w}_k)\mathbf{h}_k)}{\partial \phi^k} \cdots \mathbf{0}\right]}_{6L \times K} ,$$

$$\frac{\partial \mathbf{A}}{\partial \xi_k} = \underbrace{\left[\mathbf{0} \cdots \boldsymbol{G}(\theta^i, \phi^i, \mathbf{w}_i)\boldsymbol{e}_k \cdots \mathbf{0}\right]}_{6L \times K} , \qquad \frac{\partial \mathbf{A}}{\partial \zeta_k} = j\frac{\partial \mathbf{A}}{\partial \xi_k}$$

$(\text{for } i = 1, \ldots, K)$, and $\boldsymbol{e}_k$ is the $K \times 1$ vector with one in the k-th position and zeros elsewhere. Then we can obtain the CRB matrix using (28) easily.

SIMULATION RESULTS AND DISCUSSION

Here, we presented several experimental results for our proposed scheme using a partially calibrated array

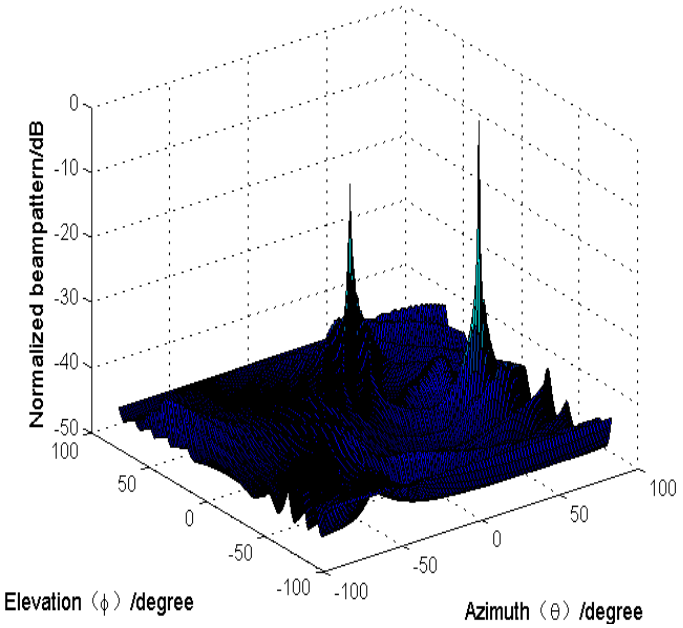

Figure 2. Normalized beam pattern using our proposed approach.

composed of multiple DEMCA subarrays. First we showed our simulation results by plotting spatial spectra of signal sources, and then we compared the performance obtained by our approach with that of the existing method (See and Nehorai, 2003) and CRB under different signal noise ratio (SNR). In order to satisfy the assumptions for the CRB, we choose an operating point where our estimate is unbiased. Throughout our simulations, we assume two uncorrelated sources impinging on the partially calibrated array of two DEMCA subarrays from the DOAs $\mathbf{\Theta}^{(1)} = [20^\circ, 40^\circ, 45^\circ, -5^\circ]$ and $\mathbf{\Theta}^{(2)} = [50^\circ, -30^\circ, 60^\circ, -60^\circ]$. Each subarray is made of x, y and z electric and magnetic component sensors arranged as a uniformly spaced circular array of half wavelength intersensor spacing. The second subarray is displaced from the first array by $[5\lambda, 5\lambda, 0]$, where λ is the signal's wavelength. All results are averaged over 1000 Monte Carlo simulations, and $T = 100$ snapshots are taken in each simulation example. For simplicity, we assume that all signal sources are of equal power σ_S^2, and input SNR is defined as $10\log_{10}(\sigma_S^2/\sigma_n^2)$.

In the first simulation, the SNR is fixed at 30 dB and 100 independent snapshots are used to estimate the array covariance matrix. As shown in Figures 2 and 3, our proposed method was able to resolve and estimate the DOAs of the signals as the MUSIC-like algorithm proposed by See and Nehorai (2003). In particular, our proposed method determined the DOA accurately as indicted by the contour plot in Figure 2, whereas the proposed MUSIC-like algorithm uses iterative manner to find $\hat{\mathbf{w}}$ and $\hat{\mathbf{h}}$ that minimize the cost function

$$J(\theta,\phi) = \mathbf{w}^H \mathbf{\Phi}^H(\theta,\phi,\mathbf{h}) \hat{E}_N \hat{E}_N^H \mathbf{\Phi}(\theta,\phi,\mathbf{h}) \mathbf{w} \quad (29)$$

$$= \mathbf{h}^H \mathbf{G}^H(\theta,\phi,\mathbf{w}) \hat{E}_N \hat{E}_N^H \mathbf{G}(\theta,\phi,\mathbf{w}) \mathbf{h} \quad . \quad (30)$$

Moreover, it requires two eigenvalue decompositions in each iteration, whereas our proposed method only need one eigen value decomposition to estimate DOAs of the signals, so the computational complexity of the MUSIC-like algorithm (See and Nehorai, 2003) is much higher than our proposed method, especially for many DEMCA subarrays. To compare the performance obtained by our approach with CRB under different SNR, we defined the root mean square error (RMSE) of the DOA estimates from 1000 Monte Carlo trials as

$$\mathrm{RMSE} = \sqrt{\sum_{n=1}^{1000}\sum_{k=1}^{N_s}((\hat{\theta}_k(n)-\theta_k)^2 + (\hat{\phi}_k(n)-\phi_k)^2) \Big/ (1000 N_s)} \quad , \quad (31)$$

where $\hat{\theta}_k(n)$, $\hat{\phi}_k(n)$ are the estimations of θ_k and ϕ_k for the n-th Monte Carlo trial respectively , and N_s is the number of all the signals.

In the second simulation, we considered the same scenario mentioned as the first simulation. Figure 4 showed the RMSE of the DOA estimates versus input SNR, and the simulation results illustrated that the proposed spectral estimators outperformed the MUSIC-like algorithm, especially for low SNR case. And the RMSE estimators of our signals, except low SNR, can

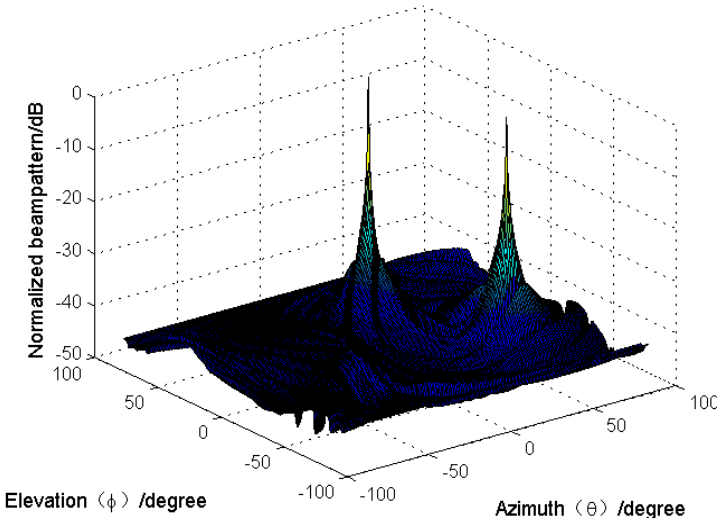

Figure 3. Normalized beam pattern with the method proposed in Sowelam and Tewfik (2000).

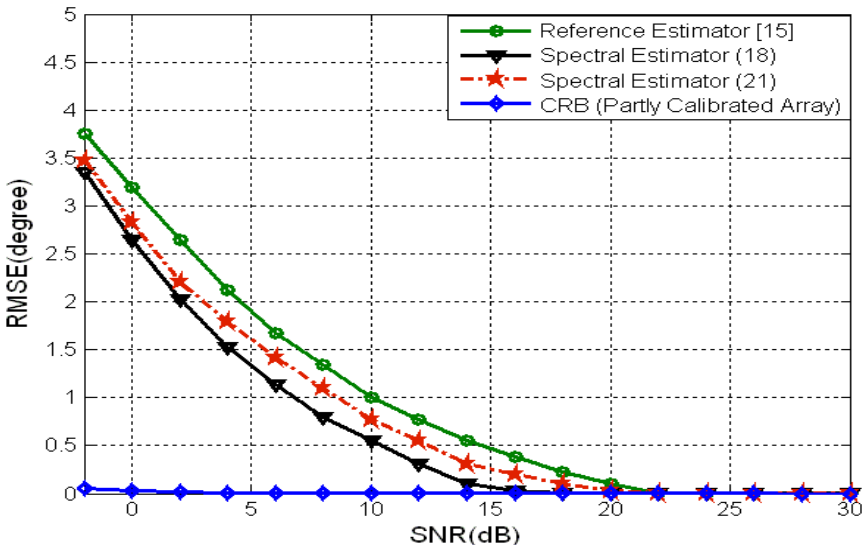

Figure 4. RMSE of the DOA estimates versus input SNR.

meet CRB. From all our examples, it showed that the proposed spectral estimators can achieve high estimation performance of multiple sources, using a partially calibrated array composed of multiple DEMCA subarrays. Moreover, low computational complexity was another advantage of the proposed method against the MUSIC-like algorithm (See and Nehorai, 2003).

CONCLUSION

In this paper, the problem of DOA estimations of electromagnetic sources using a partially calibrated array comprising multiple DEMCA subarrays was addressed under the assumption that all subarrays were well calibrated, but there were imperfections between different subarrays. An efficient dimension reduction based DOA estimation approach was proposed, which was suit for partially calibrated polarized array and array imperfection. We derived the CRB of the partially calibrated array, and presented numerical examples with combinative effect of various types of imperfections.

Finally, the simulation results illustrated the proposed method had significant accuracy and low computational complexity advantages over the existing method (See and Nehorai, 2003).

ACKNOWLEDGEMENTS

This work was supported in part by the National Natural Science Foundation of China under grant 60772146, the National High Technology Research and Development Program of China (863 Program) under grant 2008AA12Z306, the Key Project of Chinese Ministry of Education under grant 109139, and in part by the Special Fund of Central Colleges Basic Scientific Research Operating Expenses under grant E022050205.

REFERENCES

Flanagan BP, Bell KL (2000). Improved array self calibration with large sensor position errors for closely spaced sources. In Proc. Sensor Array Multichannel Signal Process. Workshop, pp. 484-488.

Friedlander B, Weiss AJ (1991). Self-calibration in high-resolution array process.in Advances in Spectrum Estimation and Array Process, S. Haykin, Ed. Englewood Cliffs: Prentice-Hall, Vol. II.

Friedlander B (1990). A sensitivity analysis of the MUSIC algorithm. IEEE Trans. Acoust Speech Signal Process, 38: 1740–1751.

Hurtado M, Xiao JJ, Nehorai A (2009). Target estimation, detection, and tracking: a look at adaptive polarimetric design. IEEE Signal Proc. Mag., 26: 42-52.

Nehorai A, Paldi E (1994). Vector sensor array proc. for electromagnetic source localization. IEEE Trans. Signal Process, 42: 376-398.

Ng BC, See CMS (1996). Sensor-array calibration using a maximum-likelihood approach. IEEE Trans. Antennas Propagat. 44: 827–835.

Pesavento M, Gershman AB, Wong KM (2001). Direction of arrival estimation in partly calibrated time-varying sensor arrays. In Proc. ICASSP. 5: 3005-3008.

Pesavento M, Gershman AB, Wong KM (2002). Direction finding in partly-calibrated sensor arrays composed of multiple subarrays. IEEE Trans. Signal Process, 50: 2103-2115.

Porat B, Friedlander B (1997). Accuracy requirements in off-line array calibration. IEEE Trans. Aerosp. Electron. Syst.., 33: 545–556.

Rockah Y, Schultheiss PM (1987). Array shape calibration using sources in unknown locations-Part 1: Far-field sources. IEEE Trans. Acoust., Speech, Signal Process, 35: 286-299.

See CMS, Gershman AB (2004). Direction-of-arrival estimation in partly calibrated subarray-based sensor arrays. IEEE Trans. Signal Process, 52: 329–338.

See CMS, Nehorai A (2003). Source localization with partially calibrated distributed electromagnetic component sensor array. IEEE Workshop on Statistical Signal Process, pp. 458-461.

See CMS, Nehorai A (1999). Distributed electromagnetic component sensor array. 7th Annual Adaptive Sensor Array Processing Workshop.

Sowelam SM, Tewfik AH (2000). Waveform selection in radar target classification. IEEE Trans. Inform. Theory, 46: 1014-1029.

Viberg M, Swindlehurst AL (1994). A Bayesian approach to auto-calibration for parametric array processing. IEEE Trans. Signal Process, 42: 3495–3507.

Zoltowski MD, Wong KT (2000). Closed-form eigenstructure-based direction finding using arbitrary but identical subarrays on a sparse uniform Cartesian array grid. IEEE Trans. Signal Process, 48: 2205-2210.

Chen H, Sezaki K, Deng P, So HC (2008). An improved DV-Hop localization algorithm with reduced location error for WSNs. IEICE Trans. Fund. E91-A(8): 2232-2236.

Chen H, Shi Q, Tan R, Poor HV, Sezaki K (2010). Mobile element assisted cooperative localization for wireless sensor networks with obstacles. IEEE Trans. Wireless Commun., 9(3): 956-963.