International Journal of the Physical Sciences Vol. 6(25), pp. 6053-6060, 23 October, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.889
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Analysis of entropy generation in a variable viscosity fluid flow between two concentric pipes with a convective cooling at the surface

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Accepted 29 July, 2011

Entropy generation rate in a variable viscosity liquid flowing steadily through two concentric cylindrical pipes with convective cooling at the pipe surface, are studied using the second law analysis. The outer system is assumed to exchange heat with the ambient following Newton's cooling law, and the fluid viscosity model varies as an inverse linear function of temperature. The resulting equations and the boundary conditions are solved using the fourth Order Runge-Kutta scheme and an efficient shooting technique. Numerical expressions for fluid velocity and temperature are derived and utilised to obtain expressions for volumetric entropy generation numbers, irreversibility distribution ratio and the Bejan number in the flow field.

Key words: Pipe flow, variable viscosity, entropy generation, convective cooling, irreversibility analysis.

INTRODUCTION

Heat transfer inside concentric pipes has many significant engineering applications. These ranges from electronic packages, industrial heat exchanges to petroleum industries (Sheng et al., 2010). Furthermore, studies related to viscous fluid with temperature dependent properties are of great importance in industries, such as, food processing, coating and polymer processing industries (Macosko, 1994; Schlichting, 2000). In industrial systems, fluid can be subjected to extreme conditions such as high temperature, pressure and shear rate. External heating, such as, the ambient temperature and high shear rates can lead to a high temperature being generated in the fluid. This may have a significant effect on the fluid properties. Fluid used in industries such as polymer fluids have a viscosity that varies rapidly with temperature and may give rise to strong feedback effects, which can lead to significant changes in the flow structure of the fluid (Sahin, 1999; Tasmin et al., 2002). Due to the strong coupling effect between the Navier-Stokes and energy equations, viscous heating also plays an important role in fluid with strong temperature dependence. Elbashbeshy and Bazid (2000) investigated the effect of temperature dependent viscosity on heat transfer over a moving surface. In their investigation, the fluid viscosity model varies as an inverse linear function of temperature. Costa et al. (2003) applied the temperature dependent viscosity model to study magma flows. The effects of temperature dependent fluid viscosity on heat transfer and thermal stability of reactive flow in a cylindrical pipe with isothermal wall was reported in Makinde (2008).

Thermodynamic irreversibility in any fluid flow process can be quantified through entropy analysis. The first law of thermodynamics is simply an expression of the conservation of energy principle. The second law of thermodynamics states that all real processes are irreversible. Entropy generation is a measure of the account of irreversibility associated with the real processes (Mirzazadeh et al., 2008; Tshehla et al., 2010). When entropy generation takes place, the quality of energy (that is, exergy) decreases (Ibanez et al., 2003; Mikinde, 2008). In order to preserve the quality of energy

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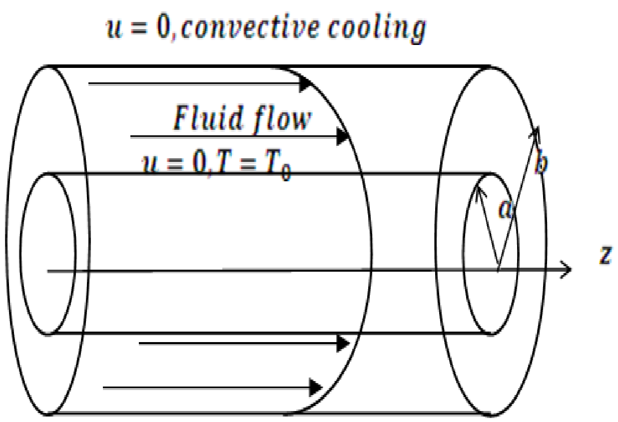

Figure 1. Geometry of the problem.

in a fluid flow process or at least to reduce the entropy generation, it is important to study the distribution of the entropy generation within the fluid volume, (Makinde et al., 2010; Sheng et al., 2010). The optimal design for any thermal system can be achieved by minimizing entropy generation in the systems. Entropy generation in thermal engineering systems destroys available work and thus reduces its efficiency.

The study of entropy generation in conductive and convective heat transfer processes has assumed considerable importance since the pioneering work of Bejan (1995) and his subsequent book on the subject Bejan (1996). Since then numerous papers have studied entropy generation in heat transfer processes (Sahin, 1999; Tasnim et al., 2002; Tshehla et al., 2010). Recently, the thermodynamics second law characteristics for variable viscosity channel flow with convective cooling at the walls are discussed by Makinde et al. (2008). It seems to the authors' knowledge that the effect of convective cooling on the entropy generation rate in a variable viscosity flow through two cylindrical pipes has not been investigated fully. Sheng et al. (2010) investigated a natural convection and entropy generation in a vertical concentric annular space. Mirzazadeh et al. (2008) investigated entropy analysis for non-linear visco-elastic fluid in concentric rotating cylinders. The results show that entropy generation number increases with increasing Brinkman number.

The scarcity of investigations, such as the problem of heat transfer and entropy generation in the flow of a variable viscosity fluid through two concentric pipes with convective cooling has motivated this study. The work in Tshehla et al. (2010) is extended to consider the flow in two concentric pipes.

MATHEMATICAL MODEL

Figure 1 presents a schematic diagram of the fluid flow and heat transfer domains. The radius of the inner cylinder is denoted *a* and the radius of the outer cylinder is denoted *b*. Flow is considered to be steady in the $\bar{z}$ -direction and length *L* under the action of a constant pressure gradient, viscous dissipation, convective cooling at the outer pipe surface. It is assumed that the pipes are long enough to neglect both the entrance and exit effects. The fluid is incompressible and the temperature dependent viscosity ($\bar{\mu}$) can be expressed as (Elbashy et al., 2000; Tasnim et al., 2002):

$$\bar{\mu} = \frac{\mu_0}{1 + m(\bar{T} - T_a)} \qquad 1$$

Where, μ_0 is the fluid dynamic viscosity at the ambient temperature T_a.

Under these conditions the continuity, momentum and energy equations governing the problem in dimensionless form may be written as (Tshehla et al., 2010);

$$\frac{\partial(ru)}{\partial z} + \frac{\partial(rv)}{\partial r} = 0 \qquad 2$$

$$\varepsilon R_e\left(u\frac{\partial u}{\partial z} + v\frac{\partial u}{dr}\right) = -\frac{\partial p}{\partial z} + 2\varepsilon^2\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\mu\left(\frac{\partial u}{\partial r} + \varepsilon^2\frac{\partial v}{\partial z}\right)\right] \qquad 3$$

$$\varepsilon^3 R_e\left(u\frac{\partial v}{\partial z} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial p}{\partial r} + \frac{2\varepsilon^2}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right) + \varepsilon^2\frac{\partial}{\partial z}\left[\mu\left(\frac{\partial u}{\partial r} + \varepsilon^2\frac{\partial v}{\partial z}\right)\right] - 2\mu\varepsilon^2\frac{v}{r^2} \qquad 4$$

$$\varepsilon \operatorname{Re}\operatorname{Pr}\left(u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r}\right) = \varepsilon^2\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \mu\Phi\,, \qquad 5$$

Where,

$$\Phi = \mathrm{Br}\left[2\varepsilon^2\left(\frac{\partial u}{\partial z}\right)^2 + 2\varepsilon^2\left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial r}\right)^2 + 2\varepsilon^2\left(\frac{v}{r}\right)^2 + 2\varepsilon^2\frac{\partial v}{\partial z}\frac{\partial u}{\partial r} + \varepsilon^4\left(\frac{\partial v}{\partial z}\right)^2\right] \qquad 6$$

The following non-dimensional quantities were employed in Equations 2 to 6:

$$r = \frac{\bar{r}}{\varepsilon L}, z = \frac{\bar{z}}{L}, u = \frac{\bar{u}}{U}, v = \frac{\bar{v}}{\varepsilon U}, \varepsilon = \frac{a}{L}, \mu = \frac{\bar{\mu}}{\mu_0}, T = \frac{\bar{T} - T_a}{T_0 - T_a}, P = \frac{a^2\bar{P}}{\mu_0 UL},$$
$$\alpha = m(T_0 - T_a), Br = \frac{\mu_0 U^2}{k(T_0 - T_a)}, \Pr = \frac{\mu_0 c_p}{k}, \operatorname{Re} = \frac{\rho U a}{\mu_0}, Bi = \frac{ah}{k}, \gamma = \frac{b}{a}. \qquad 7$$

Where, ρ is the fluid density, *k* is the thermal conductivity, $\bar{T}$ is the fluid temperature, T_0 is the inner pipe surface temperature, *U* is the velocity scale, α is the viscosity variation parameter, $\bar{u}$ is the axial velocity, $\bar{v}$ is the normal velocity, c_p is the specific heat at constant pressure, $\bar{P}$ is the pressure, *Pr* is the Prandtl number, *Br* is the Brinkman number, *Bi* is the Biot number, *h* is the heat transfer coefficient, *Re* is the Reynolds number, $\bar{x}$ and $\bar{y}$ are distances measured in streamwise and normal direction, respectively. Since the pipes are narrow and the aspect ratio 0<ε<<1, the lubrication approximation based on an asymptotic simplification of the governing Equations 2 – 6, is invoked and we obtain,

$$\frac{du}{dr}=\left(\frac{A}{r}-\frac{rG}{2}\right)(1+\alpha T) \qquad 8$$

$$\frac{dp}{dr}+O\left(\varepsilon^{2}\right)=0 \qquad 9$$

$$\frac{d^{2}T}{dr^{2}}+\frac{1}{r}\frac{dT}{dr}+\frac{Br}{(1+\alpha T)}\left(\frac{du}{dr}\right)^{2}=0 \qquad 10$$

Where $\mu = 1/(1+\alpha T)$ and $G=-\partial P/\partial z$ is the constant axial pressure gradient. The corresponding boundaries conditions at the inner pipe surface and the outer pipe surface are the usual no slip condition for the fluid velocity. However, the outer pipe surface exchanges heat with the ambient, following the Newton's cooling law and we obtain:

$u = 0$, $T = 1$ at $r = 1$ 11

And the regularity of the solution along the pipe centreline, that is,

$u = 0$, $\frac{dT}{dr}=-BiT$, at $r=\gamma$, 12

Where, $\gamma > 1$.

Solution method

The resulting Equations 8 and 10 cannot be solved analytically. These set of equations are combined with the boundary conditions, Equations 11 and 12, must be solved numerically by applying the Runge-Kutta fourth-order integration scheme coupled with shooting iteration technique. This basically involves transforming the governing boundary valued problem into a system of first order differential equations and then integrated numerically. The iterative process continues until the resulting initial valued problem at a given boundary conditions $r = \gamma$ are satisfied.

Entropy analysis

The theory of entropy production goes back to the work of Clausius and Kelvin's (Drost and Zaworski, 1988) on the irreversible aspects of the second law of thermodynamics. Thereafter, several authors have generalised the theory of entropy production to various aspect of engineering systems. However, the entropy production resulting from combined effects of fluid friction and temperature differences has remained untreated by classical thermodynamics, which motivates many researchers to conduct analyses of fundamental and applied engineering problems based on second law analysis. The general equation for the entropy generation per unit volume is given by Bejan (1995, 1996), Makinde et al. (2010), Sahin (1999) and Tasnim et al. (2002):

$$S^{m}=\frac{k}{T_{a}^{2}}\left(\nabla\bar{T}\right)^{2}+\frac{\bar{\mu}}{T_{a}}\Phi \qquad 13$$

The first term in Equation 13, is the irreversibility due to heat transfer and the second term is the entropy generation due to viscous dissipation. Equation 13 can be easily integrated from $\bar{r}=a$ to $\bar{r}=b$ to give the total entropy generated in the pipe flow as follows.

$$S^{T}=\int_{a}^{b}S^{m}2\pi L\bar{r}d\bar{r} \qquad 14$$

Using Equations 13 and 14, the entropy generation number and total entropy generated is expressed in dimensionless form as,

$$Ns=\frac{a^{2}S^{m}}{k}=\left(\frac{\partial T}{\partial r}\right)^{2}+\frac{Br}{1+\alpha T}\left(\frac{\partial u}{\partial r}\right)^{2}+O(\varepsilon^{2}) \qquad 15$$

$$N_{T}=\frac{S^{T}}{2\pi Lk}=\int_{1}^{\gamma}Nsrdr\,. \qquad 16$$

In Equation 15, the first term can be assigned as N_1 and the second term appears due to viscous dissipation as N_2, that is;

$$N_{1}=\left(\frac{\partial T}{\partial r}\right)^{2},\ N_{2}=\frac{Br}{1+\alpha T}\left(\frac{\partial u}{\partial r}\right)^{2} \qquad 17$$

In order to have an idea whether fluid friction dominates over heat transfer irreversibility or vice-versa, Bejan (1996) defined the irreversibility distribution ratio as $\Phi = N_2/N_1$. Heat transfer dominates for $0 \leq \Phi < 1$ and fluid friction dominates when $\Phi > 1$. The contribution of both heat transfer and fluid friction to entropy generation are equal when $\Phi = 1$. In many engineering designs and energy optimisation problems, the contribution of heat transfer entropy, N_1, to overall entropy generation rate, Ns, is needed. As an alternative to irreversibility parameter, the Bejan number (Be) is defined as;

$$Be=\frac{N_{1}}{Ns}=\frac{1}{1+\Phi} \qquad 18$$

Clearly, the Bejan number ranges from 0 to 1. $Be = 0$, is the limit where the irreversibility is dominated by fluid friction effects and $Be = 1$ corresponds to the limit where the irreversibility due to heat transfer by virtue of finite temperature differences dominates. The contribution of both heat transfer and fluid friction to entropy generation are equal when $Be = 1/2$. The expressions for Equations 13 - 18 can be obtained using computer algebra packages such as MAPLE, MATLAB or MATEMATICA.

RESULTS AND DISCUSSION

In order to study the influence of all parameters involved in the present problem on the flow and thermal field along with entropy generation characteristics, a selected set of graphical results are presented in Figures 2 - 13. Moreover, it is noteworthy that a positive increase in the parameter value of α indicates a decrease in the fluid viscosity. The convective cooling in the flow system is enhanced by increasing the Biot number (Bi).

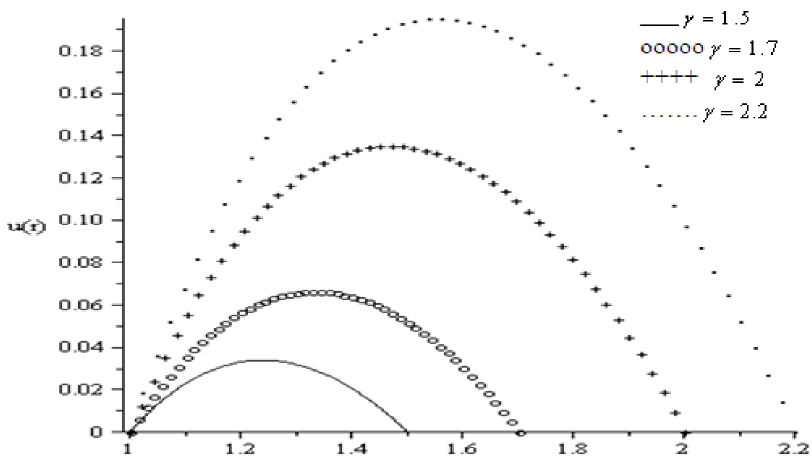

Figure 2. Effect of increasing the gap between the concentric pipes on velocity profiles for α= 0.1, *Br* = 1, *Bi*= 1, G=1.

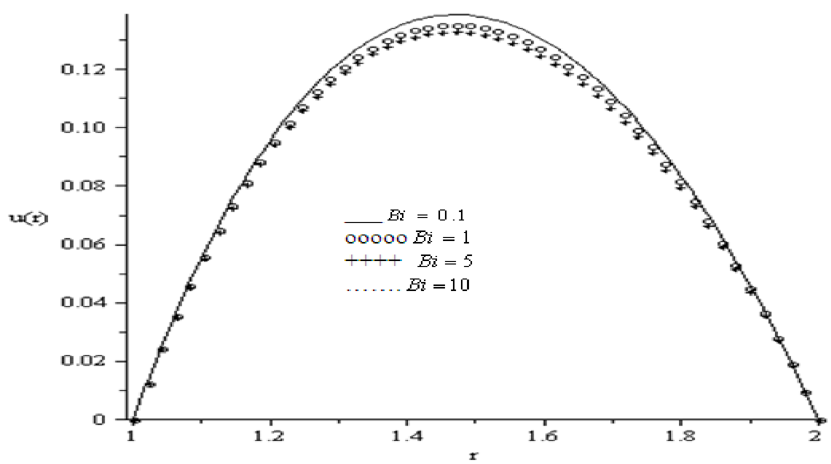

Figure 3. Effect of increasing convective cooling on velocity profiles for α= 0.1, *Br* = 1, γ= 2, G=1.

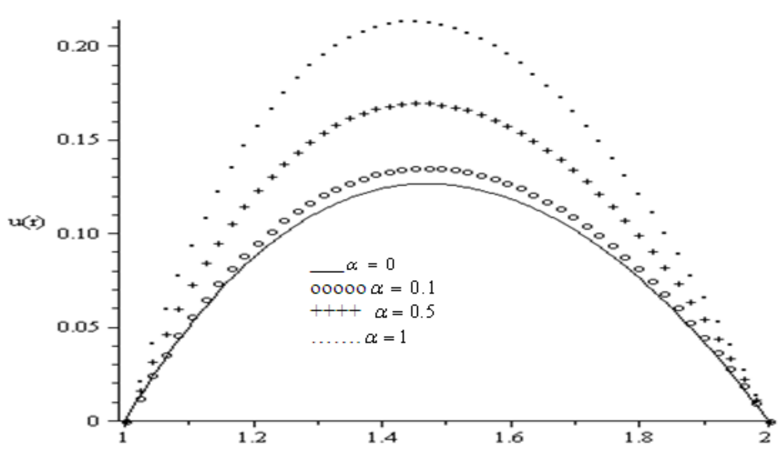

Figure 4. Effect of decreasing in fluid viscosity on velocity profiles for Bi= 1, *Br* = 1, γ= 2, G=1.

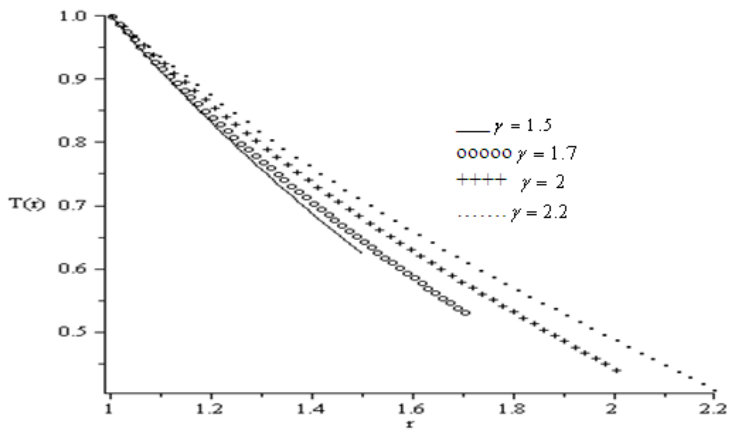

Figure 5. Effect of increasing the gap between the concentric pipes on temperature profiles for α= 0.1, Br = 1, Bi= 1, G=1.

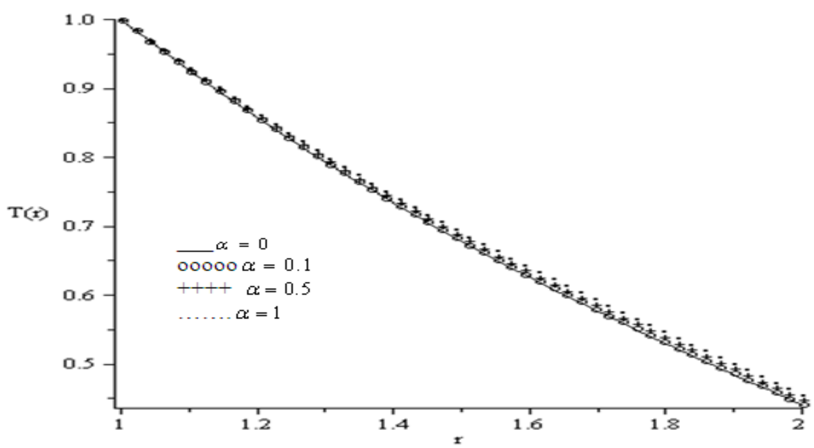

Figure 6. Effect of decreasing in fluid viscosity on temperature profiles for Bi= 1, *Br* = 1, γ= 2, G=1.

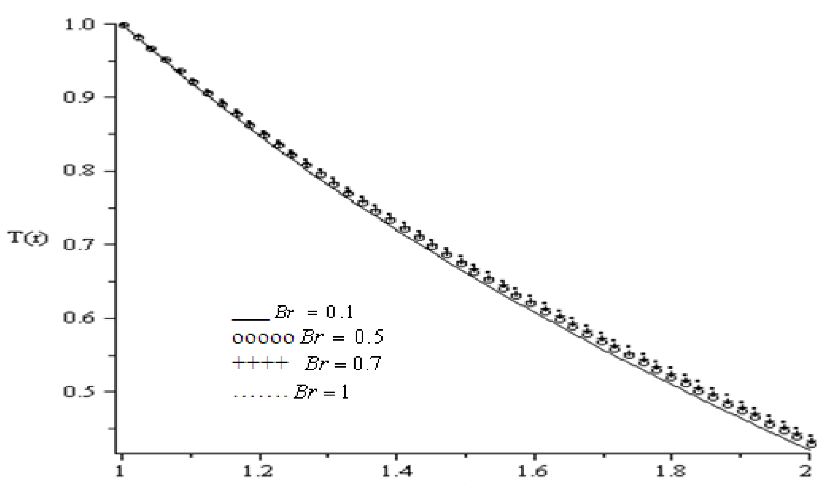

Figure 7. Effect of viscous dissipation on temperature profiles for Bi= 1, α = 0.1, γ= 2, G =1.

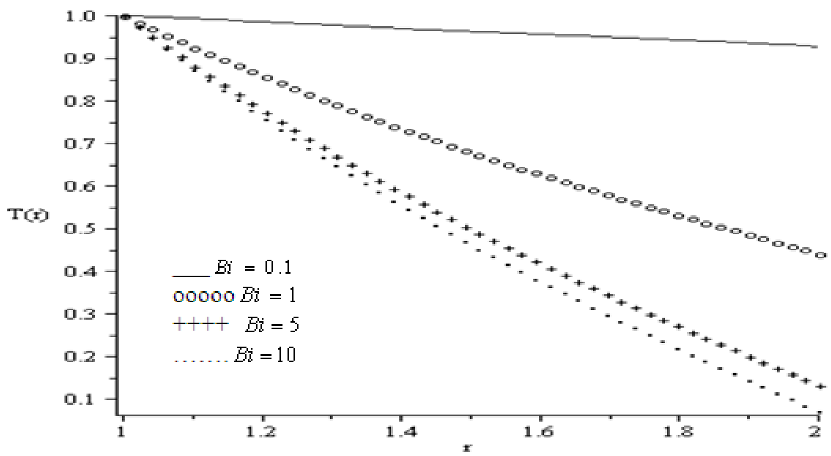

Figure 8. Effect of increasing convective cooling on temperature profiles for α= 0.1, *Br* = 1, γ= 2, G=1.

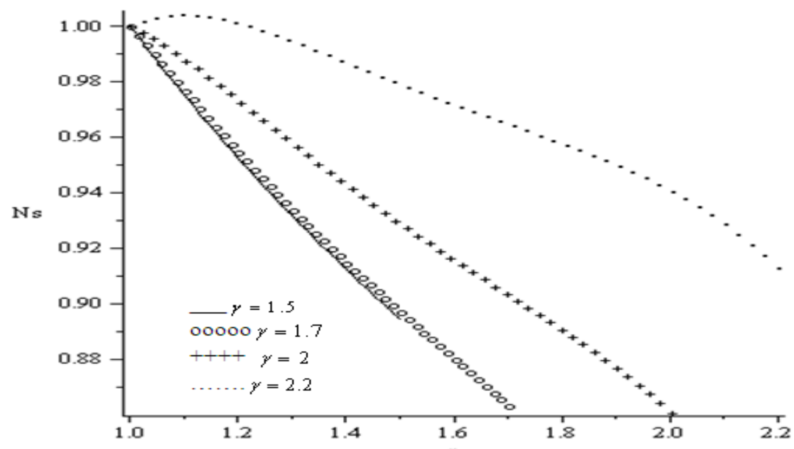

Figure 9. Effect of increasing the gap between the concentric pipes on entropy generation rate for α= 0.1, *Br* = 1, *Bi*= 0.1, G=1.

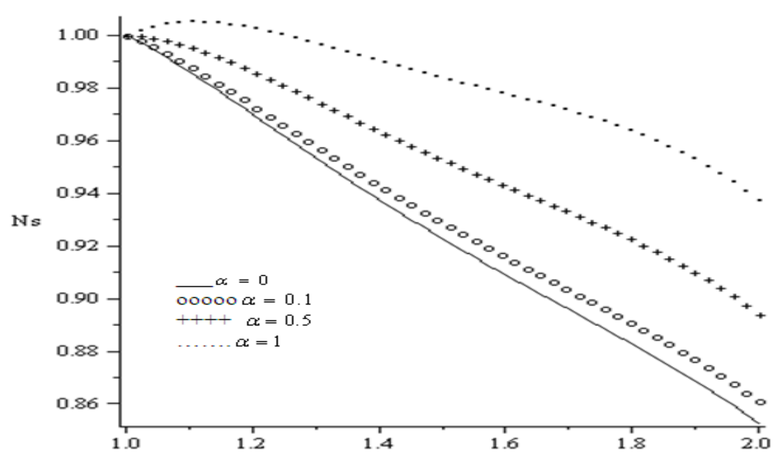

Figure 10. Effect of decreasing in fluid viscosity on entropy generation rate for Bi= 0.1, *Br* = 1, γ= 2, G=1.

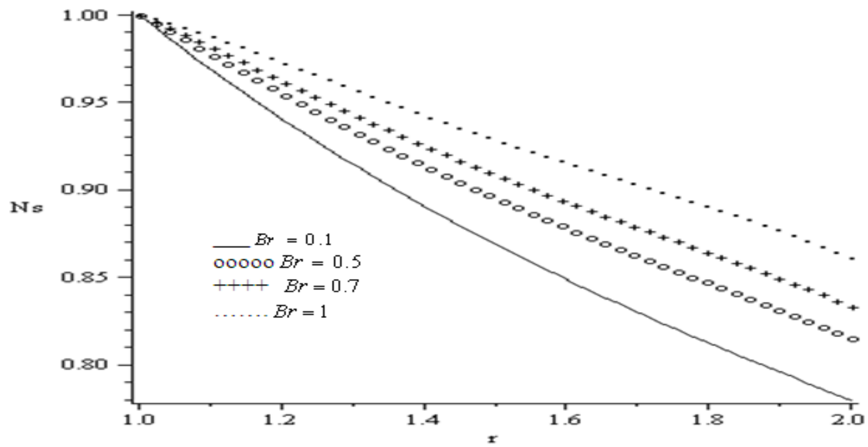

Figure 11. Effect of viscous dissipation on entropy generation rate for Bi= 0.1, α = 0.1, γ = 2, G =1.

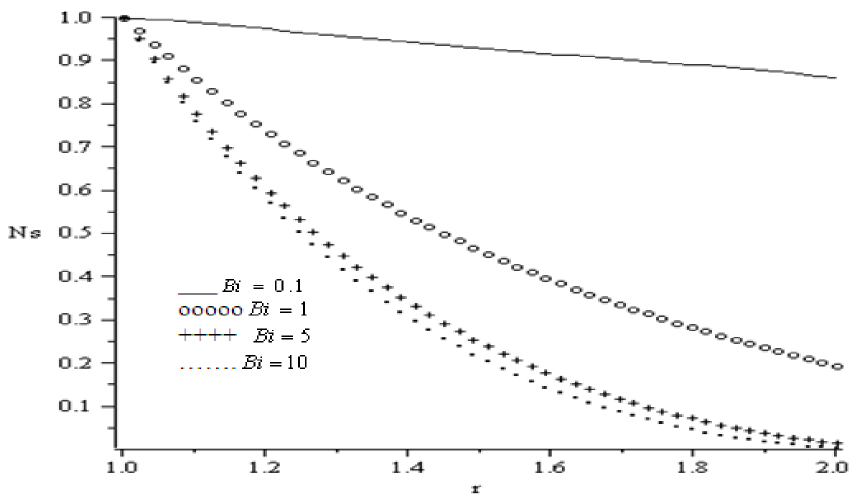

Figure 12. Effect of increasing convective cooling on entropy generation rate for α= 0.1, Br = 1, γ= 2, G=1.

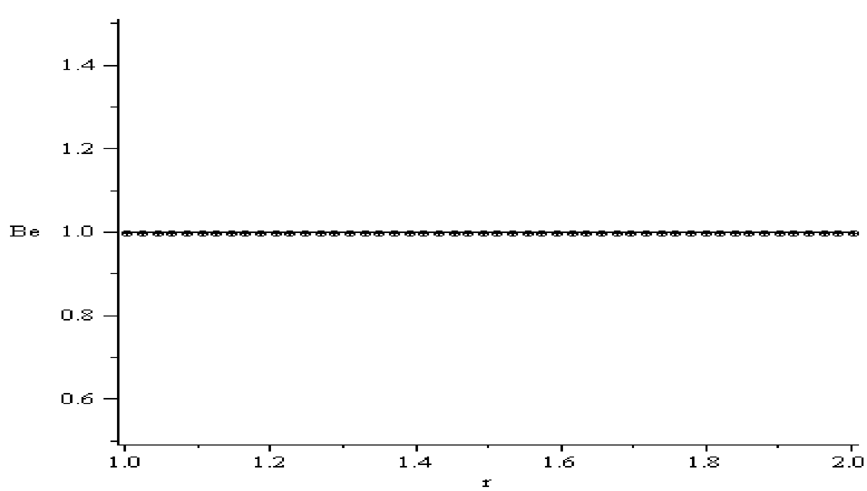

Figure 13. Bejan number for different values of embedded parameters.

Velocity and temperature profiles

Figures 2-4, shows the axial velocity distributions for increasing values of γ, *Bi* and α. The velocity profile is parabolic and increases with an increase in the gap between the two concentric pipes (Figure 2). In Figure 3, we observed that an increase in the Biot number (Bi) due to convective cooling causes a decrease in the velocity profile. The velocity distribution for increasing values of α is shown in Figure 4. It is interesting to note that the fluid velocity profile increases as its viscosity decreases. This can be attributed to the fact that the fluid has becomes lighter and then flow faster. A comparison of the effects of increasing values of *Bi* and α on the velocity profiles shows perfect agreements with the earlier observation of Makinde (2008) and Tshehla et al. (2010). Figures 2-4, shows the axial velocity distributions for increasing values of γ, *Bi* and α. The velocity profile is parabolic and increases with an increase in the gap between the two concentric pipes (see Fig.2). In Fig. 3, we observed that an increase in the Biot number (Bi) due to convective cooling causes a decrease in the velocity profile. The velocity distribution for increasing values of α is shown in figure 4. It is interesting to note that the fluid velocity profile increases as its viscosity decreases. This can be attributed to the fact that the fluid has becomes lighter and then flow faster. A comparison of the effects of increasing values of *Bi* and α on the velocity profiles shows perfect agreements with the earlier observation of Makinde (2008) and Tshehla et al. (2010).

Entropy generation rate

In Figures 9-12, the entropy generation rates in the transverse direction for various parametric values are illustrated. It is noteworthy that entropy generation rate is at the highest in the region around the inner pipe surface and lowest at the outer pipe surface. We observed that the entropy generation rate increases as the fluid viscosity decreases. Similar trend is observed with an increase in viscous heating (*Br*) and the gap between the two concentric pipes. This is in perfect agreement with the entropy generation results reported by Tasnim and Mahmud (2002) and Mirzazadeh et al. (2008). In their papers they observed that entropy generation number increases by increasing group parameter which depends strongly on the Brinkmann number (*Br*) and the temperature difference. Moreover, increasing values of Biot number (*Bi)* due to a convective cooling result to a decrease in entropy generation rate.

Bejan number

Figure 13 illustrate the Bejan (*Be*) number. It was observed that the Bejan number is 1.0 for all parameter values. This implies that the heat transfer irreversibility dominates the fluid friction irreversibility in the flow system.

Conclusions

Mathematical analysis has been developed for velocity, temperature and entropy generation number in a steady flow of a variable viscosity fluid between two concentric pipes under the action of a constant pressure gradient. We observed that both the fluid velocity and temperature increase with increasing values of γ, α and *Br*, and decreases with increasing values of Biot number, *Bi*. The inner pipe surface act as a strong concentrator of irreversibility due to high temperature gradients. A decrease in the parameter values of of γ, α, *Br* will reduce the entropy generation in the flow field.

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