International Journal of the Physical Sciences Vol. 6(30), pp. 6852 - 6860, 23 November, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.742
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Comparison of two synchronization methods for fractional order double scroll chaotic system

Cao-Yuan Ma*, Jian-Hua Liu and Chong-Lin Wang

School of Information and Electrical Engineering, China University of Mining and Technology, Xuzhou 221008, P. R. China.

Accepted 26 July, 2011

Synchronization of fractional-order chaotic system has received considerable attention for many research activities in recent years. In this Letter, we introduced a double scroll of fractional-order chaotic system. For the purpose of synchronization, we consider the driven response method and the one-way coupling method. Both the theoretical proof and numerical simulations show the effectiveness of the two methods. Moreover, we compare the two methods together, and obtain their features respectively.

Key words: Fractional-order, chaos, synchronization, driven response, one-way coupling.

INTRODUCTION

Chaos theory is a very interesting phenomenon which has provided us a new way of viewing the universe and is an important tool to understand the world we live in. Chaotic behaviors are useful in many real-world applications such as secure communication (Khadra et al., 2005), mathematics (Liu and Yang, 2010), time series analysis (Zhang et al., 2008), biology (Ma et al., 2009; Chen et al., 2011), circuit (Chen et al., 2011), human brain dynamics (Schiff et al., 1994) and heart beat regulation (Brandt and Chen, 1997), and so on.

Fractional calculus dates from three hundred years ago (Elwakil and Zahran, 1999; Jumarie, 2001; Shahiri et al., 2010; Chen et al., 2011), for many years they were not used in physics and engineering. One possible explanation of such unpopularity could be the multiple on equivalent definitions of fractional derivatives. Another difficulty is that fractional derivatives have complex geometrical interpretation because of their non-local character (Riewe, 1997). But it was applied into physics and engineering in recent ten years (Zhang and Small, 2006; Zhang et al., 2010; Chen et al., 2011). It was found that many systems in interdisciplinary fields can be described by the fractional differential equations, such as Lorenz system (Yang and Zeng, 2010), Chen's system (Li and Peng, 2004), Lü's system (Daftardar-Gejji and Bhalekar, 2010), Duffing system (Gao and Yu, 2005), the other fractional-order chaotic systems were described in many others works (Zhou et al., 2009; Wang et al., 2010; Wu and Lee, 2010; Swingle et al., 2011; Kurulay et al., 2010; Mohyud-Din et al., 2011). Due to the lack of appropriate mathematical methods, fractional-order dynamic systems were studied only marginally in the design and practice of control systems in the last few decades. However, in the recent years, emergence of effective methods in differentiation and integration of non-integer order equations makes fractional-order systems more and more attractive for the systems synchronization community.

In recent years, chaos synchronization (Chen, 2005; Kiani-B et al., 2009; Ge and Hsu, 2008; Song et al., 2010; Wei and Yang, 2009; Lin et al., 2011; Dadras and Momeni, 2010; Chen et al., 2011) has received ever increasing attention. In Kiani-B et al. (2009), a fractional chaotic communication method, which used an extended fractional Kalman filter, was presented. In Ge and Hsu (2008), chaos excited, chaos synchronizations of generalized van der-Pol systems with integral and fractional order were studied. In Song et al. (2010), a special kind of

*Corresponding author. E-mail: ieee307 ieee307@163.com

Abbreviation: FDEs, Fractional differential equations.

nonlinear chaotic oscillator, the Qi oscillator, was studied. All of the synchronization methods have their advantages and disadvantages. We should choose the perfect one according to the applied conditions. In this letter, we proposed two synchronization methods, the driven response synchronization method and the unidirectional coupling synchronization method. Both of them can make synchronization come true.

FRACTIONAL CALCULUS

Definition of fractional derivatives

There are several different definitions for fractional derivatives of order q (q >0), the Grünwald-Letnikov, the Riemann-Liouville and the Caputo definitions are three most commonly used ones. For a wide class of functions, the Grünwald-Letnikov definition and the Riemann-Liouville definition are equivalent. However, when modeling real-world phenomena with fractional differential equations (FDEs), the Caputo fractional derivative is more popular than the Riemann-Liouville definition of fractional derivative.

The Caputo derivative is defined by

$$ {}_{C}D^{q} f(t) = \frac{1}{\Gamma(n-q)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{q-n+1}} d\tau \quad (1)$$

where $n-1 < q < n$.

For the initial conditions for the FDEs with the Caputo derivative are in the same form as integer-order derivatives which have well understood physical meanings. Hence, we choose the Caputo derivative through this letter. For more details on the geometric and physical interpretation for FDEs of both the Riemann-Liouville and Caputo types. Hereafter, we use the notation $\frac{d^q}{dt^q}$ to denote the Caputo fractional derivative operator ${}_{C}D^{q}$.

Numerical calculation of the fractional differential equations

The approximate numerical techniques for FDEs have been developed in the literature which are numerically stable and can be applied to both linear and nonlinear FDEs. Recently, Deng (2009) also proposed an improved predictor-corrector approach in which the numerical approximation is more accurate and the computational cost is largely reduced.

The fractional predictor-corrector algorithm is based on the analytical property that the following FDE:

$$\frac{d^q x(t)}{dt^q} = f(t, x(t)), \quad 0 \le t \le T,$$

$$x^{(k)}(0) = x_0^{(k)}, \quad k=0,\ldots, m-1 (m=[q])$$

It is equivalent to the Volterra integral equation (Diethelm and Ford, 2005):

$$x(t) = \sum_{k=0}^{m-1} x_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q)} \int_0^t (t-s)^{(q-1)} f(s, x(s)) ds \quad . \quad (2)$$

Set $h = T/N$, $t_n = nh$, n = 0, 1, …, N ∊ Z^+. Then Equation 5 can be discretized as

$$x_h(t_{n+1}) = \sum_{k=0}^{m-1} \frac{t_{n+1}^k}{k!} x_0^{(k)} + \frac{h^q}{\Gamma(q+2)} f(t_{n+1}, x_h^p(t_{n+1})) + \frac{h^q}{\Gamma(q+2)} \sum_{j=0}^{n} a_{j,n+1} f(t_j, x_h(t_j)) \quad (3)$$

Where

$$a_{j,n+1} = \begin{cases} n^{q+1} - (n-q)(n+1)^q, & j=0, \\ (n-j+2)^{q+1} + (n-j)^{q+1} - 2(n-j+1)^{q+1}, & 1 \le j \le n, \\ 1, & j=n+1. \end{cases} \quad (4)$$

$$x_h^p(t_{n+1}) = \sum_{k=0}^{m-1} \frac{t_{n+1}^k}{k!} x_0^{(k)} + \frac{1}{\Gamma(q)} \sum_{j=0}^{n} b_{j,n+1} f(t_j, x_h(t_j)) \quad (5)$$

In which

$$b_{j,n+1} = \frac{h^q}{q} ((n+1-j)^q - (n-j)^q) \quad (6)$$

Therefore, the estimation error of this approximation is

$$\max_{j=0,1,\ldots,N} \left| x(t_j) - x_h(t_j) \right| = O(h^p) \quad (7)$$

Where $p = min(2, 1+q)$.

Stability theory

Some stability theorems on fractional-order systems and their related results are introduced. The first theorem is given for commensurate fractional-order linear systems.

Theorem 1

The following linear system:

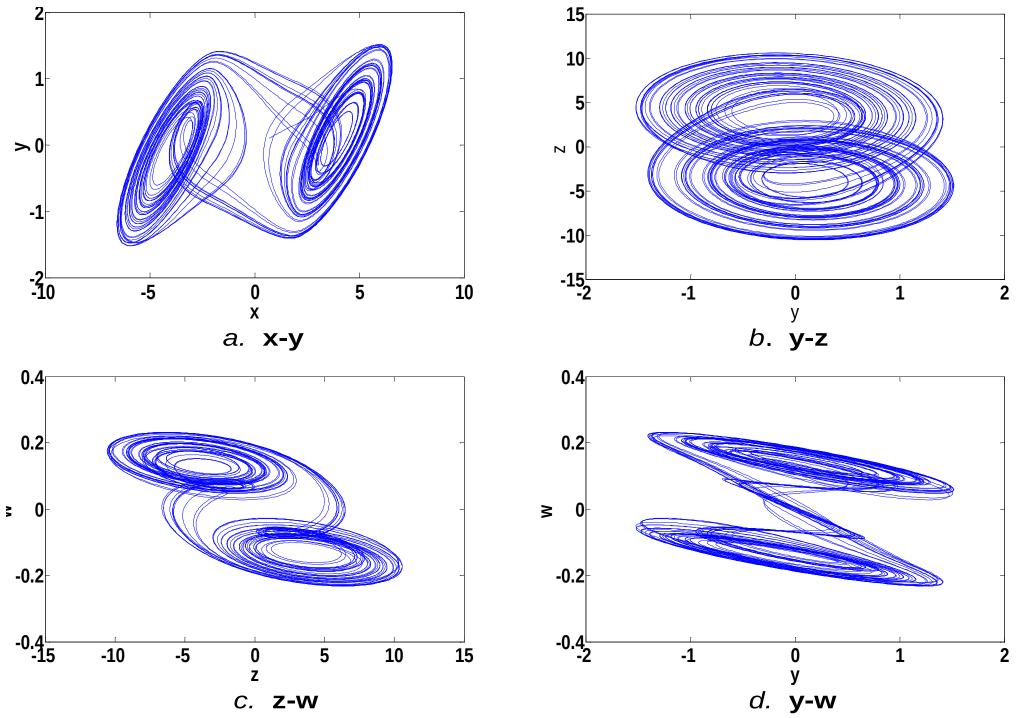


Figure 1. Two-dimensional state trajectory.

$$\frac{d^q x}{dt^q} = Ax, x(0) = x_0 \tag{8}$$

Where 0<q<1, $x \in R^n$, and $A \in R^{n \times n}$. The system is asymptotically stable if and only if $|\arg(\lambda)| > q\pi / 2$ is satisfied for all eigenvalues of matrix A. Also, this system is stable if and only if $|\arg(\lambda)| \geq q\pi / 2$ is satisfied for all eigenvalues of matrix A with those critical eigenvalues satisfying $|\arg(\lambda)| = q\pi / 2$ having geometric multiplicity of one. The geometric multiplicity of an eigenvalue λ of the matrix A is the dimension of the subspace of vectors *v* in which $Av = \lambda v$.

Theorem 2

The following commensurate fractional-order system:

$$\frac{d^q x}{dt^q} = f(x) \tag{9}$$

Where 0<q<1, $x \in R^n$. The equilibrium points of system (9) are calculated by solving the following equation: $f(x) = 0$. These points are locally asymptotically stable if all eigenvalues λ_i of the Jacobian matrix $J = \partial f / \partial x$ evaluated at the equilibrium points satisfy: $|\arg(\lambda)| > q\pi / 2$.

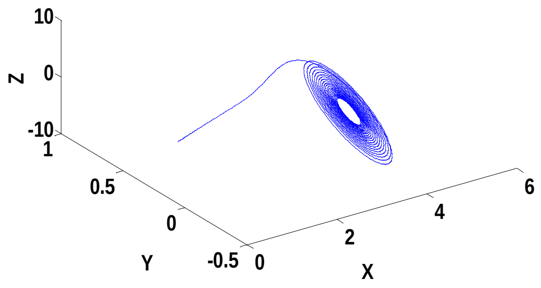
MATHEMATICAL MODEL

Observation of chaotic dynamics

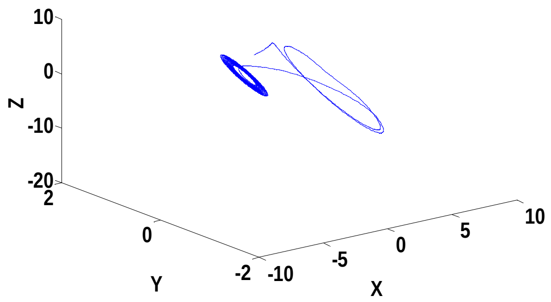
The double scroll of integral order hyper-chaos system can be described as follows.

$$\begin{cases} \frac{dx}{dt} = 10\left[y - 0.38(-|x+2| + |x-2| + 1.086x)\right] \\ \frac{dy}{dt} = x - y + z \\ \frac{dz}{dt} = -15(y - w) \\ \frac{dw}{dt} = -0.05(z + 27.333w) \end{cases} \tag{10}$$

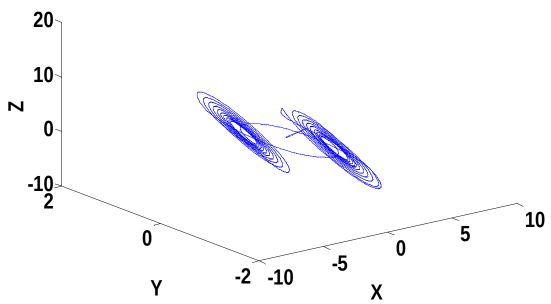
We take initial value (x, y, z, w)=(0.1, 0.1, 0.1, 0.1). The two-dimensional state trajectory is shown in Figure 1. There are two opposite scroll modes in the integral order



a. α =0.97



b. α =0.98



c. α =0.99

Figure 2. Three-dimensional state trajectory.

hyper-chaos system through Figure 1. Now, let us introduce its fractional version as follows:

$$\begin{cases} \dfrac{d^{\alpha}x}{dt^{\alpha}} = 10\left[y - 0.38(-|x+2| + |x-2| + 1.086x)\right] \\ \dfrac{d^{\alpha}y}{dt^{\alpha}} = x - y + z \\ \dfrac{d^{\alpha}z}{dt^{\alpha}} = -15(y - w) \\ \dfrac{d^{\alpha}w}{dt^{\alpha}} = -0.05(z + 27.333w) \end{cases} \tag{11}$$

Where 0<α≤1, and the system (11) is integral order system when α=1. We take initial value (x, y, z, w)=(0.1, 0.1, 0.1, 0.1) Figure 2. (*a*, *b*, *c*) is the three-dimensional phase diagram whenα=0.97, α=0.98, α=0.99.

We can know that system (11) is more and more closer to double scroll, when α is more and more close to 1. And α=0.98 is the transition value for the system (11) from single scroll to double scroll.

Local stability

The fractional system (11) has three equilibrias and their corresponding eigenvalues are:

$E_1 = (0,0,0,0)$, $E_2 = (0.01,0.37, 0.01,-0.36)$,
$E_3 = (-0.01,-0.37, -0.01,0.36)$.

For equilibria E_1 (0,0,0,0), system (11) is now linearized, the Jacobian matrix is defined as

$$J_1 = \begin{bmatrix} -4.13 & 10 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & -15 & 0 & 15 \\ 0 & 0 & -0.05 & -1.37 \end{bmatrix}$$

To gain its eigenvalues, we let

$$|\lambda I - J_1| = 0$$

These eigenvalues that corresponding to equilibrium E_1 (0,0,0,0) are respectively obtained as follows:

$$\lambda_1 = -5.53, \lambda_2 = 0.13 + 3.44i, \lambda_3 = 0.13 - 3.44i, \lambda_3 = -1.23.$$

$$q > \frac{2}{\pi}\arctan\frac{|\mathrm{Im}(\lambda)|}{|\mathrm{Re}(\lambda)|} = \frac{2}{\pi}\arctan\frac{|\pm 3.44i|}{0.13} = 0.97595$$

For equilibria $E_2 = (0.01,0.37, 0.01,-0.36)$, system (11) is now linearized, the Jacobian matrix is defined as:

$$J_2 = \begin{bmatrix} 4.714 & 10 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & -15 & 0 & 15 \\ 0 & 0 & -0.05 & -1.37 \end{bmatrix}$$

To gain its eigenvalues, we let

$$|\lambda I - J_2| = 0$$

These eigenvalues that corresponding to equilibrium $E_2 = (0.01, 0.37, 0.01, -0.36)$ are respectively obtained as follows:

$$\lambda_1 = 5.79, \lambda_2 = -1.00 + 3.46i, \lambda_3 = -1.00 - 3.46i, \lambda_3 = -1.43.$$

$$q > \frac{2}{\pi}\arctan\frac{|\mathrm{Im}(\lambda)|}{|\mathrm{Re}(\lambda)|} = \frac{2}{\pi}\arctan\frac{|\pm 3.46i|}{1} = 0.82089$$

For equilibria $E_3 = (-0.01, -0.37, -0.01, 0.36)$, system (11) is now linearized, the Jacobian matrix is defined as:

$$J_3 = \begin{bmatrix} 4.714 & 10 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & -15 & 0 & 15 \\ 0 & 0 & -0.05 & -1.37 \end{bmatrix}$$

To gain its eigenvalues, we let

$$|\lambda I - J_3| = 0$$

These eigenvalues that corresponding to equilibrium $E_3 = (-0.01, -0.37, -0.01, 0.36)$ are respectively obtained as follows:

$$\lambda_1 = 5.79, \lambda_2 = -1.00 + 3.46i, \lambda_3 = -1.00 - 3.46i, \lambda_3 = -1.43.$$

$$q > \frac{2}{\pi}\arctan\frac{|\mathrm{Im}(\lambda)|}{|\mathrm{Re}(\lambda)|} = \frac{2}{\pi}\arctan\frac{|\pm 3.46i|}{1} = 0.82089$$

It means that when q>0.97595 system (11) with commensurate fractional-order and above parameters has the necessary condition for exhibiting a chaotic attractor, which is useful for further numerical simulation.

DRIVEN RESPONSE SYNCHRONIZATION

In order to observe the two fractional hyper-chaos synchronization behavior, we take the fractional hyper-chaotic system as the drive system. The subscript of four variable states was expressed as *d*. The drive system is defined as follows:

$$\begin{cases} \frac{d^\alpha x_d}{dt^\alpha} = 10\left[y_d - 0.38(-|x_d + 2| + |x_d - 2| + 1.086x_d)\right] \\ \frac{d^\alpha y_d}{dt^\alpha} = x_d - y_d + z_d \\ \frac{d^\alpha z_d}{dt^\alpha} = -15(y_d - w_d) \\ \frac{d^\alpha w_d}{dt^\alpha} = -0.05(z_d + 27.333w_d) \end{cases} \quad (12)$$

Response system is a subsystem that contains the state variables (y, z) (The subscript of state variables expressed as r. The response system is defined as follows:

$$\begin{cases} \frac{d^\alpha y_r}{dt^\alpha} = x_d - y_r + z_r \\ \frac{d^\alpha z_r}{dt^\alpha} = -15(y_r - w_d) + u \end{cases} \quad (13)$$

Where *u* is the control input, in which $u = -(z_r - z_d)$. We define synchronization error as:

$$\begin{cases} e_2 = y_r - y_d \\ e_3 = z_r - z_d \end{cases} \quad (14)$$

Subtracting system (10) from system (11) yields the following error system:

$$\begin{cases} \frac{d^\alpha e_2}{dt^\alpha} = -e_2 + e_3 \\ \frac{d^\alpha e_3}{dt^\alpha} = -15e_2 - e_3 \end{cases} \quad (15)$$

Theorem 1

System (12) and system (13) can achieve synchronization through the synchronous controller. That is to say, $\lim_{n\to\infty} e_2 = 0$, $\lim_{n\to\infty} e_3 = 0$.

Proof: Formula 15 can be written as:

$$\frac{d^\alpha}{dt^\alpha}\begin{pmatrix} e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -15 & -1 \end{pmatrix}\begin{pmatrix} e_2 \\ e_3 \end{pmatrix} = A\begin{pmatrix} e_2 \\ e_3 \end{pmatrix} \quad (16)$$

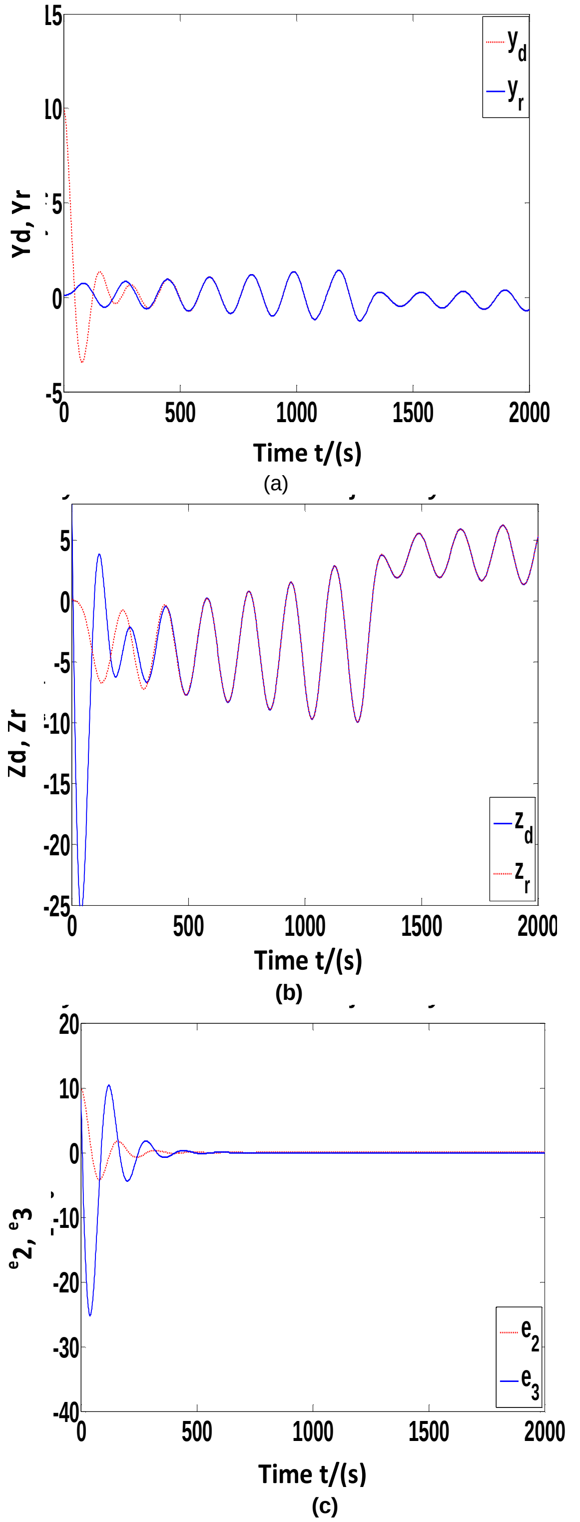


Figure 3. Synchronous state trajectory of system (13) and (14) and the error process of evolution curve. (a) Synchronous state trajectory of y_d and y_r (b) Synchronous state trajectory of z_d and z_r (c) Error evolution curves of e_2 and e_3.

So the two eigenvalues of A are $\lambda_1 = -1 + 3.87i$, $\lambda_2 = -1 - 3.87i$. And then $|\arg(\lambda_{1,2})| = 0.58\pi > \alpha\pi/2$. So the zero point of error system (15) is asymptotically stable (Jumarie, 2001). That is to say: $\lim_{n\to\infty} e_2 = 0$, $\lim_{n\to\infty} e_3 = 0$.

The above analysis shows that the fractional order drive system (Jumarie, 2001) and fractional response system (13) reach synchronization in the driven signal (x_d, w_d). Now we use the predictor-corrector algorithm method for numerical simulation. We take 0.01 as time step and (0.1, 0.1, 0.1, 0.1, 10, 10) for the initial state value (x_d, y_d, z_d, w_d, y_r, z_r). We take α=0.99 in the simulation. The simulation result is shown in Figure 3a, b and c.

We can know system (12) and (13) reach the synchronous state through the simulation result. It confirms that the driven response synchronization is effective.

Synchronization via one-way coupling method

We also take system (12) as the drive system. The response system is defined as follows:

$$\begin{cases} \dfrac{d^\alpha x_r}{dt^\alpha} = 10\left[y_r - 0.38(-|x_r+2| + |x_r-2| + 1.086x_r)\right] \\ \dfrac{d^\alpha y_r}{dt^\alpha} = x_r - y_r + z_r - k_2(y_r - y_d) \\ \dfrac{d^\alpha z_r}{dt^\alpha} = -15(y_r - w_r) - k_3(z_r - z_d) \\ \dfrac{d^\alpha w_r}{dt^\alpha} = -0.05(z_r + 27.333w_r) \end{cases} \quad (17)$$

Where k_2, k_3 are the coupling strength. We define the synchronous error as follows:

$$\begin{cases} e_1 = x_r - x_d \\ e_2 = y_r - y_d \\ e_3 = z_r - z_d \\ e_4 = w_r - w_d \end{cases} \quad (18)$$

Subtracting system (12) from system (17) yields the following error system:

$$\begin{cases} \frac{d^{\alpha}e_1}{dt^{\alpha}} = 10e_2 - 4.1268e_1 + 3.8(|x_r+2| - |x_r-2| - |x_d+2| + |x_d-2|) \\ \frac{d^{\alpha}e_2}{dt^{\alpha}} = e_1 + e_3 - (k_2+1)e_2 \\ \frac{d^{\alpha}e_3}{dt^{\alpha}} = -15(e_2 - e_4) - k_3 e_3 \\ \frac{d^{\alpha}e_4}{dt^{\alpha}} = -0.05(e_3 + 27.333e_4) \end{cases} \quad (19))$$

Theorem 2

System (12) and system (17) can achieve synchronization through the synchronous controller.

Proof: Now let us take the Laplace transformation in both sides of Equation 19. Order $E_i(s) = L(e_i(t))$ (*i*=1, 2, 3, 4). And use $L(d^{\alpha}e_i / dt^{\alpha}) = s^{\alpha}E_i(s) - s^{\alpha-1}e_i(0)$ (*i*=1, 2, 3, 4). So,

$$\begin{cases} s^{\alpha}E_1(s) - s^{\alpha-1}e_1(0) = -10E_2(s) - 4.1268E_1(s) + 3.8L(|x_r+2| - |x_r-2| - |x_d+2| + |x_d-2|) \\ s^{\alpha}E_2(s) - s^{\alpha-1}e_2(0) = E_1(s) + E_3(s) - (k_2+1)E_2(s) \\ s^{\alpha}E_3(s) - s^{\alpha-1}e_3(0) = -15(E_2(s) - E_4(s)) - k_3E_3(s) \\ s^{\alpha}E_4(s) - s^{\alpha-1}e_4(0) = -0.05(E_3(s) + 27.333E_4(s)) \end{cases} \quad (20)$$

Thus,

$$\begin{cases} E_1(s) = \frac{-10E_2(s) + 3.8L(|x_r+2| - |x_r-2| - |x_d+2| + |x_d-2|) + s^{\alpha-1}e_1(0)}{s^{\alpha} + 4.1268} \\ E_2(s) = \frac{E_1(s) + E_3(s) + s^{\alpha-1}e_2(0)}{s^{\alpha} + (k_2+1)} \\ E_3(s) = \frac{-15(E_2(s) - E_4(s)) + s^{\alpha-1}e_3(0)}{s^{\alpha} + k_3} \\ E_4(s) = \frac{-0.05E_3(s) + s^{\alpha-1}e_4(0)}{s^{\alpha} + 1.36665} \end{cases} \quad (21)$$

By the final-value theorem of the Laplace transformation, we have:

$$\begin{cases} \lim_{t\to\infty} e_1(t) = \lim_{s\to 0^+} sE_1(s) = \lim_{s\to 0^+} \frac{-10sE_2(s) + 3.8sL(|x_r+2| - |x_r-2| - |x_d+2| + |x_d-2|)}{4.1268} \\ \lim_{t\to\infty} e_2(t) = \lim_{s\to 0^+} sE_2(s) = \lim_{s\to 0^+} \frac{sE_1(s) + sE_3(s)}{k_2+1} \\ \lim_{t\to\infty} e_3(t) = \lim_{s\to 0^+} sE_3(s) = \lim_{s\to 0^+} \frac{-15s(E_2(s) - E_4(s))}{k_3} \\ \lim_{t\to\infty} e_4(t) = \lim_{s\to 0^+} sE_4(s) = \lim_{s\to 0^+} \frac{-sE_3(s)}{27.333} \end{cases}$$

Thus,

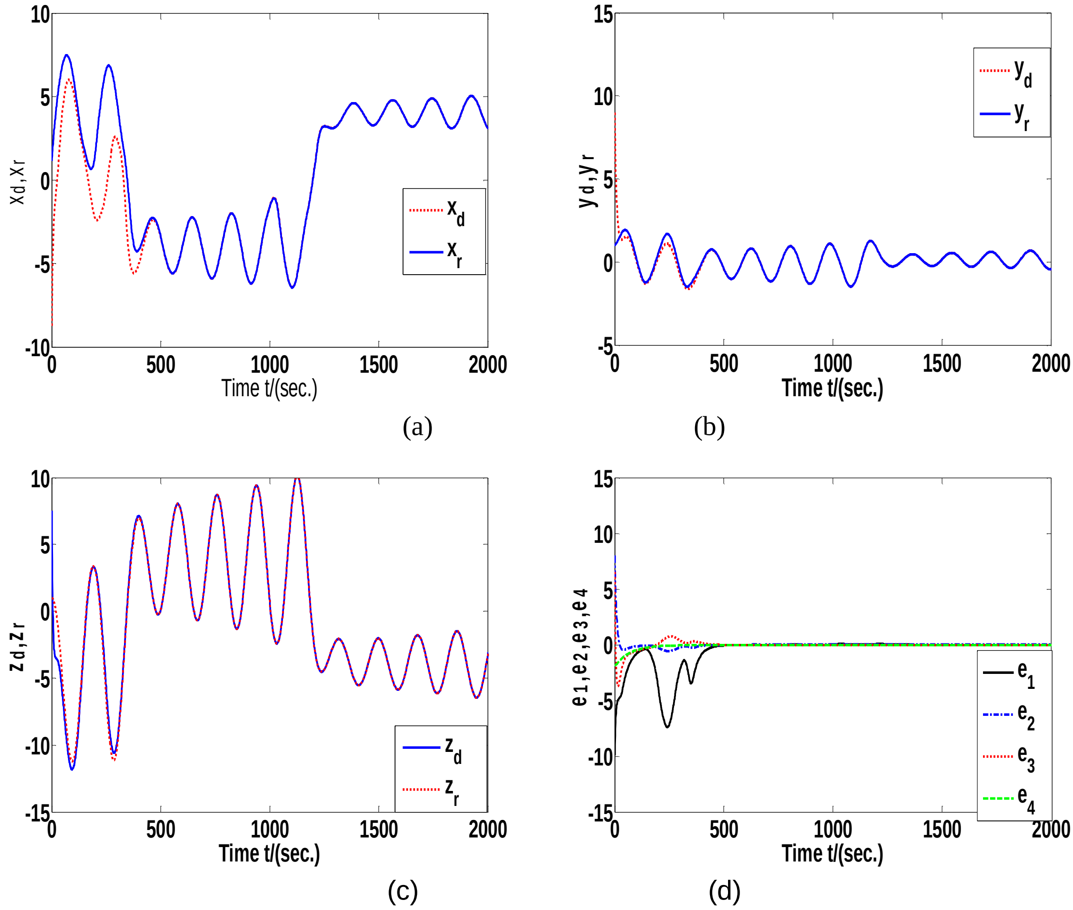


Figure 4. Synchronous state trajectory of system (13) and (18), and the error process of evolution curve. (a) Synchronous state trajectory of x_d and x_r (b) Synchronous state trajectory of y_d and y_r (c) Synchronous state trajectory of z_d and z_r (d) Error evolution curves of e_i (i=1,2,3,4).

If it is assumed $E_3(s)$ is limited, $E_1(s)$ or $E_4(s)$ is limited. Thus,

$$\begin{cases} \lim_{t\to\infty} e_1(t) = 0 \\ \lim_{t\to\infty} e_3(t) = 0 \\ \lim_{t\to\infty} e_4(t) = 0 \end{cases} \tag{23}$$

By (22) and (23), so:

$$\begin{cases} \lim_{t\to\infty} e_1(t) = 0 \\ \lim_{t\to\infty} e_2(t) = 0 \\ \lim_{t\to\infty} e_3(t) = 0 \\ \lim_{t\to\infty} e_4(t) = 0 \end{cases} \tag{25}$$

So we obtain the proof that systems (12) and (17) achieve synchronization through (24).

We obtain the synchronous result through MATLAB simulation. We take 0.01 as time step and (1, 1, 1, 1, -10, 10, 10, -1) for the initial state value (x_d, y_d, z_d, w_d, x_r, y_r, z_r, w_r). We take α = 0.99 and k_2 = k_3 = 10 in the simulation. The simulation result are shown in Figure 4a, b and c. We can also know that system (12) and (17) reach the synchronous state through the simulation result. It confirms the effectiveness of one-way coupling method.

DISCUSSION AND CONCLUSION

First, we introduced a double scroll of integral order hyper-chaotic system, which have two opposite scrolls and studied its phase trajectory map. And then the fractional order mathematical model was constructed. We also studied its phase trajectory map and we can draw the conclusion; there are two opposite scrolls in the fractional order system when q≥0.98. At last, we achieve system synchronization through the driven response method and the one-way coupling method respectively. Theoretical proof and numerical simulation confirmed the effectiveness of the two chaos synchronization methods.

Partial state variables can achieve synchronization though driven response method. However, the one-way coupling method can make all of state variables achieve synchronization. The one-way coupling method is more advantageous than the driven response method. Comparing Figures 3c and 4d, we can draw conclusions; it has better performance for the controlled system with the one-way coupling method, for its small overshoot and short transition time; for industrial applications, the driven response method has its superiority. Therefore, we should try our best to use their respective advantages according to the different requirements.

More and better synchronization method of fractional-order chaotic systems should be studied. And control and synchronization of a class of fractional-order chaotic systems may be studied. Moreover, these synchronization methods may be realized and simulated by circuit.

ACKNOWLEDGEMENT

This work is supported by “the Fundamental Research Funds for the Central Universities” of CUMT (No.2010QNB33).

REFERENCES

Brandt ME, Chen GR (1997). Bifurcation control of two nonlinear models of cardiac activity. IEEE Trans. Circ. Syst., 44(10): 1031-1034.

Chen DY, Liu YX, Ma XY, Zhang RF (2011). Control of a class of fractional-order chaotic systems via sliding mode. Nonliear Dynam. Doi: 10.1007/s11071-011-0002-x.

Chen DY, Shen T, Ma XY (2011). Sliding mode control of chaotic vibrations of spinning disks with uncertain parameter under bounded disturbance. Acta Phys. Sin, 60(5): 050505. (In Chinese)

Chen DY, Wu C, Liu CF, Ma XY, You YJ, Zhang RF (2011). Synchronization and circuit simulation of a new double-wing chaos. Nonlinear Dynam. Doi: 10.1007/s11071-011-0083-6.

Chen DY, Zhao WL, Ma XY, Zhang RF (2011). No-chattering sliding mode control chaos in Hindmarsh-Rose neurons with uncertain parameters. Comput. Math. Appl., 61(10): 3161-3171.

Chen HK (2005). Synchronization of two different chaotic systems a new system and each of the dynamical systems Lorenz Chen and Lü, Chaos Soliton. Fract., 25(5): 1049-1056.

Chen LP, Chai Y, Wu RC (2011). Lag projective synchronization in fractional-order chaotic (hyperchaotic) systems. Phys. Lett. A, 375 (21): 2099-2110.

Dadras S, Momeni HR (2010). Control of a fractional-order economical system via sliding mode. Phys. A, 389(12): 2434-2442.

Daftardar-Gejji V, Bhalekar S (2010). Chaos in fractional ordered Liu system. Comput. Math. Appl., 59(3): 1117-1127.

Deng WH (2009). Smoothness and stability of the solutions for nonlinear fractional differential equations. Nonlinear Anal.-Theor., 72(3-4): 1768-1777.

Diethelm K, Ford NJ (2005). Algorithms for the fractional calculus: A selection of numerical methods. Comput. Method Appl. M, 194(6-8): 743-773.

Elwakil SA, Zahran MA (1999). Fractional integral representation of master equation. Chaos Soliton. Fract., 10(9): 1545-1558.

Gao X, Yu JB (2005). Chaos in the fractional order periodically forced complex Duffing’s oscillators. Chaos Soliton. Fract., 24(4): 1097-1104.

Ge ZM, Hsu MY (2008). Chaos excited chaos synchronizations of integral and fractional order generalized van der Pol systems. Chaos Soliton. Fract., 36(3): 592-604.

Jumarie G (2001). Fractional master equation: non-standard analysis and Liouville-Riemann derivative. Chaos Soliton. Fract., 12(13): 2577-2587.

Khadra A, Liu XZ, Shen X (2005). Impulsively synchronizing chaotic systems with delay and applications to secure communication. Automatica, 41(9): 1491-1502.

Kiani-B A, Fallahi K, Pariz N, Leung H (2009). A chaotic secure communication scheme using fractional chaotic systems based on an extended fractional Kalman filter. Commun. Nonlinear Sci. Numer. Simulat., 14(3): 863–879.

Kurulay M, Ibrahimoglu BA, Bayram M (2010). Solving a system of nonlinear fractional partial differential equations using three dimensional differential transform method. Int. J. Phys. Sci., 5(6): 906-912.

Li C, Peng G (2004). Chaos in Chen’s system with a fractional order. Chaos Soliton. Fract., 22(2): 443-450.

Lin P, Zhou WN, Zhou L, Sun KH (2011). Chaos synchronization between two different fractional-order hyperchaotic systems. Commun. Nonlinear Sci. Numer. Simulat., 16(6): 1044-1051.

Liu YJ, Yang QG (2010). Dynamics of a new Lorenz-like chaotic system. Nonlinear. Anal. Real, 11(4): 2563-2572.

Ma J, Wang CN, Tang J, Xia YF (2009). Suppression of the spiral wave and turbulence in the excitability-modulated media. Int. J. Theor. Phys., 48(1): 150–157.

Mohyud-Din ST, Yildirim A, Usman M (2011). Homotopy analysis method for fractional partial differential equations. Int. J. Phys. Sci., 6(1): 136-145.

Riewe F (1997). Mechanics with fractional derivatives. Phys. Rev. E, 55(3): 3581-3592.

Schiff SJ, Jerger K, Duong DH, Chang T, Spano ML, Ditto WL (1994). Controlling chaos in the brain. Nature, 370(6491): 615-620.

Shahiri M, Ghaderi R, Ranjbar A, Hosseinnia SH, Momani S (2010). Chaotic fractional-order Coullet system: Synchronization and control approach. Commun. Nonlinear Sci. Numer. Simulat, 15(3): 665-674.

Song L, Yang JY, Xu SY (2010). Chaos synchronization for a class of nonlinear oscillators with fractional order. Nonlinear Anal.-Theor., 72(5): 2326-2336.

Swingle B, Barkeshli M, McGreevy J, Senthil T (2011). Correlated topological insulators and the fractional magnetoelectric effect. Phys. Rev. B, 83(19): 195139.

Wang ZH, Sun YX, Qi GY, Van Wyk BJ (2010). The effects of fractional order on a 3-D quadratic autonomous system with four-wing attractor. Nonlinear Dynam, 62(1-2): 139-150.

Wei ZC, Yang QG (2009). Controlling the diffusionless Lorenz equations with periodic parametric perturbation. Computers & Mathematics with Applications, 58(10): 1979-1987.

Wu GC, Lee EWM (2010). Fractional variational iteration method and its application. Phys. Lett., A 374(25): 2506-2509.

Yang QG, Zeng CB (2010). Chaos in fractional conjugate Lorenz system and its scaling attractors. Commun. Nonlinear Sci. Numer. Simulat., 15(12): 4041-4051.

Zhang J, Small M (2006). Complex network from pseudoperiodic time series: topology versus dynamics. Phys. Rev. Lett., 96(23): 238701.

Zhang J, Sun J, Luo X, Zhang K, Nakamura T, Small M (2008). Characterizing topology of pseudoperiodic time series via complex network approach. Phys. D, 237(22): 2856–2865.

Zhang J, Zhou CS, Xu XK, Small M (2010). Mapping from structure to dynamics: A unified view of dynamical processes on networks. Phys. Rev. E, 82(2): 026116.

Zhou P, Wei LJ, Cheng XF (2009). A novel fractional-order hyperchaotic system and its synchronization. Chin. Phys. B, 18(7): 2674-2679.