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Full Length Research Paper

A comparative simulation of mobile WiMAX physical layer performance for Zero-Force (ZF) and Minimum Mean Square Error (MMSE) channel equalizers

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Mobile WiMAX is a broadband wireless solution that enables convergence of mobile and fixed broadband network through a common wide area broadband radio access technology and flexible network architecture. The aim of this paper is the performance evaluation of 802.16e system using different channel equalizers at the receiver module for different communication and modulation technique for different bandwidth. We analyze the Symbol Error Rate (SER) of the wireless communication channel (SUI -3 channel with AWGN) for using zero-force (ZF) and Minimum Mean Square Error (MMSE) equalizers at Fast Fourier Transform (FFT) size 256, 512 and 1024, respectively for BPSK, QPSK, 16-QAM and 64-QAM modulation. The simulation includes symbol error rate versus signal to noise ratio performance predictions. In our simulation we use deterministic Random Number Generator (RNG) algorithm to generate the random input value. It is concluded that the performance of the MMSE equalizer is comparable to or slightly better than the ZF equalizer.

Key words: Fast Fourier Transform (FFT), Symbol Error Rate (SER), zero force, Minimum Mean Square Error (MMSE), bit error rate.

INTRODUCTION

Along with the advance of communication technology, the need for ubiquitous access to the Internet is IEEE 802.16e (2006) is a global broadband wireless access standard capable of delivering high data rates to fixed users as well as portable and mobile ones over long distance (Kim, 2009). The mobile WiMAX air interface adopts orthogonal frequency division multiple access (OFDMA) for improved multi-path performance in non-line-of sight (NLOS) environment. Mobile WiMAX extends the OFDM PHY layer (Figure 1) to support terminal mobility and multiple–access. The resulting technology is known as scalable OFDMA. Data streams to and from individual users are multiplexed to groups of sub channel on the downlink and uplink. By adopting a scalable PHY architecture, mobile WiMAX is able to support a wide range of bandwidths. The scalability is implemented by varying the FFT size from 128 to 512, 1024 and 2048 (Table 1) to support channel bandwidths of 1.25, 5, 10 and 20 MHz respectively. This paper analyses the performance of mobile WiMAX in terms of the BER, SER (Symbol Error Rate) as a function of Signal-to-Noise-Ratio (SNR) (Kim, 2009; Ahmadi, 2009).

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MOBILE WiMAX PHYSICAL LAYER

The mobile WiMAX standard builds on the principles of OFDM by adopting a Scalable OFDMA-based PHY layer (SOFDMA). SOFDMA supports a wide range of operating bandwidths to flexibly address the need for various spectrum allocation and application requirements (Yaghoobi, 2004). To guarantee a fixed OFDMA symbol duration, the FFT is amplified with the increase of operating bandwidth. The increase in FFT maintains a flat sub carrier frequency spacing of 10.94 kHz as shown in Tables 1 and 2. As the basic resource unit is fixed, the impact of bandwidth scaling is minimized to the upper layers (Xiao 2008; Arafat, 2010).

For producing higher code rates, the channel coding stage includes randomization, convolutional coding and puncturing. Native code rate is ½ for convolutional coding. FEC techniques typically use error correcting codes that can detect with high probability the error location. The Forward Error Control (FEC) consists of a Reed-Solomon (RS) outer code and a rate-compatible Convolutional Code (CC) (Andrews et al., 2007; Arafat, 2010).

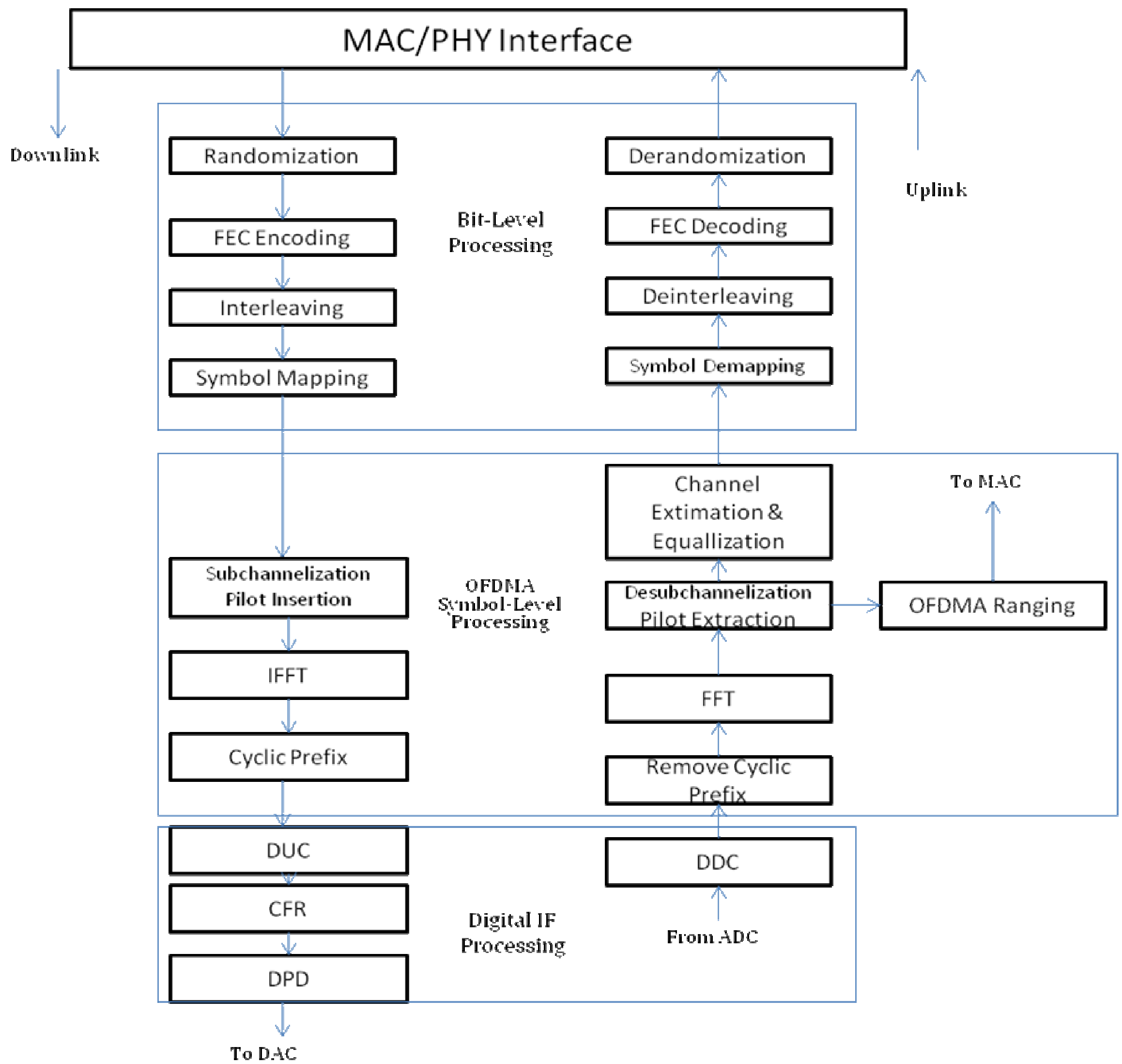

Figure 1. Physical layer.

Table 1. Parameters for OFDMA PHY.

Parameter	Value			
FFT size	128	512	1024	2048
Channel bandwidth (MHz)	1.25	5	10	20
Subcarrier frequency spacing (kHz)	10.94			
Useful symbol period	91.4			
Guard time	1/32,1/6,1/8,1/4			

CHANNEL EQUALIZER

We equalize the channel response using an equalizer. For the mobile WiMAX performance simulation we use the zero-force block equalizer and minimum mean square error equalizer at the receiver module of channel equalizer.

At baseband RF or IF, an equalizer can be implemented where most equalizers are implemented digitally after A/D conversion, since such filters are small, cheap, easily tune able and very power efficient. The goal of equalization is to mitigate the effects of ISI. However, this goal must be balanced so that in the process of removing ISI, the noise power in the received signal is not

Table 2. OFDMA parameters used in mobile WiMAX simulator.

Parameter	Value	
Channel bandwidth (MHz)	5	
Sampling frequency F_s(MHz)	5.6	
Sampling period $1/F_s$(µs)	0.18	
Subcarrier frequency spacing $\Delta f=F_s/N_{FFT}$(kHz)	10.94	
Useful symbol period $T_b=1/\Delta f$(µs)	91.4	
Guard Time $T_g=T_b/8$(µs)	11.4	
OFDMA symbol duration $T_s=T_b+T_g$		
	DL PUSC	**UL PUSC**
Number of used subcarrier(N_{used})	421	409
Number of pilot subcarriers	60	136
Number of data subcarriers	360	272
Number of subchannels	15	17
Number of users(N_{users})	3	3

enhanced (Alim, 2007).

Zero-force block equalizer

The zero-forcing equalizer removes all ISI, and is ideal when the channel is noiseless. However, when the channel is noisy, the zero-forcing equalizer will amplify the noise greatly at frequencies *f* where the channel response H(j2π*f*) has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely. Zero-forcing equalizers ignore the additive noise and may significantly amplify noise for channels with spectral nulls.

Mathematical model for zero forcing (ZF) equalizers

The samples $\{y_n\}c$ input to the equalizer can be represented based on the discretized combined system response $f(t) = h(t) * g^*(t)$ as

$$Y(z) = D(z)F(z) + N_g(z) \tag{1}$$

where $Ng(z)$ is the power spectrum of the white noise after passing through the matched filter G□(1/z□) and

$$F(z) = H(z)G_m^*\left(1/z^*\right) = \sum_n f(nT_s)z^{-n} \tag{2}$$

The zero-forcing equalizer removes all ISI introduced in the combined response *f*(*t*). From (1) we see that the equalizer to accomplish this is given by

$$H_{ZF(z)} = \frac{1}{F(z)} \tag{3}$$

This is the discrete-time equivalent to the analog equalizer 4.29 described above, and it suffers from the same noise enhancement properties (Figure 2). Specifically, the power spectrum *N*(*z*) is given by:

$$N(z) = N_g(z)\left|H_{ZF}(z)\right|^2 = \frac{N_0\left|G_m^*\left(1/z^*\right)\right|^2}{\left|F(z)\right|^2} = \frac{N_0\left|G_m^0\left(1/z^*\right)\right|^2}{\left|H(z)\right|^2\left|G_m^*\left(1/z^*\right)\right|^2} = \frac{N_0}{\left|H(z)\right|^2} \tag{4}$$

The noise power will be significantly increased, as is seen from 4, if the channel *H*(*z*) is sharply attenuated at any frequency within the bandwidth of interest, as is common on frequency-selective fading channels (Ohno, 2004). This motivates an equalizer design that better optimizes between ISI mitigation and noise enhancement. One such equalizer is the MMSE equalizer, described in the next section. The ZF equalizer defined by *HZF* (*z*) = 1/*F*(*z*) may not be implementable as a finite impulse response (FIR) filter. Specifically, it may not be possible to find a finite set of coefficients *w–L, . . , wL* such that

$$w_{-L}z^{L} + \cdots + w_{L}z^{-L} = \frac{1}{F(z)} \tag{5}$$

In this case we find the set of coefficients *{wi}* that best approximates the zero-forcing equalizer. It needs to be noted here that this is not straightforward since the approximation must be valid for all values of *z*. There are many ways we can make this approximation. One technique is to represent *HZF* (*z*) as an infinite impulse response (IIR) filter, 1/*F*(*z*) and then set $w_i = c_i$. It can be shown that this minimizes at *z* = *ejω*. Alternatively, the tap weights can be set to minimize the peak distortion (worst-case ISI). Finding the tap weights to minimize peak distortion is a convex optimization problem and can be solved by standard techniques, e.g. the method of steepest descent (Falconer, 2002; Kok, 2004; Sankar, 2008).

$$\left|\frac{1}{F(z)} - \left(w_{-L}z^{L} + \cdots + w_{L}z^{-L}\right)\right|^2 \tag{6}$$

Minimum Mean Square Error (MMSE) equalizer

A more balanced linear equalizer in this case is the minimum

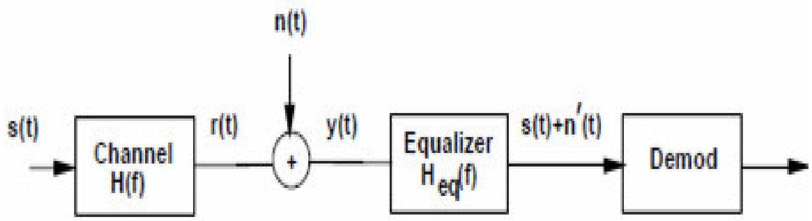

Figure 2. Analog equalizer illustrating enhancement.

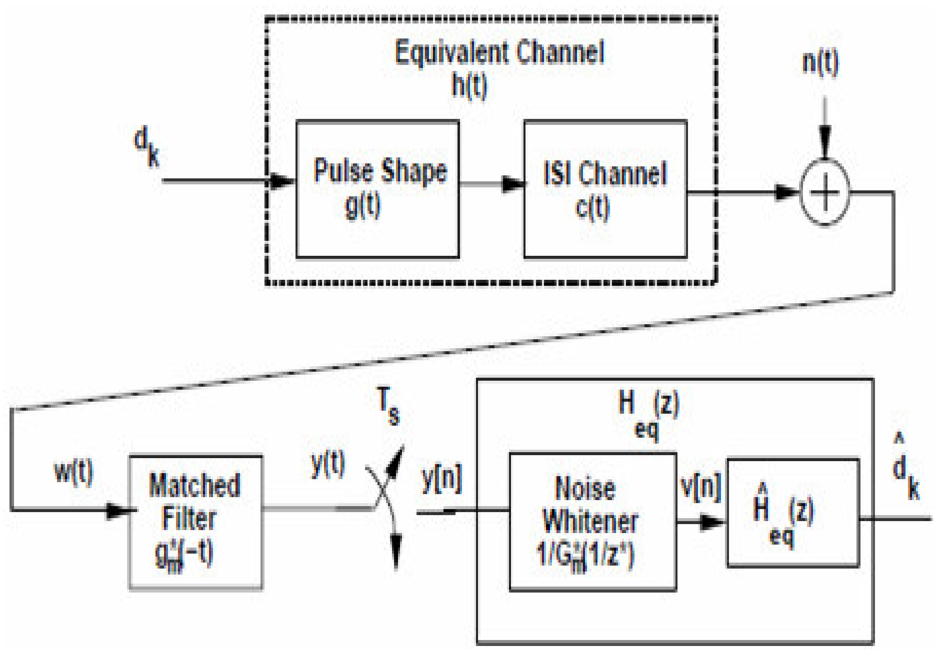

Figure 3. MMSE equalizer with noise whitening filter.

mean-square error equalizer, which does not usually eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output.

Minimum-Mean-Square Error (MMSE) equalizers minimize the mean-square error between the output of the equalizer and the transmitted symbol. They require knowledge of some auto and cross-correlation functions, which in practice can be estimated by transmitting a known signal over the channel.

Mathematical model for Minimum Mean Square Error (MMSE) equalizer

MMSE equalization aims to minimize the average mean square error (MSE) between the transmitted symbol d_k and its estimate ˆ d_k at the output of the equalizer. In other words, the *{wi}*'s are chosen to minimize E[d_k – ˆ d_k]2. Since the MMSE is a linear equalizer, its output ˆ d_k is a linear combination of the input samples *y*[*k*]

$$\hat{d}_k = \sum_{i=-L}^{L} w_i \, y[k-i] \tag{7}$$

In this way, for linear estimation, obtaining the optimal filter coefficients *{wi}* becomes a standard problem. This is known as a standard Weiner filtering problem if the noise input to the equalizer is white. However, because of the matched filter *g*m*(–*t*) at the receiver front end, the noise input to the equalizer is not white but colored with power spectrum *N*0/*G* m*(1/*z**)/2. Against this backdrop, to apply known techniques for optimal linear estimation, the filter *Heq*(*z*) was expanded into two components, a noise whitening component 1/*G* m*(1/*z*□) and an ISI removal component ˆ*Heq*(*z*), as shown in Figure 3.

As it can be anticipated by the name itself, the noise whitening filter whitens the noise such that the noise component output from this filter has a constant power spectrum. Since the noise input to this receiver has power spectrum N_0/*G8 m*(1/*z**)/2, the appropriate noise whitening filter is 1/*G*m*(1/*z**). The noise power spectrum at the output of the noise whitening filter is then N_0/*G* m*(1/*z**)/2/|*G* m*(1/*z**)/2 = N_0. Note that the filter 1/*G* m*(1/*z**) is not the only filter that will whiten the noise, and another noise whitening filter with more desirable properties (like stability) may be chosen. It might seem odd at first to introduce the matched filter *g * m*(–*t*) at the receiver front end only to cancel its effect in the equalizer. However, the matched filter is meant to maximize the SNR prior to sampling. By removing the effect of this matched filter through noise whitening after sampling, the design of ˆ$Heq(z)$ can merely be simplified to minimize MSE (Li Youming; Leshem 2005; Schniter 2008) **.** If the noise whitening filter does not yield optimal performance then its effect would be cancelled by the ˆ$Heq(z)$ filter design, as we will see below in the case of IIR, MMSE equalizers. We assume the filter ˆ$Heq(z)$, with input vn, is a linear filter with $N = 2L + 1$ taps:

$$\hat{H}_{eq}(z) = \sum_{i=-L}^{L} w_i z^{-i} \tag{8}$$

Our goal is to design the filter coefficients $\{w_i\}$ so as to minimize $E[d_k - \hat{}\, d_k]2$. This is the same goal as for the total filter $H_{eq}(z)$, we've just added the noise whitening filter to make solving for these coefficients simpler.

Define v = ($v[k + L], v[k + L - 1] \ldots, v[k - L]) = (vk+L, vk+L-1, \ldots, vk-L$)

as a vector of inputs to the filter ˆ$H_{eq}(z)$ used to obtain the filter output

ˆ d_k and w = ($w-L, \ldots, wL$) as the vector of filter coefficients.

Then,

$$\hat{d}_k = w^T v = v^T w \tag{9}$$

Thus, we want to minimize the mean square error

$$J = E\left[d_k - \hat{d}_k\right]^2 = E\left[w^T v v^H w^* - 2\Re\{v^H w^* d_k\} + |d_k|^2\right] \tag{10}$$

Where, $M_v = E[vv^H]$ and $v_d = E[v^H d_k]$. The matrix Mv is an $N \times N$ Hermitian matrix and v_d is a length N row vector. Assume $E/d_k/2 = 1$. Then the MSE J is

$$J = w^T M_v w^* - 2\Re\{v_d w^*\} + 1 \tag{11}$$

We obtain the optimal tap vector w by setting the gradient _wJ = 0 and solving for w. From (11) the gradient is given by

$$\nabla_w J = \left(\frac{\partial J}{\partial w_{-L}}, \cdots, \frac{\partial J}{\partial w_L}\right) = 2w^T M_v - 2v_d \tag{12}$$

Setting this to zero yields $w^T M_v = v_d$ or, equivalently, that the optimal tap weights are given by

$$w_{opt} = \left(M_v^T\right)^{-1} v_d^T \tag{13}$$

It needs to be noted here that solving for wopt requires a matrix inversion with respect to the filter inputs. Thus, this computation is plagued by quite high complexity, typically on the order of $N2$ to $N3$ operations. Substituting in these optimal tap weights we obtain the minimum mean square error as

$$J_{\min} = 1 - v_d M_v^{-1} v_d^H \tag{14}$$

For an infinite length equalizer,

v = ($vn+\infty, \ldots, vn, vn-\infty$) and w = ($w-\infty, \ldots, w0, \ldots, w\infty$). Then $w^T M_v = v_d$ can be written as

$$\sum_{i=-\infty}^{\infty} w_i\left(f|j-i| + N_0\right)\delta[j-i] = g_m^*[-j], \qquad -\infty \le j \le \infty \tag{15}$$

Taking z transforms and noting that ˆ$Heq(z)$ is the z transform of the filter coefficients w yields

$$\hat{H}_{eq}(z)\left(F(z) + N_0\right) = G^*\left(1/z^*\right) \tag{16}$$

Solving for ˆ$H_{eq}(z)$ yields

$$\hat{H}_{eq}(z) = \frac{G_m^*\left(1/z^*\right)}{F(z) + N_0} \tag{17}$$

Since the MMSE equalizer consists of the noise whitening filter $1/G\square m(1/z\square)$ plus the ISI removal component ˆ$H_{eq}(z)$, we get that the full MMSE equalizer, when it is not restricted to be finite length, becomes

$$H_{eq}(z) = \frac{\hat{H}_{eq}(z)}{G_m^*\left(1/z^*\right)} = \frac{1}{F(z) + N_0} \tag{18}$$

From the above, three interesting results need to be noted here. First of all, the noise whitening filter is cancelled out by the ideal infinite length MMSE equalizer (Youming, 2005; Leshem, 2005; Schniter, 2008). Second, this infinite length equalizer is identical to the ZF filter except for the noise term $N0$, so in the absence of noise the two equalizers are equivalent. Finally, this ideal equalizer design clearly shows a good balance between inverting the channel and noise enhancement: if $F(z)$ is highly attenuated at some frequency the noise term $N0$ in the denominator prevents the noise from being significantly enhanced by the equalizer. Yet the equalizer effectively inverts $F(z)$ at frequencies where the noise power spectral density $N0$ is small compared to the composite channel $F(z)$. For the equalizer (18) it can be shown that the minimum MSE (14) can be expressed in terms of the folded spectrum $F\Sigma(f)$ as

$$J_{\min} = T_s \int_{-.5/T_s}^{.5/T_s} \frac{N_0}{F\sum(f) + N_0}\, df \tag{19}$$

SIMULATION RESULTS

In this section, the simulation results obtained will be discussed to evaluate the performance, we used varying channel models such as AWGN,SUI and differents modulation,techniques. For experimentation purposes, simulation is done in Matlab over hundred iterations with the parameters in Table 3.

To evaluate the equalizer performance in Mobile WiMAX, SUI-3 channel model were used. We also assume that the channel comes with addition of AWGN.

Table 3. Simulation parameter.

Parameter	Value
Bandwidth(FFT)	256,512,1024
Cyclic prefix	1/8
Frame duration(TDD)	5 ms
Symbol time	102.90 µs
Channel models	SUI-3

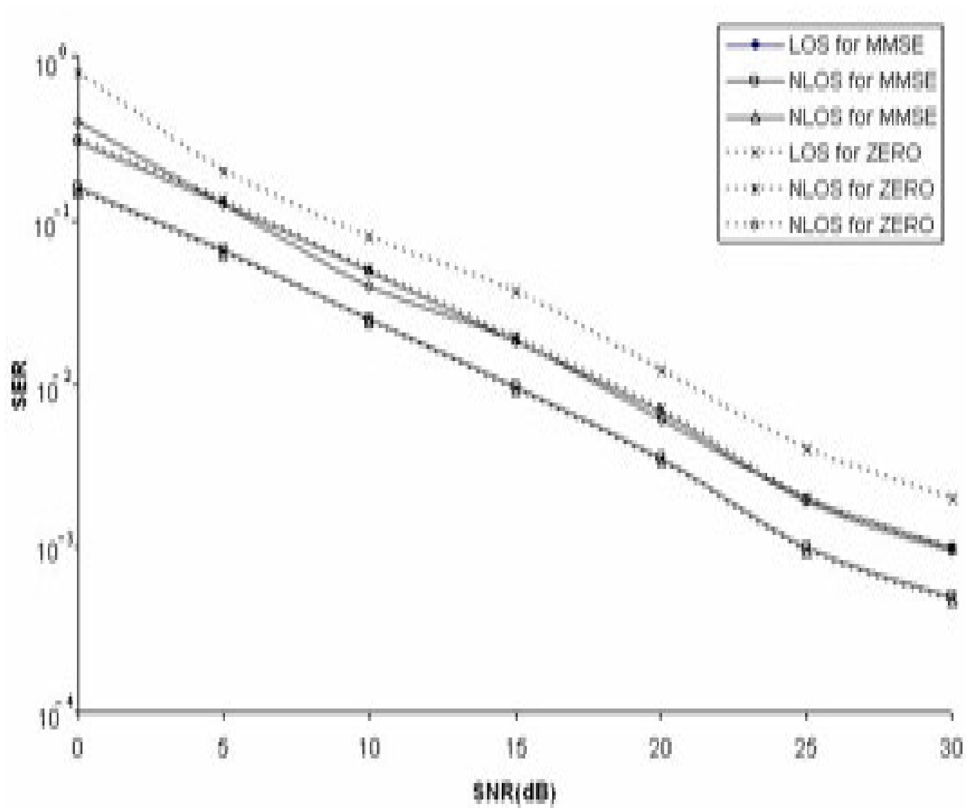

Figure 4. SER vs. SNR plot for BPSK modulation on channel SUI-3 at FFT 256.

For SUI-3 we assume following parameters: 7km is the cell size. BTS antenna height is 30 m. Receive antenna height is 6m. BTS antenna beam width is 120 degree. Receive antenna beam width is Omni-directional polarization. 90% cell coverage with 99.9% reliability at each location covered. The FFT size 256,512 and 1024 are considered in simulation for performance measurement purposes.

In Figures 4 - 15 the SER variations are analyzed versus the SNR values for different (BPSK, QPSK, 16QAM, 64QAM) modulation technique, FFT size and equalizer for the mobile WiMAX SUI-3 channel. The three channel paths consisting one unfaded LOS path and two NLOS paths which are used as a Rayleight fading channel was gotten.

In Figures 4, 5, 6 and 7 a comparison in term of SER of the zero force and MMSE equalizer is presented where the FFT size is 256 and BPSK, QPSK, 16QAM, 64QAM modulation technique are used. At the Figures 4 - 7, it is shown that the SER for MMSE equalizer is lower than the Zero force equalizer. Moreover the SER reduction is more significant with the QPSK, 16QAM and 64QAM modulation respectively. For Figures 4 and 5 it is observed that when the FFT is 256, the changes for the MMSE and ZF equalizer can be ignored. But in Figures 6 and 7, there is an important change for the different equalizers. By setting the SER at 10^{-3} the gain is equal to 2dB for MMSE for LOS and 4dB for NLOS .Again when SNR value is 13dB SER becomes zero for MMSE and when SNR>15dB the SER start increasing due to Inter

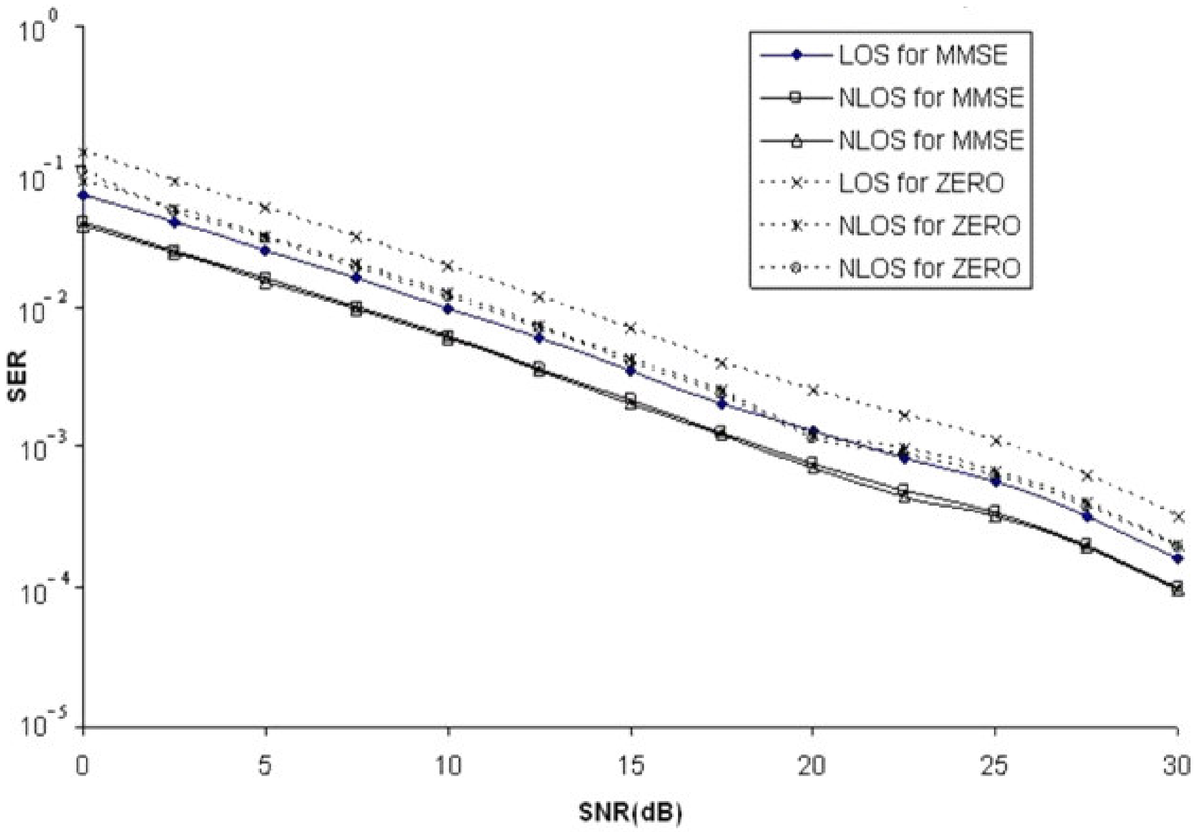

Figure 5. SER vs. SNR plot for QPSK modulation on channel SUI-3 at FFT 256.

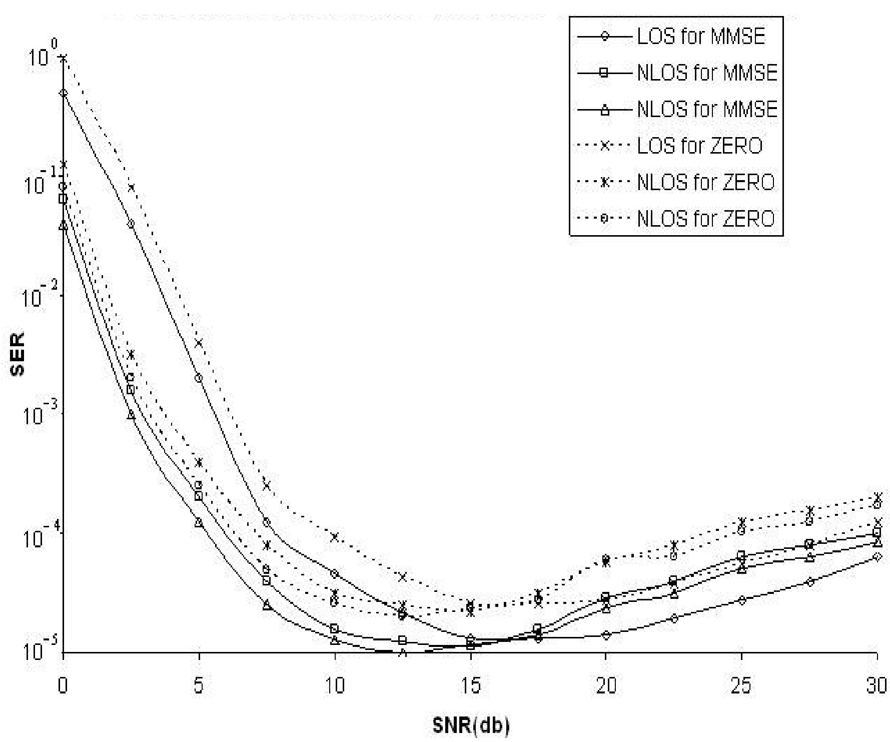

Figure 6. SER vs. SNR plot for 16QAM modulation on channel SUI-3 at FFT 256.

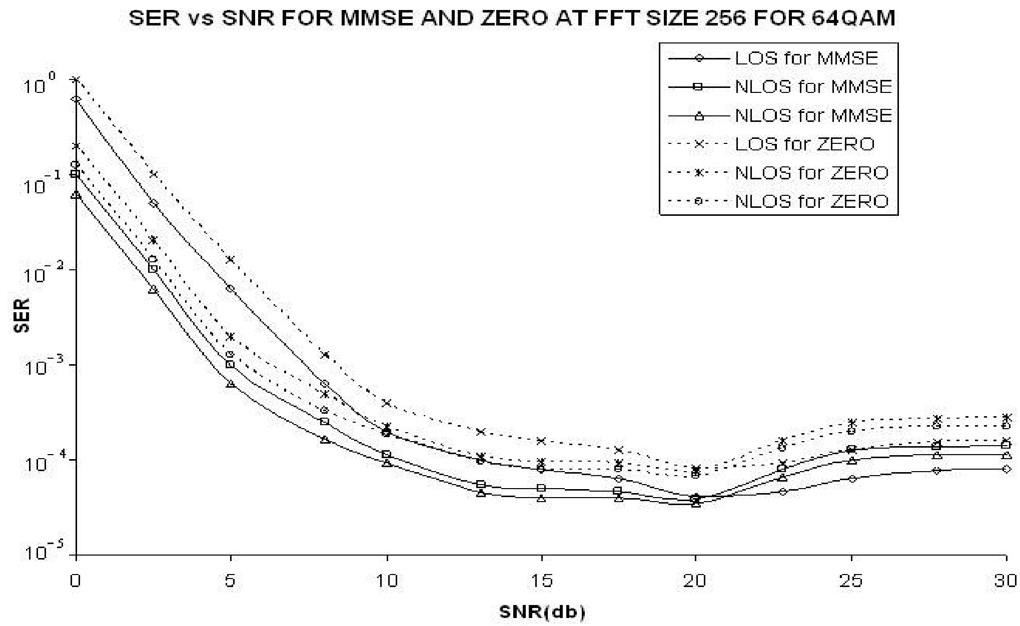

Figure 7. SER vs. SNR plot for 64QAM modulation on channel SUI-3 at FFT 256.

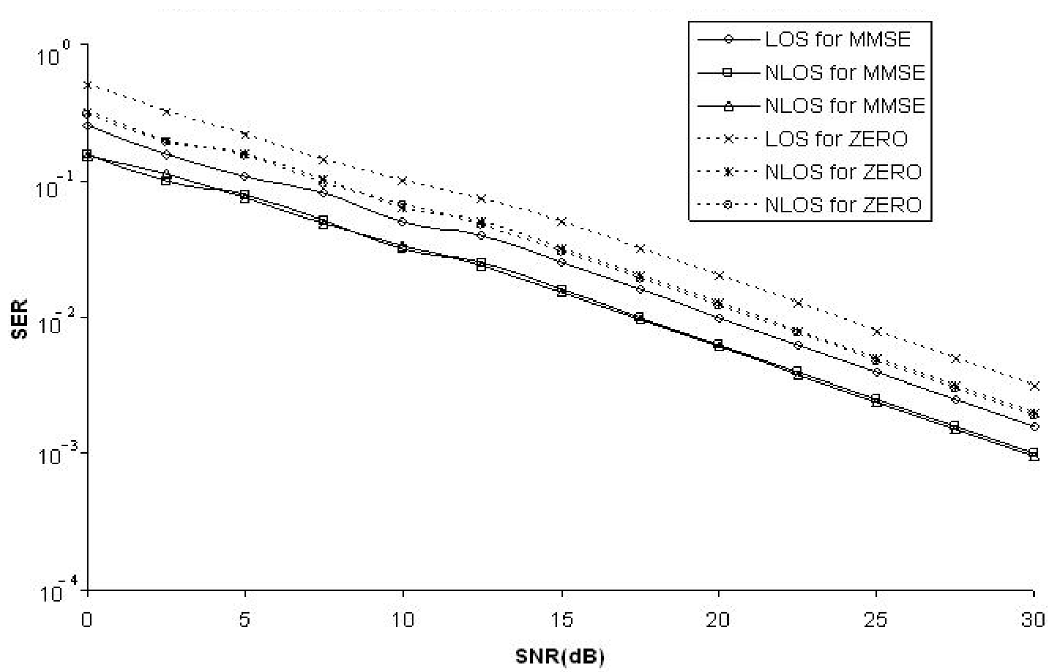

Figure 8. SER vs. SNR plot for BPSK modulation on channel SUI-3 at FFT 512.

symbol interference (ISI) .

Figures 8 - 11 shows the equalization performance for different modulation technique on SUI-3 channel where the FFT size is 512 and BPSK, QPSK, 16QAM, 64QAM modulation technique are used. Here also the SER for zero force equalizer is lower that the MMSE equalizer. And the SER reduction is more significant with the QPSK, 16QAM and 64QAM modulation respectively. At level 10^{-3} for Figures 10 and 11 the gain for zero force equalizer in LOS is 3 dB and in NLOS is 4 dB.

Figures 8 - 11 shows the equalization performance for different modulation technique on modified SUI channel

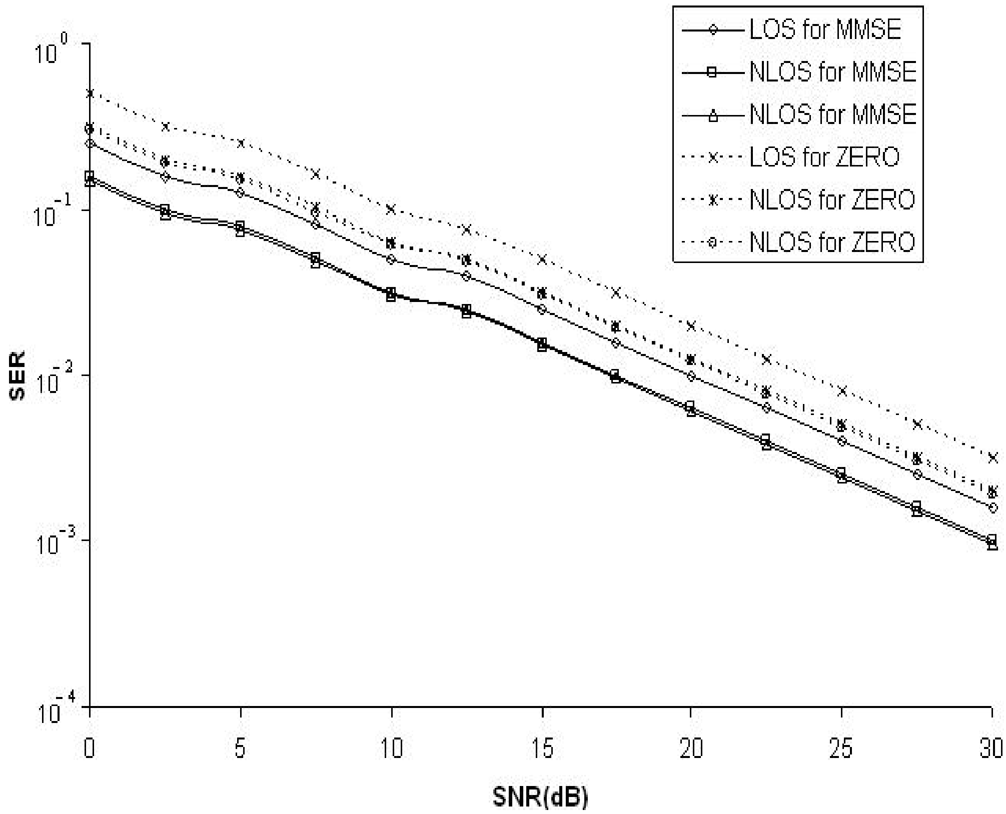

Figure 9. SER vs. SNR plot for QPSK modulation on channel SUI-3 at FFT 512.

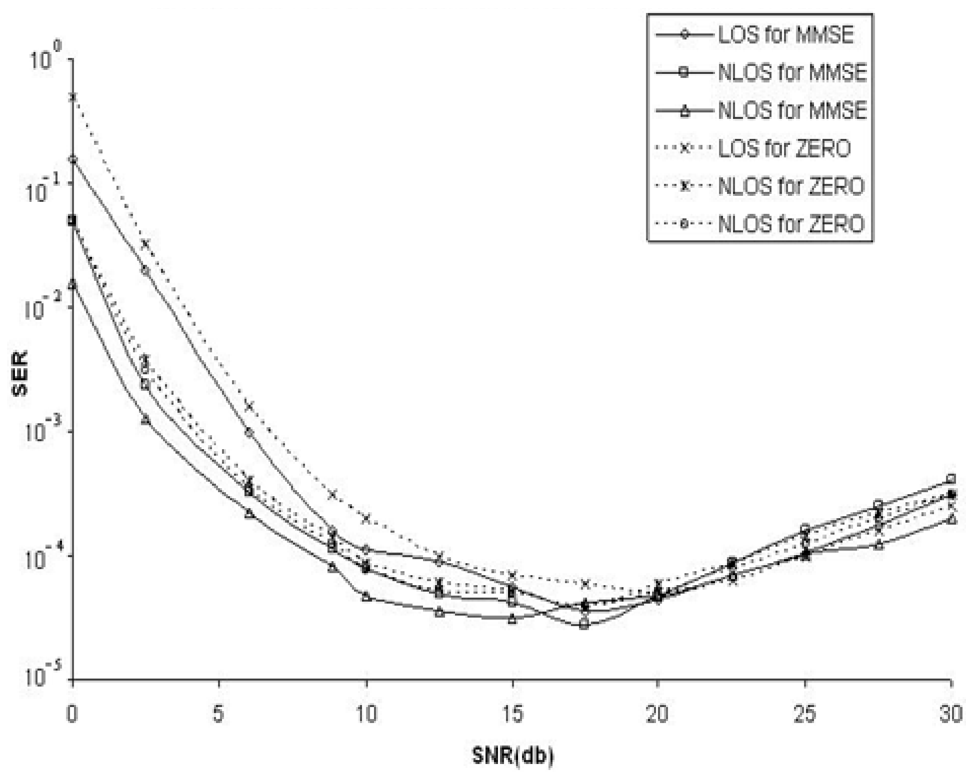

Figure 10. SER vs. SNR plot for 16QAM modulation on channel SUI-3 at FFT 512.

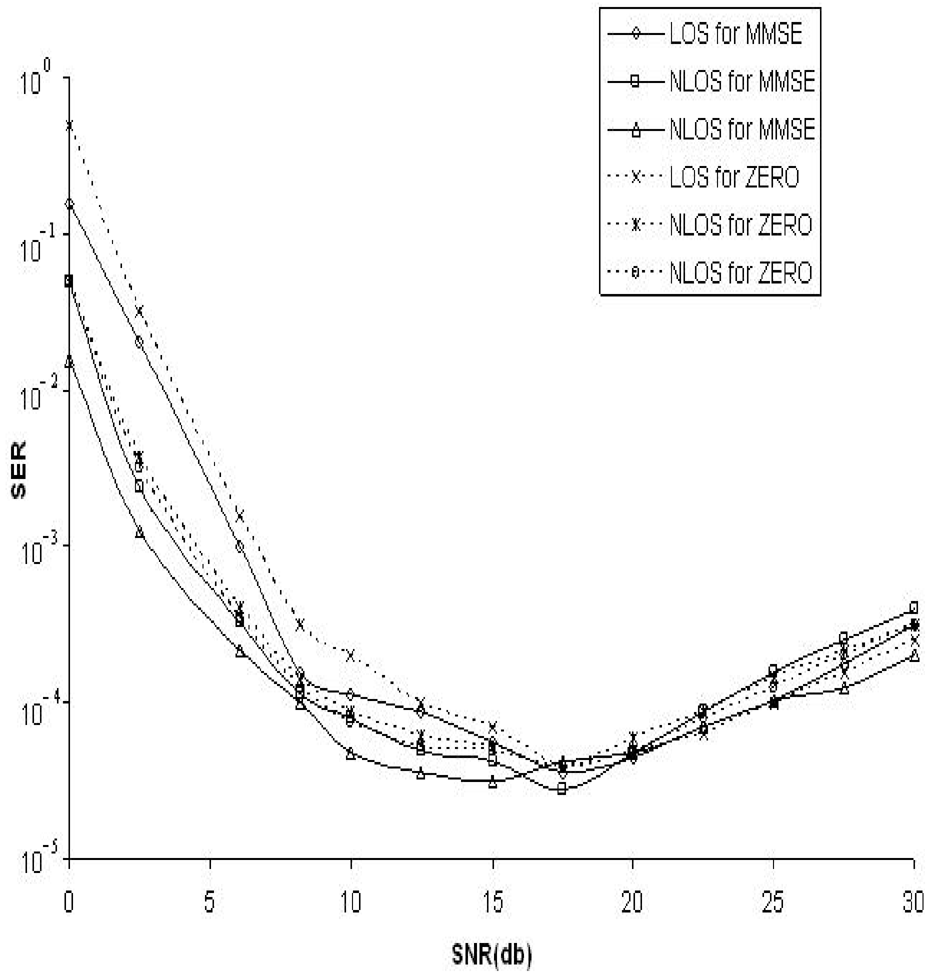

Figure 11. SER vs. SNR plot for 64QAM modulation on channel SUI-3 at FFT 512.

where the FFT size is 512 and BPSK, QPSK, 16 QAM, 64 QAM modulation techniques are used. Here also the SER for MMSE equalizer is lower than the Zero force equalizer. The SER reduction is more significant with the QPSK, 16 QAM and 64 QAM modulation respectively .At level 10^{-3} for the Figures 10 and 11 the gain for MMSE equalizer for LOS is 3 dB and for NLOS is 4 dB.

There is a cross over point for both 16 QAM and 64 QAM modulations (Figures 10 and 11). When SNR reach at 15 to 20 dB the error rate start increasing again. That means the receiver can tolerate SNR< 20dB for 16QAM and 64QAM modulation for any FFT size.

Figures 12, 13, 14 and 15 also shows the equalization performance for different modulation technique on modified sui channel where the FFT size is 1024 and BPSK, QPSK, 16QAM, 64QAM modulation technique are used. From the figure we found that the symbol error rate for zero force is less than the MMSE equalizer. The SER reduction is more significant with the QPSK, 16QAM and 64QAM modulation, respectively. At an SER level of 10^{-3} for the figures 14 and 15 the gain for MMSE equalizer in LOS is 5 dB and in NLOS is 6 dB.

DISCUSSION

The key observations from the above figures can be summarized as follows: (1) It is observed that SER is less for higher FFT size such as 1024 compared to lower FFT size such as 256 and 512. (2) For 16QAM and 64QAM the error rate decreases till a certain SNR value, after that error rate start increasing due to the ISI tolerance limit at receiver. (3) The curves corresponding to the 16 QAM and 64 QAM have identical slopes and the slopes of the SER curves are reduce with higher code rate. (4) For BPSK and QPSK modulation at any bandwidth (FFT size) the performance changes for MMSE and Zero Force equalizer are inconsequential. (5) Compared to the zero forcing equalizer case, the MMSE equalizer results in improvements at any SER point. (6) The MMSE equalizer is therefore the better method for equalizing the received symbols for the SOFDMA based Mobile WiMAX

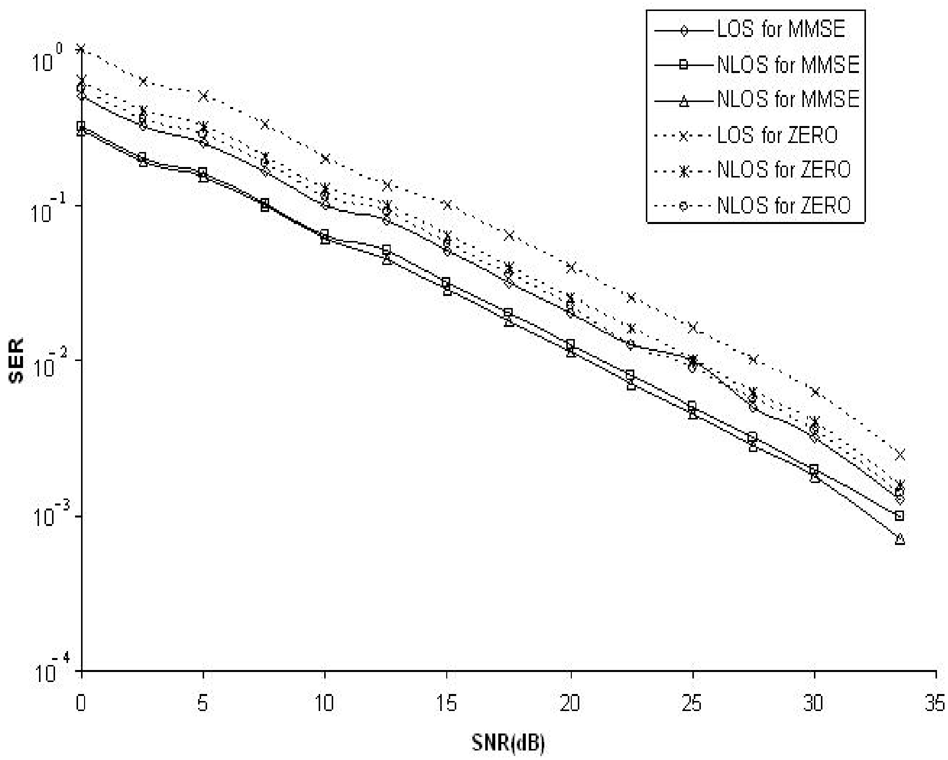

Figure 12. SER vs. SNR plot for BPSK modulation on channel SUI-3 at FFT 1024.

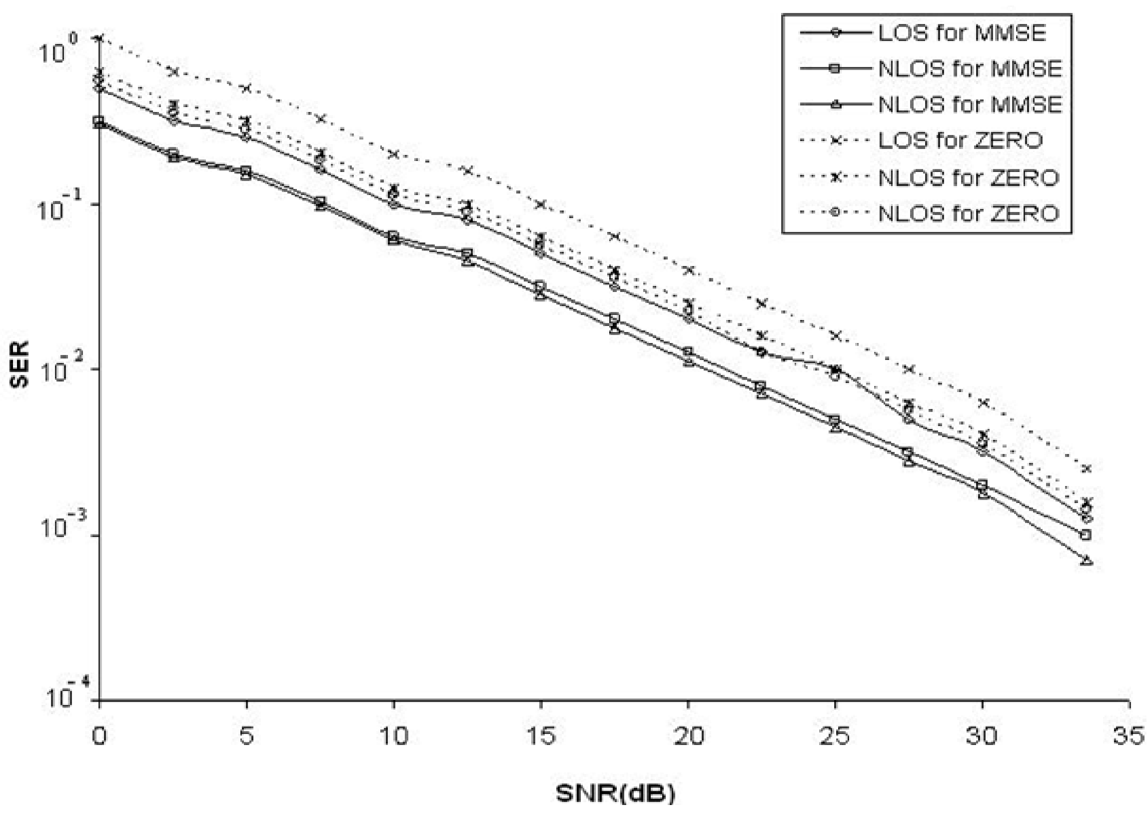

Figure 13. SER vs. SNR plot for QPSK modulation on channel SUI-3 at FFT 1024.

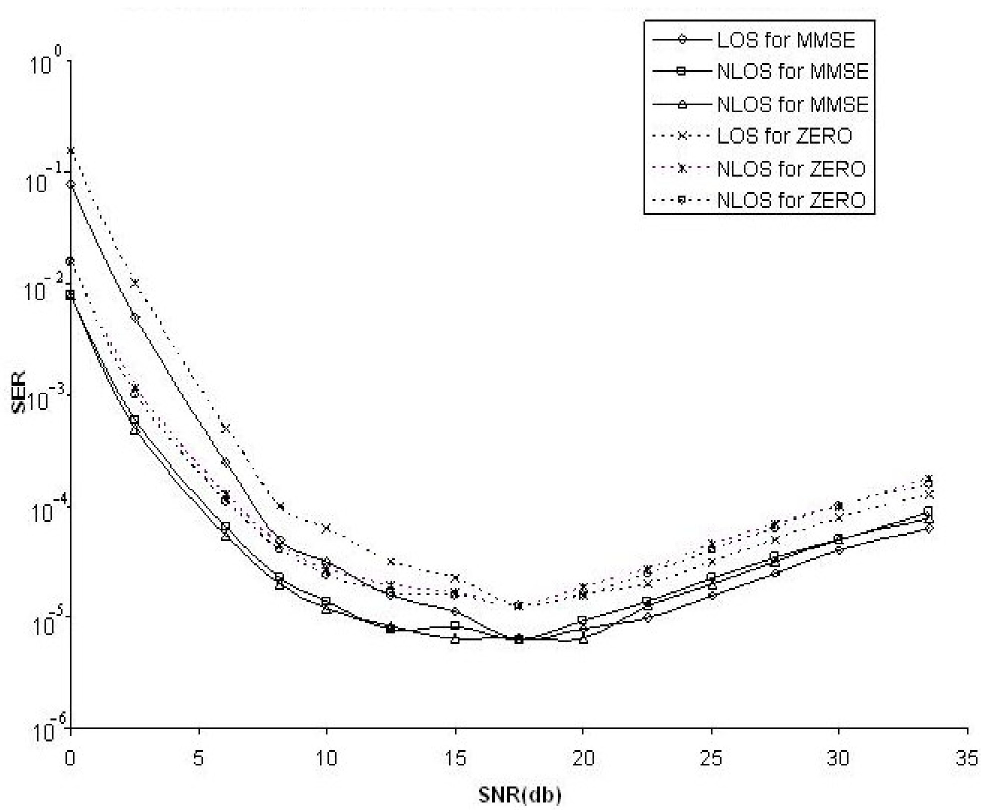

Figure 14. SER vs. SNR plot for 16QAM modulation on channel SUI-3 at FFT 1024.

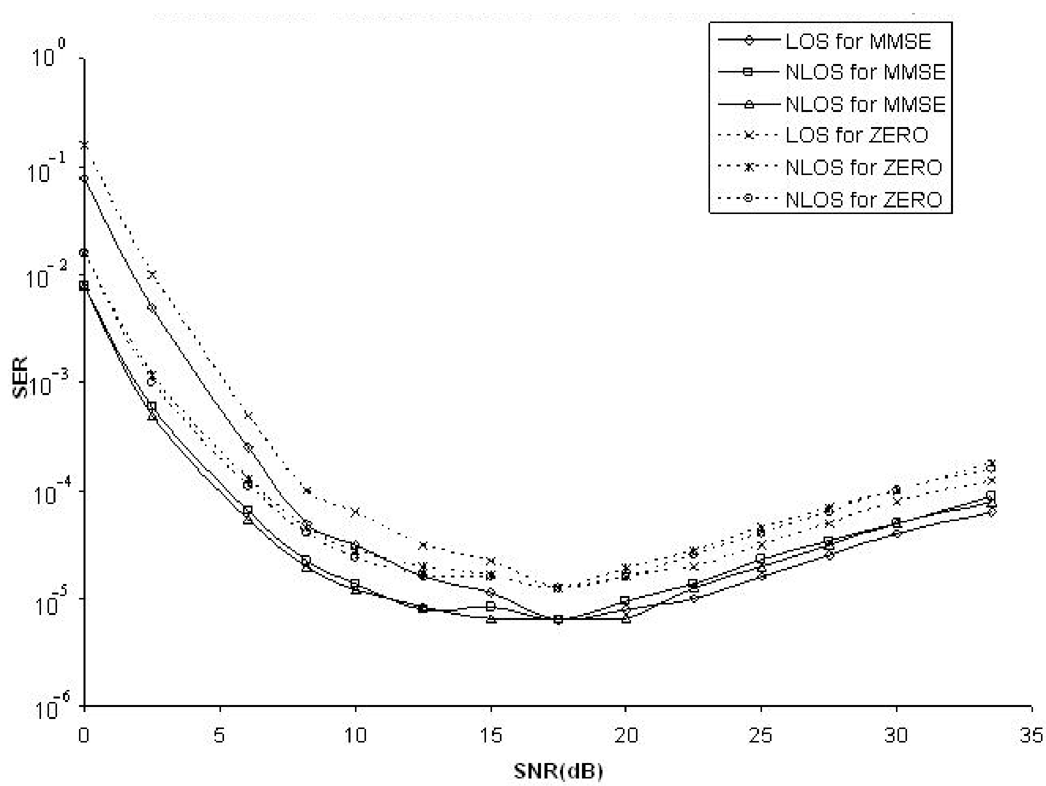

Figure 15. SER vs. SNR plot for 64QAM modulation on channel SUI-3 at FFT 1024.

physical layer.

Conclusion

This paper has analysed the performance differences between ZF and MMSE equalizers of mobile WiMAX physical layer on an SUI-3 channel at different FFT sizes and modulation techniques as assisted by Mobile IP (Internet protocol) for mobility management. Analysis demonstrated that performance of Mobile WiMAX with the MMSE equalizer is comparable to or slightly better than the ZF equalizer under almost all modulations, FFT sizes and channel conditions.

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