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Full Length Research Paper

Coefficient distribution of the stability function of a high order integrator

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This study considers the stability function of a high order integrator analysed by Onianwa and Aashikpelokhai (2007) for the sole purpose of having a sound knowledge of the way in which the coefficients of the stability function $s(u,v)=\left[\sum_{r=0}^{11}(2.12-1-r)!\binom{11}{r}\bar{h}^r\right]\left[\sum_{r=0}^{12}(-1)^r(2.12-1-r)!\binom{12}{r}\bar{h}^r\right]^{-1}$ **is distributed. The study enables us to draw the conclusion as an alternative proof that** $\|s(u,v)\|$ **lies in the unit ball in** R^3**. It also studied and established that the integrator is consistent and convergent.**

Key words: Consistency, convergence, open unit ball in R^3.

INTRODUCTION

The researchers considered the initial value problem

$$y' = f(x,y),\ y(a) = y_0,\ x\in [a,b\},\ y\in R \tag{1}$$

whose solution may be stiff and have singularities, but with continuous derivative of high order. This paper deals with the coefficient distribution of the stability function of a high order rational integrator. The methods in this class of integrators are L-stable; for this high order, the stability function s(u,v), given by (10) lies in the unit ball in R^3. Consequently, the researchers study the way the coefficients of the underlying numerator A(u,v)+iB(u,v) and denominator C(u,v)+iD(u,v) are distributed. The consistency and convergence of the integrator are also examined.

PROPERTIES OF THE INTEGRATORS: (CONSISTENCY AND CONVERGENCE)

Generally, the concept of consistency is very important in the sense that it controls the magnitude of the local truncation error committed at every integration step and is also crucial to the convergence of the method.

Convergence is an important tool in multistep methods in that it guarantees that by using a sufficiently small step and accurate computation, the numerical solution can be made arbitrarily close to the true solution. It is also a desirable property any numerical integration formula must posses. Consequently, Dahlquist (1956, 1959) established the following theorems which will guide numerical analysts in the formulation of new integration formulas.

Theorem 1

The multistep method $\sum_{j=0}^{k}\alpha_j y_{n+j} = h\sum_{j=o}^{k}\beta_j f_{n+j}$ is convergent if and only if it is consistent and zero stable.

Theorem 2

A zero stable multistep method (Theorem 4) is at best of the order, P = $\begin{cases}\mathrm{k+1,\ if\ \ k\ odd}\\ \mathrm{k+2,\ if\ \ k\,even}\end{cases}$

However, the proof of these theorems is available in

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the original papers of Dahlquist and Henrici (1962). Ideally, one will normally wish for a k – step multistep methods to be of highest possible order, but these goals are not always reachable due to Dahlquist barrier.

Thus according to Fatunla (1988), a one - step method $y_{n+1} = y_n + h_n\phi(x_n; y_n; h)$ where $\phi(x, y; h)$ is the increment function and h_n, the mesh size is said to be convergent if for an arbitrary initial vector y_0 and an arbitrary point x ∈ [a, b], the global error;

$$E_{n+1} = y_{n+1} - y(x_{n+1}) \quad (2)$$

Satisfies the following relationship:

$$\lim_{h\to 0}\left[\max_{n\to\infty} e_n\right] = 0$$

Provided x is always a mesh point.

Theorem 3

Consistency and convergence

The numerical integration formula given by

$$y_{n+1} = P_{11}(x_{n+1})[Q_{12}(x_{n+1})]^{-1}. \quad (3)$$

where,

$$P_{11} = \sum_{j=0}^{11} P_j x_{n+1}^j \quad (4)$$

and

$$Q_{12} = 1 + \sum_{j=1}^{12} q_j x_{n+1}^j \quad (5)$$

is convergent.

Proof

Recall that a one-step numerical integrator $y_{n+1} = y_n + h\phi(x_n, y_n, h_n)$ is convergent ⇔ it is consistent (Lambert, 1976).

Now if we assume that $P_0 = y_n$ and for the case k = 12 in (9), we have by considering

$$y_{n+1} = \left[p_0 + p_1x_{n+1} + p_2x_{n+1}^2 ... + p_{11}x_{n+1}^{11}\right]\left[1 + q_1x_{n+1} + q_2x_{n+1}^2 ... + q_{12}x_{n+1}^{12}\right]^{-1} \quad (6)$$

Re-arranging (6), we have

$$y_{n+1} - y_n = \left[p_{11}(x_{n+1}) - y_n q_{12}(x_{n+1})\right]\left[q_{12}(x_{n+1})\right]^{-1}$$

Therefore,

$$\frac{y_{n+1} - y_n}{h} = \left[p_{11}(x_{n+1}) - y_n q_{12}(x_{n+1})\right]\left[q_{12}(x_{n+1})\right]^{-1} h^{-1} \quad (7)$$

Taking limits of (7) as $h \to 0$, we have

$$\phi(x_n; y_n; 0) = \lim_{h\to 0}\left[\frac{y_{n+1} - y_n}{h}\right]$$
$$= (p_1 - q_1y_n)x_{n+1}$$
$$= hy_n'$$
$$= (p_1 - q_1p_0)x_{n+1}$$

where $y_n = p_0$

$$\Rightarrow \frac{hy_n'}{x_{n+1}} = p_1 - q_1y_n \quad (8)$$

Which implies that $\phi(x_n, y_n, 0) = y_n^{(1)} = f(x_n, y_n)$.

This establishes the proof. Hence the study concludes that the method is consistent. This also agrees with Lambert (1973), Dahlquist (1963), Fatunla (1988) and Aashikpelokhai (1991), consequently stable and convergent as in Henrici (1962).

ANALYSIS OF THE STABILITY FUNCTION

Stability is a vital element in every numerical computation of any integrator. Lambert (1976) opined that stability implies the existence of a positive h* such that for h ∈(0,h*), stable propagation of error will occur. Because of the difficulty in analyzing rational functions as compared with the linear functions, very little theoretical results are available in the case of rational integration methods.

By and large, one major concern is the stability of results at poles of singular problems. Therefore, some of these schemes are those of Lambert and Shaw (1965), Lambert (1974), Luke and Wimp (1975), Niekerk (1987), Fatunla (1982, 1986, 1994), Fatunla and Aashikpelokhai (1994) and Otunta and Ikhile (1996).

Our study on the stability function in this paper have been restricted to the case k = 12. The work confirmed the difficulties in analyzing rational integrators (Fatunla, 1982, 1988; Lambert, 1973, 1995). However, the analysis of the stability function shall be restricted to the u-v plane.

Now considering $y' = \lambda y$ and substituting into the formula of the integrator, we obtain

$$\frac{y_{n+1}}{y_n} = \left[\sum_{r=0}^{11}(2.12-1-r)!\binom{11}{r}\bar{h}^r\right]\left[\sum_{r=0}^{12}(-1)^r(2.12-1-r)!\binom{12}{r}\bar{h}^r\right]^{-1} \tag{9}$$

This is the stability function. Next we designate the stability function by $s(\bar{h})$ and as usual set $\bar{h} = u + iv$ leading us to the stability function as:

$$s(u,v) = \left[\sum_{r=0}^{11}(2.12-1-r)!\binom{11}{r}(u+iv)^r\right]\left[\sum_{r=0}^{12}(-1)^r(2.12-1-r)!\binom{12}{r}(u+iv)^r\right]^{-1} \tag{10}$$

Theorem 4

Given the stability function in (10);

$$s(u,v) = \left[\sum_{r=0}^{11}(2.12-1-r)!\binom{11}{r}(u+iv)^r\right]\left[\sum_{r=0}^{12}(-1)^r(2.12-1-r)!\binom{12}{r}(u+iv)^r\right]^{-1}$$

Then $|s(u,v)|$, lies in the unit ball whenever u < 0.

Proof

Consider $s(u,v) = \dfrac{A(u,v)+iB(u,v)}{C(u,v)+iD(u,v)}$

Then $|s(u,v)| \le 1$

$$\Leftrightarrow |s(u,v)|^2 \le 1$$

This holds $\dfrac{|A(u,v)+iB(u,v)|^2}{|C(u,v)+iD(u,v)|^2} \le 1$

This in turn holds

$$\Leftrightarrow A^2(u,v)+B^2(u,v) \le C^2(u,v)+D^2(u,v)$$
$$\Leftrightarrow \{A^2-C^2\}+\{B^2-D^2\} \le 0$$

Now by setting $\phi(u,v) = A(u,v)+iB(u,v)$

where $A(u,v) = R_e\phi(u,v)$

$$B(u,v) = I_m\phi(u,v)$$

and

$$\phi(u,v) = \sum_{r=0}^{11}(2.12.1-r)!\binom{11}{r}(u+iv)^r, \quad i=\sqrt{-1} \tag{11}$$

We then obtain by isolating the real parts from the imaginary parts in (11) as follows;

$$\phi_1(u,v) = A = \left[(2.12-1-0)!\binom{11}{0}\binom{0}{0} + (2.12-1-1)!\binom{11}{1}\binom{1}{0}\right]u + (2.12-1-2)!\binom{11}{2}\binom{2}{0}u^2$$

$$-(2.12-1-2!\binom{11}{2}\binom{11}{2}v^2 + (2.12-1-3)!\binom{11}{3}\binom{3}{}u^3 - (2.12-1-3)!\binom{11}{3}\binom{3}{2}uv^2$$

$$+(2.12-1-4)!\binom{11}{4}\binom{4}{0}u^4 - (2.12-1-4)!\binom{11}{4}\binom{4}{2}u^2v^2 + (2.12-1-4)!\binom{11}{4}\binom{4}{4}v^4$$

$$+(2.12-1-5)!\binom{11}{5}\binom{5}{0}u^5 - (2.12-1-5)!\binom{11}{5}\binom{5}{2}u^3v^2 + (2.12-1-5)!\binom{11}{5}\binom{5}{4}uv^4$$

$$
\begin{aligned}
&+(2.12-1-6)!\binom{11}{6}\binom{6}{0}u^{6}-(2.12-1-6)!\binom{11}{6}\binom{6}{2}u^{4}v^{2}+(2.12-1-6)!\binom{11}{6}\binom{6}{4}u^{2}v^{4}\\
&-(2.12-1-6)!\binom{11}{6}\binom{6}{6}v^{6}+(2.12-1-7)!\binom{11}{7}\binom{7}{0}u^{7}-(2.12-1-7)!\binom{11}{7}\binom{7}{2}u^{5}v^{2}\\
&+(2.12-1-7)!\binom{11}{7}\binom{7}{4}u^{3}v^{4}-(2.12-1-7)!\binom{11}{7}\binom{7}{6}uv^{6}+(2.12-1-8)!\binom{11}{8}\binom{8}{2}u^{8}\\
&-(2.12-1-8)!\binom{11}{8}\binom{8}{2}u^{6}v^{2}+(2.12-1-8)!\binom{11}{8}\binom{8}{4}u^{4}v^{4}-(2.12-1-8)!\binom{11}{8}\binom{8}{6}u^{2}v^{6}\\
&+(2.12-1-8)!\binom{11}{8}\binom{8}{8}v^{8}+(2.12-1-9)!\binom{11}{9}\binom{9}{0}u^{9}-(2.12-1-9)!\binom{11}{9}\binom{9}{2}u^{7}v^{2}\\
&+(2.12-1-9)!\binom{11}{9}\binom{9}{4}u^{5}v^{4}-(2.12-1-9)!\binom{11}{9}\binom{9}{6}u^{3}v^{6}+(2.12-1-9)!\binom{11}{9}\binom{9}{8}uv^{8}\\
&+(2.12-1-10)!\binom{11}{10}\binom{10}{0}u^{10}-(2.12-1-10)!\binom{11}{10}\binom{10}{2}u^{8}v^{2}+(2.12-1-10)!\binom{11}{10}\binom{10}{4}u^{6}v^{4}\\
&-(2.12-1-10)!\binom{11}{10}\binom{10}{6}u^{4}v^{6}+(2.12-1-10)!\binom{11}{10}\binom{10}{8}u^{2}v^{8}-(2.12-1-10)!\binom{11}{10}\binom{10}{10}v^{10}\\
&+(2.12-1-11)!\binom{11}{11}\binom{11}{0}u^{11}-(2.12-1-11)!\binom{11}{11}\binom{10}{2}u^{9}v^{2}+(2.12-1-11)!\binom{11}{11}\binom{10}{4}u^{7}v^{4}\\
&-(2.12-1-11)!\binom{11}{11}\binom{11}{6}u^{5}v^{6}+(2.12-1-11)!\binom{11}{11}\binom{10}{8}u^{3}v^{8}-(2.12-1-11)!\binom{11}{11}\binom{11}{10}uv^{10}
\end{aligned}
\tag{12}
$$

Next is

$$
\begin{aligned}
\phi_1(u,v)=B=\;&(2.12-1-1)!\binom{11}{0}\binom{1}{1}v+(2.12-1-2)!\binom{11}{2}\binom{2}{1}uv+(2.12-1-3)!\binom{11}{3}\binom{3}{1}u^{2}v\\
&-(2.12-1-3)!\binom{11}{3}\binom{3}{3}v^{3}+(2.12-1-4)!\binom{11}{4}\binom{4}{1}u^{3}v-(2.12-1-4)!\binom{11}{4}\binom{4}{3}uv^{3}\\
&+(2.12-1-5)!\binom{11}{5}\binom{5}{1}u^{4}v-(2.12-1-5)!\binom{11}{5}\binom{5}{3}u^{2}v^{3}+(2.12-1-5)!\binom{11}{5}\binom{5}{5}v^{5}\\
&+(2.12-1-6)!\binom{11}{6}\binom{6}{1}u^{5}v-(2.12-1-6)!\binom{11}{6}\binom{6}{3}u^{3}v^{3}+(2.12-1-6)!\binom{11}{6}\binom{6}{5}uv^{5}\\
&+(2.12-1-7)!\binom{11}{7}\binom{7}{1}u^{6}v-(2.12-1-7)!\binom{11}{7}\binom{7}{3}u^{4}v^{3}+(2.12-1-7)!\binom{11}{7}\binom{7}{5}u^{2}v^{5}\\
&-(2.12-1-7)!\binom{11}{7}\binom{7}{7}v^{7}+(2.12-1-8)!\binom{11}{8}\binom{8}{1}u^{7}v-(2.12-1-8)!\binom{11}{8}\binom{8}{3}u^{5}v^{3}\\
&+(2.12-1-8)!\binom{11}{8}\binom{8}{5}u^{3}v^{5}-(2.12-1-8)!\binom{11}{8}\binom{8}{7}uv^{7}+(2.12-1-9)!\binom{11}{9}\binom{9}{1}u^{8}v
\end{aligned}
$$

$$-(2.12-1-9)!\binom{11}{9}\binom{9}{3}u^6v^3+(2.12-1-9)!\binom{11}{9}\binom{9}{5}u^4v^5+(2.12-1-9)!\binom{11}{9}\binom{9}{7}u^2v^7$$
$$+(2.12-1-9)!\binom{11}{9}\binom{9}{9}v^9+(2.12-1-10)!\binom{11}{10}\binom{10}{1}u^9v-(2.12-1-10)!\binom{11}{10}\binom{10}{3}u^7v^3$$
$$+(2.12-1-10)!\binom{11}{10}\binom{10}{5}u^5v^5-(2.12-1-10)!\binom{11}{10}\binom{10}{7}u^3v^7+(2.12-1-10)!\binom{11}{10}\binom{10}{9}uv^9$$
$$+(2.12-1-11)!\binom{11}{11}\binom{11}{1}u^{10}v-(2.12-1-11)!\binom{11}{11}\binom{11}{3}u^8v^3+(2.12-1-11)!\binom{11}{11}\binom{11}{5}u^6v^9$$
$$+(2.12-1-11)!\binom{11}{11}\binom{11}{7}u^4v^7+(2.12-1-11)!\binom{11}{11}\binom{11}{9}u^2v^9-(2.12-1-11)!\binom{11}{11}\binom{11}{11}v^{11}. \quad (13)$$

Similarly, we set

$$C(u,v)=R_e\psi(u,v)$$
$$D(u,v)=I_m\psi(u,v)$$

where $\psi(u,v)=\sum_{r=0}^{12}(-1)^r(2.12-1-r)!\binom{12}{r}(u+iv)^r$

We then obtain by isolation all the real parts from the imaginary parts, we have

$$\psi_1(u,v)=C=(2.12-1-0)!\binom{12}{0}\binom{0}{0}-(2.12-1-1)!\binom{12}{1}\binom{1}{0}u+(2.12-1-2)!\binom{12}{2}\binom{2}{0}u^2$$
$$-(2.12-1-2)!\binom{12}{2}\binom{2}{2}v^2-(2.12-1-3)!\binom{12}{3}\binom{3}{0}u^3+(2.12-1-3)!\binom{12}{3}\binom{3}{2}uv^2$$
$$+(2.12-1-4)!\binom{12}{4}\binom{4}{0}u^4-(2.12-1-4)!\binom{12}{4}\binom{4}{2}u^2v^2+(2.12-1-4)!\binom{12}{4}\binom{4}{4}v^2$$
$$-(2.12-1-5)!\binom{12}{5}\binom{5}{0}u^5+(2.12-1-5)!\binom{12}{5}\binom{5}{2}u^3v^2-(2.12-1-5)!\binom{12}{5}\binom{5}{4}uv^4$$
$$+(2.12-1-6)!\binom{12}{6}\binom{6}{0}u^6-(2.12-1-6)!\binom{12}{6}\binom{6}{2}u^4v^2+(2.12-1-6)!\binom{12}{6}\binom{6}{4}u^2v^4$$
$$-(2.12-1-6)!\binom{12}{6}\binom{6}{6}v^5-(2.12-1-7)!\binom{12}{7}\binom{7}{0}u^7+(2.12-1-7)!\binom{12}{7}\binom{7}{2}u^5v^2$$
$$-(2.12-1-7)!\binom{12}{7}\binom{7}{4}u^3v^4+(2.12-1-7)!\binom{12}{7}\binom{7}{0}uv^6+(2.12-1-8)!\binom{12}{8}\binom{8}{0}u^8$$
$$-(2.12-1-8)!\binom{12}{8}\binom{8}{2}u^6v^2+(2.12-1-8)!\binom{12}{8}\binom{8}{4}u^4v^4-(2.12-1-8)!\binom{12}{8}\binom{8}{6}u^2v^6$$

$$+(2.12-1-9)!\binom{12}{9}\binom{9}{6}u^3v^6-(2.12-1-9)!\binom{12}{9}\binom{9}{8}uv^8+(2.12-1-10)!\binom{12}{10}\binom{10}{0}u^{10}$$

$$(2.12-1-10)!\binom{12}{10}\binom{10}{2}u^8v^2+(2.12-1-10)!\binom{12}{10}\binom{10}{4}u^6v^4-(2.12-1-10)!\binom{12}{10}\binom{10}{6}u^4v^6$$

$$+(2.12-1-10)!\binom{12}{10}\binom{10}{8}u^2v^8+(2.12-1-10)!\binom{12}{10}\binom{10}{10}v^{10}-(2.12-1-11)!\binom{12}{11}\binom{10}{0}u^{11}$$

$$+(2.12-1-11)!\binom{12}{11}\binom{11}{2}u^9v^2-(2.12-1-11)!\binom{12}{11}\binom{11}{4}u^7v^4+(2.12-1-11)!\binom{12}{11}\binom{11}{6}u^5v^6$$

$$-(2.12-1-11)!\binom{12}{11}\binom{11}{8}u^3v^8+(2.12-1-11)!\binom{12}{11}\binom{11}{10}uv^{10}+(2.12-1-12)!\binom{12}{12}\binom{12}{0}u^{12}$$

$$-(2.12-1-12)!\binom{12}{12}\binom{12}{2}u^{10}v^2+(2.12-1-112)!\binom{12}{12}\binom{12}{4}u^8v^4-(2.12-1-12)!\binom{12}{12}\binom{12}{6}u^6v^6$$

$$+(2.12-1-12)!\binom{12}{12}\binom{12}{8}u^4v^8+(2.12-1-12)!\binom{12}{12}\binom{12}{10}u^2v^{10}-(2.12-1-12)!\binom{12}{12}\binom{12}{12}v^{12} \quad (14)$$

While $\psi_2(u,v)=D$

$$=i\left[-(2.12-1-1)!\binom{12}{1}\binom{1}{1}v+(2.12-1-2)!\binom{12}{2}\binom{2}{1}\right]uv-(2.12-1-3)!\binom{12}{3}\binom{3}{1}u^2v$$

$$+(2.12-1-3)!\binom{12}{3}\binom{3}{3}v^3+(2.12-1-4)!\binom{12}{4}\binom{4}{1}u^3v-(2.12-1-4)!\binom{12}{4}\binom{4}{3}uv^3$$

$$-(2.12-1-5)!\binom{12}{5}\binom{5}{1}u^4v+(2.12-1-5)!\binom{12}{5}\binom{5}{3}u^2v^3-(2.12-1-5)!\binom{12}{5}\binom{5}{5}v^5$$

$$+(2.12-1-6)!\binom{12}{6}\binom{6}{1}u^5v-(2.12-1-6)!\binom{12}{6}\binom{6}{3}u^3v^3+(2.12-1-6)!\binom{12}{6}\binom{6}{5}uv^5$$

$$-(2.12-1-7)!\binom{12}{7}\binom{7}{1}u^6v+(2.12-1-7)!\binom{12}{7}\binom{7}{3}u^4v^3+(2.12-1-7)!\binom{12}{7}\binom{7}{5}u^2v^5$$

$$+(2.12-1-7)!\binom{12}{7}\binom{7}{7}v^7+(2.12-1-8)!\binom{12}{8}\binom{8}{1}u^7v-(2.12-1-8)!\binom{12}{8}\binom{8}{3}u^5v^3$$

$$+(2.12-1-8)!\binom{12}{8}\binom{8}{5}u^3v^5-(2.12-1-8)!\binom{12}{8}\binom{8}{7}uv^7-(2.12-1-9)!\binom{12}{9}\binom{9}{1}u^8v$$

$$+(2.12-1-9)!\binom{12}{9}\binom{9}{3}u^6v^3-(2.12-1-9)!\binom{12}{9}\binom{9}{5}u^4v^5+(2.12-1-9)!\binom{12}{9}\binom{9}{7}u^2v^7$$

$$-(2.12-1-9)!\binom{12}{9}\binom{9}{9}v^9-(2.12-1-10)!\binom{12}{10}\binom{10}{1}u^9v-(2.12-1-10)!\binom{12}{10}\binom{10}{3}u^7v^3$$

$$\begin{aligned}
&+(2.12-1-10)!\binom{12}{10}\binom{10}{5}u^5v^5-(2.12-1-10)!\binom{12}{10}\binom{10}{7}u^3v^7+(2.12-1-10)!\binom{12}{10}\binom{10}{9}uv^9\\
&-(2.12-1-11)!\binom{12}{11}\binom{11}{1}u^{10}v+(2.12-1-11)!\binom{12}{11}\binom{11}{3}u^8v^3-(2.12-1-11)!\binom{12}{11}\binom{11}{5}u^6v^5\\
&+(2.12-1-11)!\binom{12}{11}\binom{11}{7}u^4v^7-(2.12-1-11)!\binom{12}{11}\binom{11}{9}u^2v^9+(2.12-1-11)!\binom{12}{12}\binom{11}{11}v^{11}\\
&+(2.12-1-12)!\binom{12}{12}\binom{12}{1}\mathrm{u}^{12}\mathrm{v}^1-(2.12-1-12)!\binom{12}{12}\binom{12}{3}u^9v^3+(2.12-1-12)!\binom{12}{12}\binom{15}{5}u^7v^5\\
&-(2.12-1-12)!\binom{12}{12}\binom{12}{7}\mathrm{u}^5\mathrm{v}^7+(2.12-1-12)!\binom{12}{12}\binom{12}{9}u^3v^9+(2.12-1-12)!\binom{12}{12}\binom{15}{11}uv^{11}]
\end{aligned} \quad (15)$$

From the aforementioned, we have

$$\begin{aligned}\psi(u,v) &= \psi_1(u,v)+\psi_2(u,v)\\ &= C+iD\end{aligned}$$

where $\psi_1(u,v)=C$ represents the real part while $\psi_2(u,v)=D$ represents the imaginary part. Therefore, by squaring and adding the terms in (12 to 15), we obtain

$$\left\{A^2(u,v)-C^2(u,v)\right\}+\left\{B^2(u,v)-D^2(u,v)\right\}\leq 0$$

Whenever u < 0. Indeed the terms involving the invoking of u < 0 are the terms in $(A^2(u,v)+B^2(u,v)-(C^2(u,v)+D^2(u,v))$ which are positive, the rest yield < 0 independent of the sign of u (Proof established).

Conclusion

From our study, we observe that the characteristics bordering on the consistency, convergence and stability of the integrator was found to be of the desired type and that $\left|s(u,v)\right|$ was equally found to lie in the unit ball whenever u < 0.

It was observed that from the expansion that a sequence of alternating signs of the various coefficients of the parameters A (u, v), B (u, v), C (u, v) and D (u,v) was obtained. Also the pairs of A (u, v), C (u, v) and B (u, v), D (u, v) took even and odd coefficients, respectively.

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