International Journal of the Physical Sciences Vol. 5(6), pp. 793-803, June 2010
Available online at http://www.academicjournals.org/IJPS
ISSN 1992 - 1950 ©2010 Academic Journals

Full Length Research Paper

Damped free vibration analysis of a beam with a fatigue crack using energy balance method

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Accepted 20 April, 2010

In this paper, a new approach based on the energy balance method is proposed for free vibration analysis of a cracked cantilever beam by taking into account both the structural damping and the damping due to the crack. Also, by taking into account the effect of opening and closing the crack during the beam vibration, the stiffness changes at the crack location are considered to be a nonlinear amplitude-dependent function which causes the frequencies and mode shapes of the beam to vary continuously with time. The results show that neglecting the effects of structural damping and nonlinear behavior of the crack will be a source of considerable error in obtaining the dynamic response and vibration characteristics of the cracked beam. In order to validate the results obtained through the proposed method, some experimental test have been conducted.

Key words: Damped vibration, nonlinear fatigue crack, structural damping, amplitude-dependent local stiffness, superharmonic, time-dependent mode shape.

INTRODUCTION

Beams are one of the most commonly used elements in structures and machines, and fatigue cracks are the main cause of beams failure. Thus, in order to develop the nondestructive inspection and health monitoring methods, dynamic behavior and modeling of the cracked structures have been studied by many investigators. Closing effect of the crack during the vibration causes the stiffness of the structure to vary continuously with time and amplitude. Therefore, the nonlinear effects appear in the dynamic response and it affects modal parameters of the structure. In order to avoid difficulties resulting from the closing effect of the crack, many researchers have assumed that the crack remains always open during the vibration (Caddemi and Calio, 2009; Mazanoglu et al., 2009; Orhan, 2007). Cornwell et al. (1999) used strain energy method to detect and locate damage in plate-like structure. The method requires the mode shapes of the structure before and after the damage. Indeed, this method is the development of one-dimensional strain energy method by Stubbs et al. (1995) to two-dimensional structures. Yang et al., (2001) studied the influence of open cracks on the vibration behavior of a beam by using strain energy variation around the crack for single-and double-cracked beams. They used Galerkin's method to determine beam modes and frequencies. Swamidas et al. (2004) developed an open crack model using energy formulations and fracture mechanics considerations. Galerkin's method is utilized to solve for natural frequencies of uncracked and cracked beams. In order to taking into account the closing effect of the crack on the vibrational behavior of the beam, some researchers considered a contact stiffness which is added to the initial stiffness at the crack location in half period of the beam vibration (Kisa and Brandon, 2000; Benfratello et al., 2007; Foong et al., 2007). Such a model takes into account only fully open and fully closed cases of the crack and ignores partially open or closed situations. Bovsunovsky and Surace (2005) and Bovsunovskii et al. (2006) used finite element method to study the influence of the crack parameters on the system damping by bilinear stiffness model. They used a proportional damping model to describe the damping nature of the system. They claimed that nonlinear effects are more sensitive to the presence of a crack than the change in natural frequencies, or mode shapes. In order to take into account the effects of partial crack closure, Abraham and Brandon (1995) have simulated the changes in stiffness at the location of the breathing crack

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by employing many terms of the Fourier series and ignoring the damping effects of the system. They assumed that the beam stiffness varies between the stiffness of the beam with the open crack and that of the intact beam. Cheng et al. (1999) have considered a single degree of freedom model with time-varying stiffness to study the forced vibration behavior of a cracked cantilever beam. The time-varying stiffness of the beam is modeled as a simple periodic function. They obtained the forced vibration response of the cracked beam numerically using the Runge-Kutta method. Zhang and Testa (1999) have investigated closure effects on the vibration response of a fatigue cracked steel T-beam, experimentally.

Panteliou et al. (2001) considered a thermoelastic mechanism of energy dissipation in a cracked body. In another work, Curadelli et al. (2008) employed changes in system damping due to the damage for structural damage identification using the wavelet transform. Filipiak et al. (2006) studied damping effect on beam vibration forced by impact and demonstrated that influence of the air on the beam damping is negligible. It is worth noting that the modeling of the nonlinear behavior of a complicated structure due to the crack is far from straightforward.

Therefore, establishing an analytical model for vibration analysis of a cracked real structure by considering the structural damping and the nonlinear behavior of the breathing crack due to continuous changes of local stiffness and local damping at the crack location during the opening and closing the crack is very complex task. Thus, in the literature in this area, many researchers used the beam like structure to show dynamical behavior of the cracked structures.

In this work, a new approach is developed for damped free vibration analysis of a beam with a fatigue crack, considering both the distributed structural damping and the damping due to presence of the crack. It is assumed that the crack behaves as a viscous damper. Also, a nonlinear amplitude-dependent function is developed for modeling the local stiffness changes at the crack location during the vibration. In addition, by considering the experimental tests, it is shown that the local stiffness at the crack location varies continuously between two extreme values corresponding to the fully open and fully closed cases of the crack.

These extreme values are determined by the experimental tests. Damped free vibration response of the cracked beam and its frequency spectrum is obtained by the proposed method, and the results are compared with experimental tests results.

The appearance of the superharmonic component in the frequency spectrum implies the nonlinear behavior of the cracked beam. In addition, by employing the proposed method, the variation of the frequency ratio (that is, the ratio of the fundamental frequency of the cracked beam to that of the intact beam) against the crack location ratio for a given crack depth ratio is plotted and the obtained results are compared with those obtained from the linear model (open crack model) and the experimental ones. The comparison shows a good accuracy of the proposed method.

MATHEMATICAL MODELING OF THE BEAM WITH A FATIGUE CRACK

A uniform cantilever cracked beam with a length of L is shown in Figure 1a. The crack is modeled as a fatigue one with nonlinear stiffness (Figure 1b). Local stiffness at the crack location varies with time due to the crack opening and closing during the beam vibration. Therefore, the dynamic behavior of the beam is affected by the stiffness variations at the crack location. For the sake of simplicity, Cheng et al. (1999) used a SDOF model for the cracked beam. They considered the equivalent stiffness of the cracked beam as a time-varying harmonic function:

$$K(t) = K_o + \frac{1}{2}(K_c - K_o)\left[1 + \cos(\omega t)\right] \quad (1)$$

where ω is the fundamental frequency, k_c is the equivalent stiffness of the intact beam and k_o is the equivalent stiffness of the cracked beam when the crack is fully open.

In this research, unlike the abovementioned model, the cracked beam is considered as a continuous system and the local stiffness changes at the crack location due to the beam vibration is considered as an amplitude-dependent function.

Assume that A_c and A_o to be the amplitudes of a specified point on the cracked beam (e.g. free end of the beam) corresponding to the fully closed and fully open situations of the crack, respectively. Also, assume that k_c and k_o to be the local stiffness of the beam at the crack location corresponding to the amplitudes of A_c and A_o, respectively. In this case, during the oscillation of the beam at its first mode, the local stiffness at the crack location will vary continuously in the range of

$$k_c \le k \le k_o .$$

Here, by adapting Equation (1) for local stiffness, it is assumed that the local stiffness change at the crack location during the vibration is a harmonic function of the amplitude of the cracked beam, as:

$$k_A = k_o + \left(\frac{k_c - k_o}{2}\right)\left\{1 + \cos\left[\frac{\pi}{2}\left(1 - \frac{A_o^2 + A_c^2}{A_o A_c (A_o - A_c)} A + \frac{A_o + A_c}{A_o A_c (A_o - A_c)} A^2\right)\right]\right\} \quad (2)$$

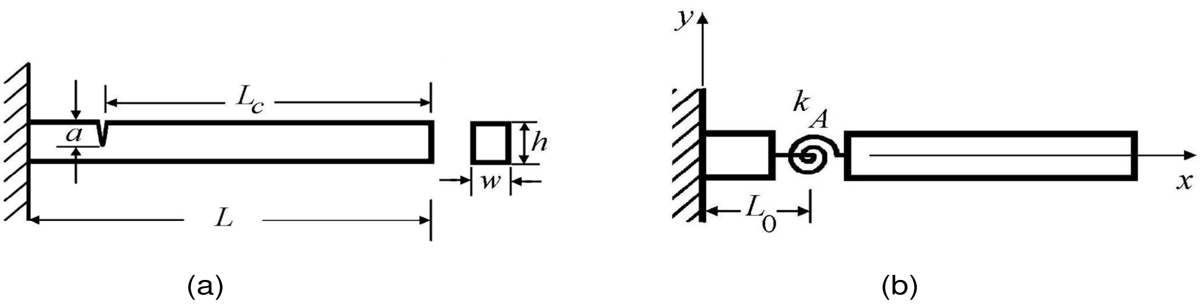


Figure 1. (a) Cantilever beam with a breathing crack; (b) a nonlinear stiffness model for the breathing crack on a cantilever beam.

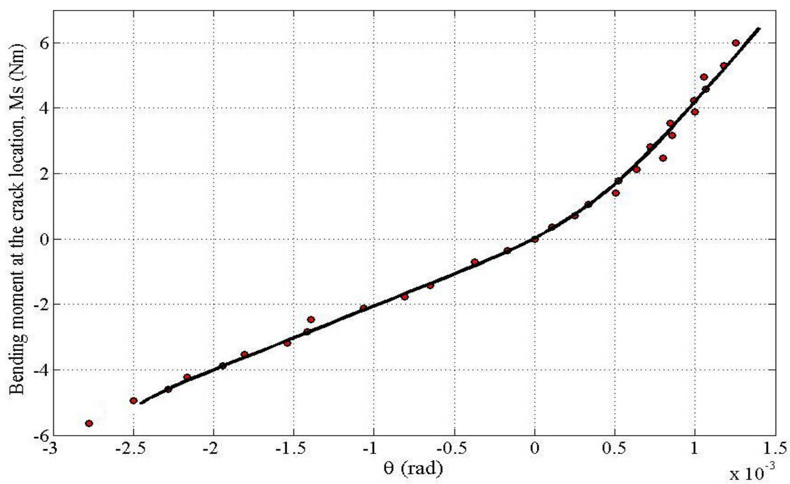


Figure 2. The variation of bending moment against the local slope difference at the crack location for a steel cracked beam with a cross-sectional area of $3.9\times6.4\ mm^2$ and a crack depth ratio of $\alpha = 0.36$ obtained through the experimental tests (●) and the curve fitted to the experimental test points (—).

where, A is the amplitude of the beam.

Unlike Equation (1) which models the cracked beam as a SDOF system with a time-varying equivalent stiffness, Equation (2) introduces the local stiffness changes at the crack location in terms of the amplitude of the cracked beam. By having k_c and k_o experimentally, and corresponding amplitudes, A_c and A_o, Equation (2) gives the local stiffness at the crack location, k_A, as a function of instant beam amplitude A.

EXPERIMENTAL MEASUREMENTS OF THE LOCAL STIFFNESS AT THE CRACK LOCATION

In this research, a servo hydraulic universal dynamic test machine (Zwick/Roell Amsler HA250) is used for initiating and propagating the fatigue crack on steel beams. In order to obtain the local stiffness of the beams at the crack location, they are subjected to bending moments. Then, the local difference between the slopes of lines normal to the two faces of the crack corresponding to the applied bending moment is obtained by measuring the angle between laser light rays originated from the two sides of the crack.

Figure 2 shows a typical test results. This figure illustrates the variation of the bending moment, M_s, against the local slope difference at the crack location, θ, for a steel beam with a cross-sectional area of $3.9\times6.4\ mm^2$ and crack depth ratio of $\alpha = \frac{a}{h} = 0.36$. The slope of the $M_s - \theta$ curve at any θ shows the local stiffness of the beam at the crack location. That is. (Cheng et al., 1999; Newman and Elber, 1988):

$$k = \frac{dM_s}{d\theta} \tag{3}$$

By examining the Figure 2 with close attention, it is observed that the curve $M_s - \theta$ consists of two straight lines with two different slopes which are connected to each other smoothly by a transition curve. The line at the left side of the transition curve corresponds to the fully open case of the crack which has a slope of $k_o = 1954.2 \frac{N.m}{rad}$. The slope of the line at the right side which corresponds to the fully closed case of the crack is $k_c = 2.85 k_o$. On the other hand, the local stiffness at the location of a fully open crack is given by (Chondros et al., 1998):

$$k_o = \frac{EI}{6\pi\left(1-\nu^2\right)h} \frac{1}{J(\alpha)} \tag{4}$$

where $J(\alpha)$ is the dimensionless local compliance function which is given by:

$$\begin{aligned} J(\alpha) = & 0.6272\alpha^2 - 0.4533\alpha^3 + 4.5948\alpha^4 - 9.9736\alpha^5 + 20.2948\alpha^6 \\ & - 33.0351\alpha^7 + 47.1063\alpha^8 - 40.7556\alpha^9 + 19.6\alpha^{10} \end{aligned} \tag{5}$$

Using Equation (4), k_0 for the mentioned beam is obtained

$$k_0 = 2061.8 \frac{N.m}{rad},$$

which differs 5.2% from the experiment.

Furthermore, Figure 2 demonstrates that the vibrational behavior of a beam with a fatigue crack depends on the state of the crack opening, and the degree of opening and closing of the crack depends on the vibration amplitude of the beam.

When the beam free vibration amplitude is high enough, the crack is alternatively fully open and fully closed, therefore, the local stiffness at the crack location will vary continuously between the two extreme values corresponding to the fully open and fully closed cases of the crack.

The beam vibration amplitude decreases gradually with time due to the system damping, therefore, the crack opens and closes partially and the range of local stiffness variation becomes smaller and smaller, so by decreasing the vibration amplitude, the local stiffness approaches to a constant value.

THE GOVERNING EQUATION OF MOTION FOR THE CRACKED BEAM WITH A NONLINEAR FATIGUE CRACK MODEL

Free vibration of a beam is significantly affected by the structural damping. Thus, in order to enhance the accuracy of the proposed method, distributed structural damping effect is considered. Also, local stiffness at the crack location is modeled as a nonlinear massless torsional spring and its stiffness variations due to opening and closing the crack during the beam vibration cause the mode shapes and natural frequencies to vary with vibration amplitude. In addition, the local damping effect due to opening and closing the crack is considered in obtaining the dynamic response of the cracked beam but its influence on the mode shapes is neglected. Therefore, the governing equations of motion for the two segments of the beam are as follows:

$$EIW_{1,xxxx}(x,t) + c_s I W_{1,xxxxt}(x,t) + mW_{1,tt}(x,t) = 0 \qquad 0 \le x \le L_0 \tag{6a}$$

$$EIW_{2,xxxx}(x,t) + c_s I W_{2,xxxxt}(x,t) + mW_{2,tt}(x,t) = 0 \qquad L_0 \le x \le L \tag{6b}$$

where E is the Young's modulus, I is the beam cross-sectional area moment of inertia, m is the mass per unit length of the beam, c_s is the structural damping coefficient and, W_1 and W_2 are the deflection functions of the beam at the left and right sides of the crack, respectively.

As mentioned before, because the local stiffness of the beam at the crack location is an amplitude-dependent function, therefore, the beam natural frequencies and its mode shape will vary continuously according to the vibration amplitude. Thus, the vibration response of the beam can be considered as follows:

$$W_1(x, A(t)) = A(t)\phi_1(x, A(t)) \qquad 0 \le x \le L_0 \tag{7a}$$

$$W_2(x, A(t)) = A(t)\phi_2(x, A(t)) \qquad L_0 \le x \le L \tag{7b}$$

where, $A(t)$ is the displacement of a specified point on the beam and $\phi_1(x, A(t))$ and $\phi_2(x, A(t))$ are amplitude-dependent eigen functions of the beam at the right and left side of the crack, respectively. Substituting Equations (7a) and (7b) into Equations (6a) and (6b), one obtains:

$$\phi_1(x, A(t)) = c_1 \cosh \lambda x + c_2 \sinh \lambda x + c_3 \cos \lambda x + c_4 \sin \lambda x \tag{8a}$$

$$\phi_2(x, A(t)) = c_5 \cosh \lambda x + c_6 \sinh \lambda x + c_7 \cos \lambda x + c_8 \sin \lambda x \tag{8b}$$

where $\lambda^4 = \frac{m\omega^2}{EI}$. ω is the circular frequency and c_i,

$i = 1, 2, ..., 8$ are unknown constants to be determined from the boundary conditions.

It is worth noting that λ and ω are time-dependent variables. The boundary conditions at both ends are:

at $x = 0$: $\phi_1(0, A) = 0$, $\phi_{1,x}(0, A) = 0$ (9)

at $x = L$: $EI\,\phi_{2,xx}(L, A) = 0$, $EI\,\phi_{2,xx}(L, A) = 0$ (10)

At the crack location, $x = L_0$, the matching conditions are as follows:

$$, \phi_{1,xxx}(L_0, A) = \phi_{2,xxx}(L_0, A)\ \phi_{1,xx}(L_0, A) = \phi_{2,xx}(L_0, A)\ ,$$

$$\phi_1(L_0, A) = \phi_2(L_0, A) \quad (11)$$

$$EI\phi_{1,xx}(L_0, A) = k_A\left[\phi_{2,x}(L_0, A) - \phi_{1,x}(L_0, A)\right]$$

Equations (8a) and (8b) and the boundary and matching conditions (Equations (9) to (11)) constitute an Eigen value problem with time-varying eigenvalues and eigenfunctions. The characteristic determinant is:

$$\begin{vmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cosh\lambda L_0 & \sinh\lambda L_0 & -\cos\lambda L_0 & -\sin\lambda L_0 \\ 0 & 0 & 0 & 0 & \sinh\lambda L_0 & \cosh\lambda L_0 & \sin\lambda L_0 & -\cos\lambda L_0 \\ \cosh\lambda L_0 & \sinh\lambda L_0 & \cos\lambda L_0 & \sin\lambda L_0 & -\cosh\lambda L_0 & -\sinh\lambda L_0 & -\cos\lambda L_0 & -\sin\lambda L_0 \\ \cosh\lambda L_0 & \sinh\lambda L_0 & -\cos\lambda L_0 & -\sin\lambda L_0 & -\cosh\lambda L_0 & -\sinh\lambda L_0 & \cos\lambda L_0 & \sin\lambda L_0 \\ \sinh\lambda L_0 & \cosh\lambda L_0 & \sin\lambda L_0 & -\cos\lambda L_0 & -\sinh\lambda L_0 & -\cosh\lambda L_0 & -\sin\lambda L_0 & \cos\lambda L_0 \\ \lambda\cosh\lambda L_0 + \frac{k_A}{EI}\sinh\lambda L_0 & \lambda\sinh\lambda L_0 + \frac{k_A}{EI}\cosh\lambda L_0 & -\lambda\cos\lambda L_0 - \frac{k_A}{EI}\sin\lambda L_0 & -\lambda\sin\lambda L_0 + \frac{k_A}{EI}\cos\lambda L_0 & -\frac{k_A}{EI}\sinh\lambda L_0 & -\frac{k_A}{EI}\cosh\lambda L_0 & \frac{k_A}{EI}\sin\lambda L_0 & -\frac{k_A}{EI}\cos\lambda L_0 \end{vmatrix} \quad (12)$$

In order to have a non-trivial solution for c_i 's, the determinant must be zero, that is:

$$\left|\Delta\left(\omega, L_0, A(t)\right)\right| = 0 \quad (13)$$

Solving this characteristic equation will give the eigenvalue λ and the mode shapes $\phi_1(x, A(t))$ and $\phi_2(x, A(t))$ corresponding to the amplitude $A(t)$ in terms of time t . i.e., for a given A , the local stiffness at the crack location, the natural frequency and the mode shape corresponding to A are determined. By considering both distributed structural damping and local viscous damping due to the crack and obtaining the time intervals corresponding to the successive movements of the beam from a given position to a neighboring position, the damped free response of the beam can be obtained.

MECHANICAL ENERGY BALANCE APPROACH

As mentioned before, for obtaining the beam vibration response, taking into account the mechanical energy loss due to the distributed structural damping and the local damping due to the crack, the mechanical energy of the cracked beam is calculated in every moment. The mechanical energy of the cracked beam, E_b , at a given time t can be written as:

$$E_b = E_K + E_P + E_S \quad (14)$$

where E_K , E_P and E_S denote the kinetic energy, the elastic strain energy and the energy stored at the crack location, respectively. When the specified point of the cracked beam during the vibration is moved from a position A_{j-1} to its neighboring position A_j , the kinetic energy of the cracked beam can be calculated as:

$$E_{K_j} = \frac{m}{2}\left[\int_0^{L_0}\left(\frac{A_{j-1}\phi_1(x, A_{j-1}) - A_j\phi_1(x, A_j)}{\Delta t}\right)^2 dx + \int_{L_0}^{L}\left(\frac{A_{j-1}\phi_2(x, A_{j-1}) - A_j\phi_2(x, A_j)}{\Delta t}\right)^2 dx\right] \quad (15)$$

In this equation Δt is the time required for moving the specified point of the cracked beam from the position A_{j-1} to the position A_j . Also, the elastic strain energy of the beam corresponding to the position A_j can be written in the form:

$$E_{P_j} = \frac{EI}{2}\left[\int_0^{L_0}\left(A_j\phi_1''(x, A_j)\right)^2 dx + \int_{L_0}^{L}\left(A_j\phi_2''(x, A_j)\right)^2 dx\right] = A_j^2 U_P(A) \quad (16)$$

The potential energy stored at the crack location depends on the magnitude of the slope discontinuity of the mode shapes at the two sides of the crack. The slope discontinuity at the crack location is given by:

$$\theta(A) = A\Theta(A) \quad (17)$$

where $\Theta(A) = \left[\phi_{2,x}(L_0, A) - \phi_{1,x}(L_0, A)\right]$.

Equation (17) shows slope discontinuity at the crack location corresponding to the vibration amplitude of A . On the other hand, the nonlinear relation between the magnitude of the bending moment at the crack location, M_s , and the slope discontinuity at the crack location, θ , is given by:

$$M_s = k_A\ \theta(A) \quad (18)$$

The area under the curve $M_s - \theta$ is the stored potential energy at the crack location, that is,

$$E_S = \int_0^{\theta} M_s d\theta \tag{19}$$

On the other hand, the curve $M_s - \theta$ may be obtained by experimental tests. Therefore, one can plot the variations of the stored energy at the crack location against the variations of slope discontinuity $\theta(A)$. In order to avoid repetitive and tedious integration, we can obtain the stored energy at the crack location, E_S , for some values of θ, and then by interpolation, one can obtain E_S as a function of θ . Here, we use a fifth order polynomial interpolation as:

$$E_S = a_1\theta^5 + a_2\theta^4 + a_3\theta^3 + a_4\theta^2 + a_5\theta + a_6 \tag{20}$$

where a_i , $i = 1, 2, ..., 6$ are the coefficients of the polynomial. Finally, by using Equations (15), (16) and (20) one can obtain the total mechanical energy of the cracked beam corresponding to the given amplitude of $A(t)$. When the beam moves from one position to another neighboring position, the total mechanical energy of the beam decreases because of the distributed structural damping and the local damping at the crack location. The energy dissipation caused by the distributed structural damping in the time interval $[0,t]$ can be obtained as:

$$\begin{aligned} E_{c_s} &= \int_0^t \left[\int_0^{L_0} c_s I \left(\frac{\partial^3 W_1(x,t)}{\partial t \partial x^2} \right)^2 dx + \int_{L_0}^{L} c_s I \left(\frac{\partial^3 W_2(x,t)}{\partial t \partial x^2} \right)^2 dx \right] dt \\ &= c_s I \int_0^t \left[\left(\frac{dA}{dt} \right)^2 \left(\int_0^{L_0} \phi_1''^2(x,A) dx + \int_{L_0}^{L} \phi_2''^2(x,A) dx \right) \right] dt \end{aligned} \tag{21}$$

The energy dissipation in a fatigue crack has a complex mechanism. Because of this complexity, the local damping due to the crack is neglected in bending vibration analysis of the cracked structures by many researchers (Chondros et al., 1998; Cornwell et al., 1999; Kisa and Brandon, 2000; Chondros et al., 2001; Orhan, 2007; Mazanoglu et al., 2009). In this research, it is assumed that the energy-dissipation mechanism of the crack is viscous. Therefore, the energy dissipation at the crack location in time interval $[0,t]$ can be expressed as:

$$E_{c_c} = \int c_c \dot{\theta} d\theta = \int_0^t c_c \dot{\theta}^2 dt = c_c \int_0^t \left(\frac{dA}{dt} \right)^2 \left[\phi_2'(x,A) - \phi_1'(x,A) \right]^2 dt \tag{22}$$

where, c_c is the viscous damping coefficient.

The coefficients of c_s and c_c are obtained by the experimental tests. Thus, when the beam moves between two neighboring positions, the amount of energy dissipation in the system can be written as:

$$\Delta E_{d_j} = c_s I \left[\left(\frac{dA}{dt} \right)^2 \left(\int_0^{L_0} \phi_1''^2(x,A) dx + \int_{L_0}^{L} \phi_2''^2(x,A) dx \right) \right] \Delta t_j + c_c \left(\frac{dA}{dt} \right)^2 \Theta_j^2(A) \Delta t_j \tag{23}$$

Here, a step by step method is introduced to obtain the damped free vibration response of the cracked beam. Suppose that at $t = 0$ the displacement of a given point on the beam, e.g. the free end of the beam, is A_1 and the beam is released in the direction in which the crack is opening. At this case, the mechanical energy of the beam is E_1 .

Consider the beam passes the successive positions $A_j = A_1 + (j-1)\Delta A$, $j = 1, 2, 3, ..., N$, in which ΔA is the incremental change in amplitude. By solving Equation(13) in every step, one can obtain the second derivatives of the mode shapes functions, ϕ_1'' and ϕ_2'' , and the slope discontinuity at the crack location, $\theta(A_j)$, corresponding to each A_j .

Thus, the mechanical energy of the beam can be calculated in terms of the known amplitudes A_j . Then, by taking into account the energy dissipation and applying the mechanical energy balance, the time required the beam to move from A_{j-1} to A_j is obtained. When the beam reaches $A_{extr} = A_N$, the kinetic energy of the beam vanishes and the beam moves in the opposite direction. In a similar way, one can obtain the time duration for every incremental movements of the beam, therefore, $A_j = A_{extr} - (j-1)\Delta A$ can be calculated.

This procedure is continued until the total mechanical energy of the beam, due to the energy dissipation in each step, approaches to zero. The energy balance relation between the two neighboring successive amplitudes of the beam is:

$$E_{P_{j-1}} + E_{K_{j-1}} + E_{S_{j-1}} = E_{K_j} + E_{P_j} + E_{S_j} + \Delta E_{d_j} \tag{24}$$

The left hand side of Equation (24) corresponds to the mechanical energy of the beam at the previous step $j-1$ which is known. Therefore, using Equation(24), the time required the beam moves from A_{j-1} to A_j , i.e. Δt_j , can be calculated from the following equation:

$$A_j^2\frac{EI}{2}\left[\int_0^{L_0}\left(\phi_1''(x,A_j)\right)^2dx+\int_{L_0}^{L}\left(\phi_2''(x,A_j)\right)^2dx\right]+a_1\theta_j^5+a_2\theta_j^4+a_3\theta_j^3+a_4\theta_j^2+a_5\theta_j+a_6$$
$$+\frac{m}{2\Delta t_j^2}\{A_j^2\left[\int_0^{L_0}\phi_1^2(x,A_j)dx+\int_{L_0}^{L}\phi_2^2(x,A_j)dx\right]+A_{j-1}^2\left[\int_0^{L_0}\phi_1^2(x,A_{j-1})dx+\int_{L_0}^{L}\phi_2^2(x,A_{j-1})dx\right]$$
$$-2A_{j-1}A_j\left[\int_0^{L_0}\phi_1(x,A_j)\phi_1(x,A_{j-1})dx+\int_{L_0}^{L}\phi_2(x,A_j)\phi_2(x,A_{j-1})dx\right]\}$$
$$-c_sI\left[\left(\frac{dA}{dt}\right)^2\left(\int_0^{L_0}\phi_1''^2(x,A)dx+\int_{L_0}^{L}\phi_2''^2(x,A)dx\right)\right]\Delta t_j-c_c\left(\frac{dA}{dt}\right)^2\Theta_j^2(A)\Delta t_j$$
$$-E_{P_{j-1}}-E_{K_{j-1}}-E_{S_{j-1}}=0 \tag{25}$$

Hence, the time required that the given point on the beam moves from A_1 to A_j, is:

$$t_j = \sum_{i=2}^{j}\Delta t_{i-1} \tag{26}$$

In this way, the damped free response of the cracked beam can be obtained by plotting A_j against the time t_j.

The effect of the distributed structural damping on the free vibration of an intact beam

The equation of motion for a beam with structural damped is given by (Humar, 1990):

$$EIW_{,xxxx}(x,t)+c_sIW_{,xxxxt}(x,t)+mW_{,tt}(x,t)=0 \qquad 0\le x\le L \tag{27}$$

By applying the separation of variables method, the solution of Equation (27) may be considered as:

$$W(x,t)=A_0\phi(x)e^{-\zeta\omega t}\sin(\omega\sqrt{1-\zeta^2}\,t+\psi) \tag{28}$$

where, $\zeta=\frac{c_s\omega}{2E}$ and, A_0 and ψ are constants. For the first vibration mode, the natural frequency of the beam will be

$$\omega=\left(\frac{1.875}{L}\right)^2\sqrt{\frac{EI}{m}}.$$

Thus, the structural damping coefficient, c_s, for the first mode can be expressed as:

$$c_s=0.5689\zeta L^2\sqrt{\frac{mE}{I}} \tag{29}$$

For determining the structural damping coefficient, free responses of an intact cantilever beam with a cross-sectional area of $3.9\times6.4\,mm^2$ and two different lengths $L=40cm$ and $L=18cm$ obtained experimentally. A point-to-point laser vibrometer (OMETRON VH300+) is used to measure free response at the free end of the beam during the vibration and signal analyzer (B&K, type 3109) is used to extract the experimental results. By fitting a decaying exponential function, $A_oe^{-\zeta\omega t}$, to local maximum points of the experimental data, the damping ratios, ζ, are obtained 0.0008 and 0.0013, respectively (Figure 3).

THE EFFECT OF CRACK PARAMETERS ON THE DAMPED VIBRATIONAL BEHAVIOR OF THE CRACKED BEAM

By employing the proposed method described in section 4, a quantitative and qualitative evaluation of crack parameters effects on the vibrational behavior of the cracked beam is investigated. For the steel cantilever beam considered, the length is $18\,cm$, the rectangular cross-section is $3.9\times6.4mm^2$, and the crack parameters are $\beta=\frac{L_c}{L}=0.81$, $\alpha=0.36$. The local stiffness at the crack location corresponding to the fully open case of the crack is $k_o=1954.2\frac{N.m}{rad}$ and that of corresponding to the fully closed one is $k_c=2.85k_o$. By applying the proposed method, the variation of the natural frequency against the vibration amplitude is calculated (Figure 4). The examination of the results demonstrate that for a given vibration amplitude, the range of the natural frequency variation depends on the crack parameters. An increase in the crack depth and/or approaching the crack location to the clamped end of the beam results in an increase in the range of natural frequency changes. In addition, as the initial amplitude of the beam becomes smaller, the range of the natural frequency changes will be smaller, too. For estimation of the local damping effect of the crack, a comparison is made between the free responses of the cracked and the intact beams.

For this purpose, free responses of the beam having the crack depth ratio of 0.36 are obtained for two different lengths of $L=40cm$ and $L=18cm$, with crack location ratios of $\beta=0.37$ and $\beta=0.81$, respectively. By inspecting the responses, it is concluded that only a small portion of the total damping is caused by the crack.

For two mentioned cases and considering the distributed structural damping and the local damping at the crack location, free responses of the cracked beam are obtained using the proposed method. (Figures 5a and 5b). Figures 6a and 6b illustrate free responses of the

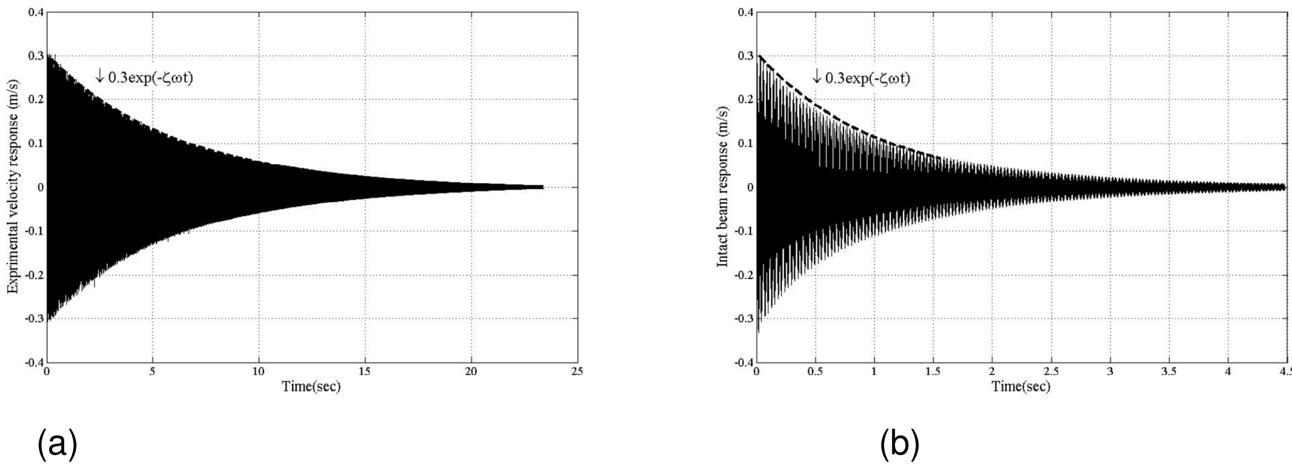


Figure 3. Experimental response of the intact cantilever beam with a length of (a) $L = 40cm$; (b) $L = 18cm$.

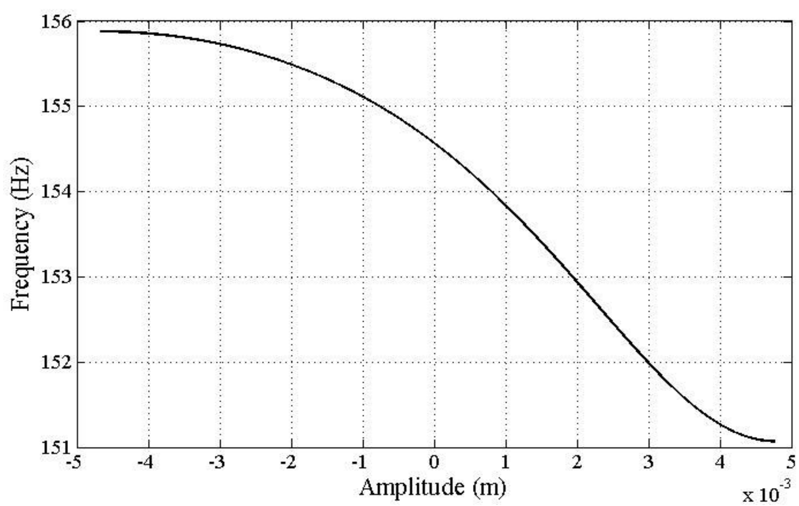
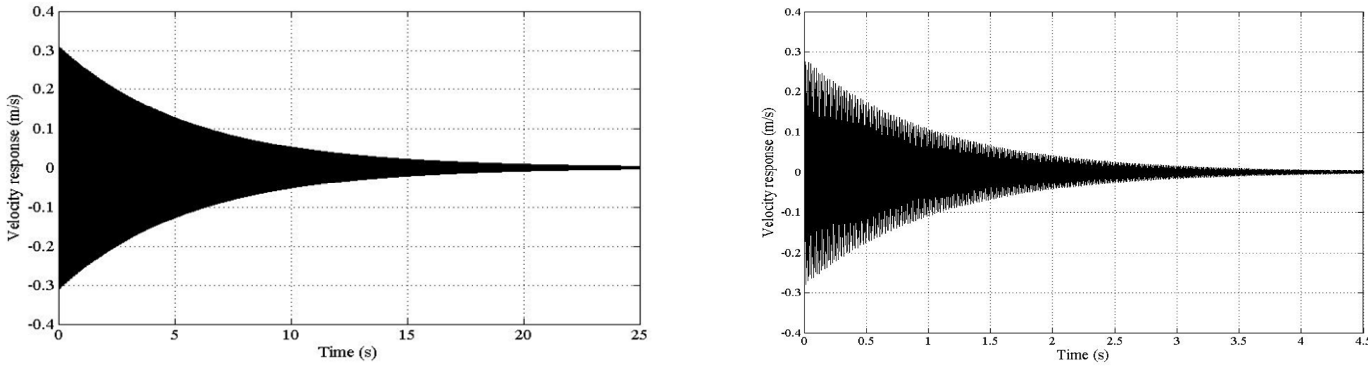


Figure 4. The range of the natural frequency changes of the cracked beam with the crack parameters of $\alpha = 0.36$ and $\beta = 0.81$ against the vibration amplitude.

mentioned cracked beams obtained by the experimental measurements. The comparison between Figures 5 and 6 indicates that there is a good agreement between the theoretical and the experimental results.

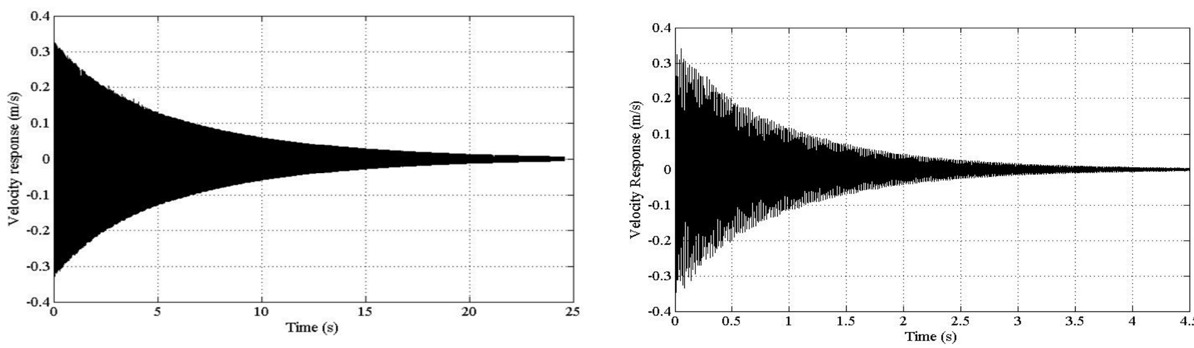
Also, the results show that approaching the crack to the clamped end of the beam causes to more decay of the cracked beam response. The spectra of the free responses illustrated in Figures 5 and 6 are shown in Figures 7a and 7b, respectively. Figures 7a and 7b reveal that the frequencies obtained from the proposed method are in agreement with those of experimental results. Also, these figures indicate that the harmonic components are appeared in the spectrum which reflects the nonlinearity due to the crack. In Figure 7b, in both curves, there are two picks at the frequencies of $154\ Hz$ and $308\ Hz$. The first pick corresponds to the fundamental frequency of the cracked beam and the second one is the first harmonic component of the fundamental frequency, which shows the superharmonic phenomenon. The superharmonics are weak in Figure 7a. Comparing Figures 7a and 7b illustrates that for a beam with a given crack depth, approaching the crack location to the clamped end of the beam, results in an increase in the amplitude of the superharmonic.

As mentioned in section 5, for investigating the vibrational behavior of the cracked structures, in order to avoid complexity, many researchers have neglected damping effects (Chondros et al., 1998; Cornwell et al.,



(a) (b)

Figure 5. The free responses obtained by the proposed method for the cracked beam with a crack depth ratio of $\alpha = 0.36$ and with two different crack location ratios of (a) $\beta = 0.37$ and (b) $\beta = 0.81$.



(a) (b)

Figure 6. The free responses obtained by the experiment for the cracked beam with a crack depth ratio of $\alpha = 0.36$ and with two different crack location ratios of (a) $\beta = 0.37$ and (b) $\beta = 0.81$.

(a) (b)

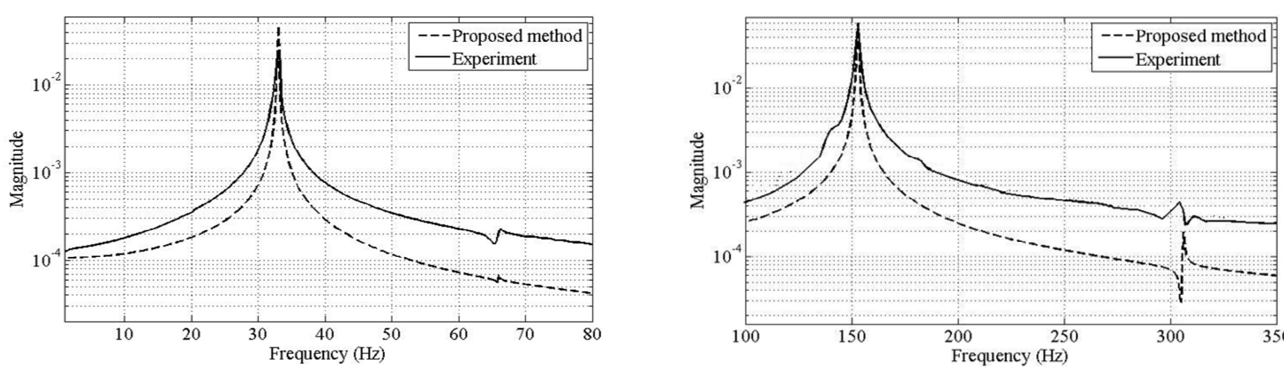


Figure 7. The spectra of the free responses obtained by the proposed analytical method (- - -) and the experiment (___) for the cracked beam with a crack depth ratio of $\alpha = 0.36$ and with two different crack location ratios of (a) $\beta = 0.37$ and (b) $\beta = 0.81$.

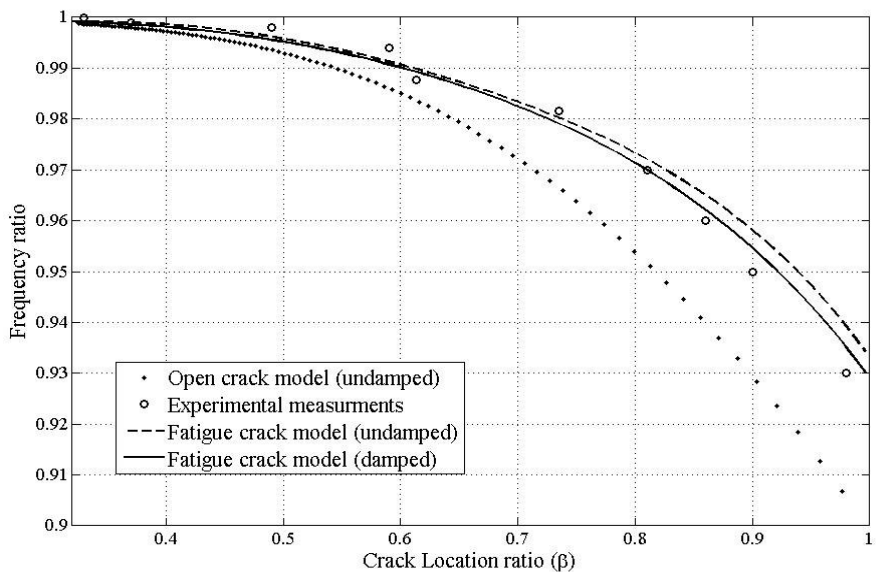


Figure 8. Variation of the fundamental frequency ratio of the cracked cantilever beam against the crack location ratio for the crack depth ratio of $\alpha = 0.36$ obtained from the proposed method based on the fatigue crack model by taking into account the effects of the system damping (___), fatigue crack model without taking into account the effects of the system damping (- - -), undamped open crack model (. . .) and the experimental results (o).

1999; Kisa and Brandon, 2000; Chondros et al., 2001; Orhan, 2007; Mazanoglu et al., 2009). However, the experimental evidences show that the damping effects can not be ignored in free vibration analysis of the structures (Figure 6). Therefore, the main purpose of the present work is to propose a more realistic and reasonable model for the cracked beam by improving the assumptions, that is, considering a continuous model for the cracked beam and taking into account the nonlinear behavior of the crack as well as the effects of the structural damping and the local damping due to the crack. To this end, Kelvin-Voigt damping model (Humar, 1990) is used for taking into consideration the distributed structural damping of the beam, and the local damping at the crack is assumed to be viscous. However, it should be mentioned that the construction of an exact damping model for a cracked structure due to the complexity of the damping mechanisms is very difficult task. This complexity comes from the fact that in cracked metallic materials, in addition to linear viscoelastic effects, there are other mechanisms of energy dissipation such as, plastic deformation, internal Coulomb damping, air damping and the other nonlinearities. Therefore, in figures 7a and 7b, the sources of difference between the spectra corresponding to the responses obtained through the proposed theoretical method and the experiment are due to the abovementioned effects and some other uncertainties.

In Figure 8, the variation of the frequency ratio against the crack location ratio, is plotted for both fatigue crack (nonlinear model) and open crack (linear model). In this Figure, the small circles show the results obtained by the experimental measurements. The solid curve is corresponding to the fatigue crack model by taking into account the effects of the distributed structural damping and the local damping of the crack. The dashed and dotted curves are corresponding to the fatigue crack model and the open crack model, respectively. For plotting these two curves, the damping effects are neglected.

By examining this figure, it is concluded that the results obtained from the proposed method based on the nonlinear model (the solid curve) agree well with the experimental results (the hollow circles). Moreover, the results illustrate that the frequency ratio is affected by the system damping. Therefore, in practical cases, the damping effects on the vibrational behavior of the beam can not be neglected. In addition, Figure 8 shows that for a given crack depth ratio, the frequency reduction in the open crack model (linear model with no energy dissipation) is more than that of the breathing one. This result has been previously proved for the cracked beams (Cheng et al., 1999; Chondros et al., 2001).

Conclusion

In this research, a new approach for free vibration analysis of the cracked cantilever beam with a breathing crack by taking into account the effects of the distributed structural damping and the local damping of the crack is presented. The experimental test results show that the beam local stiffness at the crack location varies continuously as a nonlinear function between the two extreme values of k_o and k_c due to opening and closing the crack. Therefore, local stiffness at the crack location changes continuously during the beam vibration. Furthermore, the experimental results show that the stiffness of the cracked beam with a fully closed crack is different from that of intact one. However, some researchers have been assumed that the stiffness of the cracked beam for the case of fully closed crack is equal to that of the intact beam (Abraham and Brandon, 1995; Cheng et al., 1999; Loutridis et al., 2005; Douka and Hadjileontiadis, 2005).

A main advantage of the proposed model is that it makes it possible to obtain continuous variations of the natural frequency and mode shape of the cracked beam during the vibration. Also, using the model and employing the proposed method, one can obtain the damped free vibration response of the beam with a breathing crack. Whereas the frequency monitoring is used for assessment of the structural integrity and safety, ignoring the system damping effects may lead to significant errors. Many researches ignore damping effects on vibration analysis of the cracked structures. Moreover, to avoid the nonlinearity, many researchers assumed that the crack remains always open or they considered the bilinear frequency in order to take into account the effect of the crack closure during the system vibration. This research shows that ignoring the nonlinearity due to the crack and the damping effects, lead to inaccurate results.

One of the features of a nonlinear system is the appearance of the superharmonics of its fundamental frequency in the vibration response. Therefore, presence of the superharmonics of the fundamental frequency in the response spectra of the cracked beam reveals the nonlinear dynamic behavior of the cracked beam, which can be used as a crack indicator in structural health monitoring applications. It is worth noting that, in real structures it is usually difficult to obtain a clear interpretation of a FFT, and superharmonics are difficult to identify. However, the nonlinearity due to the variation of the beam stiffness during the vibration always gives rise to superharmonics of the fundamental frequency, and the level of their amplitudes and clarity in FFT depends on the intensity of the nonlinearity. In addition, free responses and their corresponding spectra obtained by the experiment are in agreement with those of obtained by proposed method. Also, the results show that for a beam with a given crack depth, approaching the crack location to the clamped end of the beam increases in the damping of the beam free response.

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