

*Full Length Research Paper*

# Equivalence principle of light's momentum harmonizing observation from quantum theory to cosmology

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Unlike in Newtonian mechanics, in the theory of relativity, the coordinate axis is not fixed, and the time axis serves as the background for the observation. In other words, the theory of relativity is background independent (BI). The quantum theory created in the micro-world, in contrast, is background dependent (BD). Efforts to unify both by BI have not yielded much success. In addition, it is recognized that we cannot consider gravity in a quantum theory by BD. This paper focuses on neither the BI method based on the invariant speed of light and the inertial mass observed only in the uniform field nor the method to consider gravity in an existing quantum theory. By using the invariant mass and the speed of light in the frame of reference of the free space of the electromagnetism as well as the observed variable mass and wave speed, a method to link them to the equivalence of light's momentum was explained. In addition, some cases in which we can observe space from a micro-world by the equivalence principle were examined.

**Key words:** Background independent, gravity, free space, electromagnetism, invariant mass, variable mass, speed of light, wave speed.

## INTRODUCTION

In this paper, given the established principles of electromagnetism, they were logically and rationally integrated with those of mechanics. The electromagnetic correlation between the speed and energy of light was defined and interrelated with the mechanics of the energy, momentum, mass, and speed of light. The question of whether gravity should be taken into consideration in the electromagnetic analysis of free space and vacuums is also addressed. Moreover, a deviation was made from the approach of the theory of relativity, which unifies electromagnetism and mechanics by disregarding gravity and assuming that the speed of light is constant. The two fields were unified by considering the interrelation of gravity and mass and

assuming a fixed speed of light in free space. This assumption is the most important aspect of the proposition of this paper.

Firstly, the known facts of classical physics were stated, including (1) the speed and energy of light waves, and (2) the mass and velocity of a material in free space. Some unique terms and symbols used in this paper were also defined, highlighting the difference between particle velocity ( $v$ ) and wave speed ( $w$ ) and went further to discuss the following issues about gravity, based on the assumptions already made:

i. Gravitational mass and the light wave speed under gravity.

## Propagation of a light wave )))))))

**Figure 1.** Propagation of a light wave.

- ii. Mass and velocity of a material under gravity.
- iii. Mass and speed of photons under gravity.
- iv. The speed of light and the wave speed are different physical quantities.
- v. Matter wave and uncertainty relation.

Finally, a conclusion from the abovementioned considerations was derived, and the correlation of speed, mass, momentum, energy, and quantum in the context of logical and rational integration of electromagnetism and mechanics were summarized. This integration is based on the interrelation of the physical quantities in terms of the equivalence principle by the momentum of light.

### SPEED AND ENERGY OF LIGHT WAVES WITHOUT TAKING GRAVITY INTO CONSIDERATION

In electromagnetism, free space is a virtual space where no matter exists. It is defined by physical constants such as the speed of light in a vacuum (free space), magnetic permeability of a vacuum (free space), and permittivity of a vacuum (free space). Maxwell's equation of the speed of propagation of an electromagnetic field is as follows:

$$C = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{1}$$

where C is the speed of light, and  $\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of a vacuum (free space), respectively (Figure 1). The correlation between the electromagnetic wave energy (E), absolute value of the momentum of light (P), and speed of light (C) can be derived from the theory of electromagnetism alone:

$$E = PC \tag{2}$$

Furthermore, in the MKSA system or the International System of Units (SI), the absolute refractive index  $n_0$  is obtained by dividing the speed of light in a vacuum (free space) (C) by the light wave speed in a medium (w). In other words, it is given by the phase speed.

$$n_0 = \frac{C}{w} = \sqrt{\frac{\epsilon \mu}{\epsilon_0 \mu_0}} \tag{3}$$

where  $\mu, \epsilon$  are the permeability and permittivity of the medium, respectively. It can be observed from the above that the light wave speed in a medium changes and is refracted relative to the speed of light in a vacuum (free space).

### VELOCITY AND INERTIAL MASS OF MATTER WITHOUT TAKING GRAVITY INTO CONSIDERATION

According to the observations of Kaufmann's experiments on bending beta rays (Boorse and Motz, 1966), the inertial mass (m') varies when the velocity (v) of the center of gravity of an object's rest mass ( $m_0$ ) changes (Figure 2).

$$\frac{m'}{m_0} = \frac{C}{\sqrt{C^2 - v^2}} = \frac{1}{\sqrt{1 - v^2/C^2}} \tag{4}$$

The momentum of light (P) is obtained from the correlation among the speed of light in free space (C), particle velocity (v), and wave speed (w):

$$P = m_0 C = m' \sqrt{C^2 - v^2} = m' w \tag{5}$$

The energy (E) is obtained by multiplying the momentum of light by the speed of light (C):

$$E = PC = m_0 C^2 = m_0 (v^2 + w^2) = m' w C \tag{6}$$

### GRAVITATIONAL MASS AND LIGHT WAVE SPEED UNDER GRAVITY

If we consider free space as the standard, Earth's substances would exert a gravitational influence on the matter field of the surface. If the propagation speed of the light waves observed on Earth's surface is (w) and the gravitational potential on Earth's surface is given by  $2\phi = 2GM/r$ , then the sum (Equation 8) of the gravitational potential corresponds to the speed of light (C) in free space. It can thus be determined from the equation for the fixed speed of light (Equation 1) that the light wave speed decreases with increasing gravity (Figure 3).

$$w = f\lambda \tag{7}$$

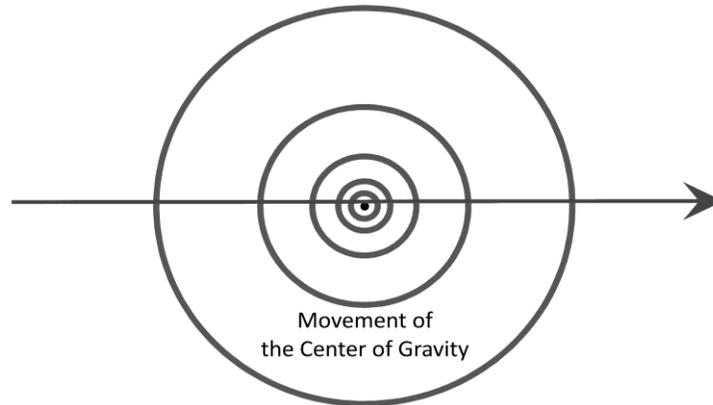


Figure 2. Movement of mass.

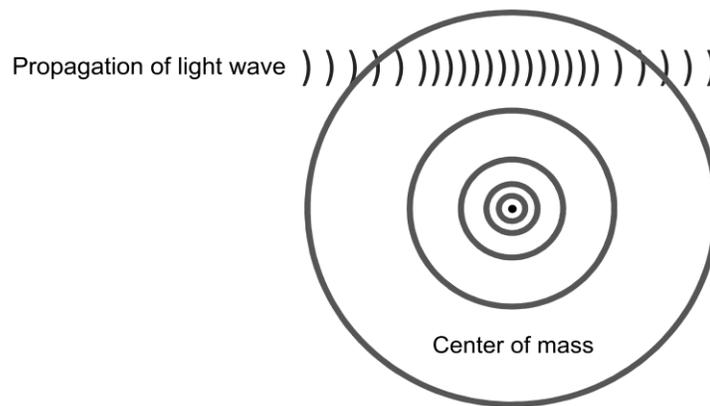


Figure 3. Perturbation between a light wave and a mass.

where  $f$  is the frequency of the wave and  $\lambda$ , the wavelength

$$C^2 = w^2 + 2\varphi \tag{8}$$

Gravitational mass ( $M$ ) is the product of the material density ( $C^2 - w^2$ ) and the radius ( $r$ ) of the earth divided by the constant ( $2G$ ). Therefore, there is no mass ( $M = 0$ ) if there is no volume ( $r = 0$ ).

$$M = (C^2 - w^2)r/2G = v_2^2 r/2G \tag{9}$$

$$C^2 = w^2 + 2\varphi = v_2^2 + w^2 \tag{10}$$

This is equivalent to the potential ( $v_2^2 = 2GM/r$ ) of the second cosmic velocity ( $v_2$ ).

**MATERIAL MASS AND VELOCITY UNDER GRAVITY**

If the object's gravitational mass ( $M$ ) is considered to be equal to the inertial mass ( $m$ ) of its particle motion ( $v^2, 2\varphi$ ), then the difference between the rest mass under gravity ( $m_0$ ) and the inertial mass ( $m'$ ) can be obtained from the following:

$$C^2 = (v^2 + w^2) + 2\varphi \tag{11}$$

$$\frac{m'}{m_0} = \frac{\sqrt{C^2 - 2\varphi}}{\sqrt{C^2 - v^2 - 2\varphi}} = \frac{w_0}{w'} \tag{12}$$

$$n_0 = \frac{m}{M} = \frac{C}{\sqrt{C^2 - v^2 - 2\varphi}} = \frac{C}{w} \tag{13}$$

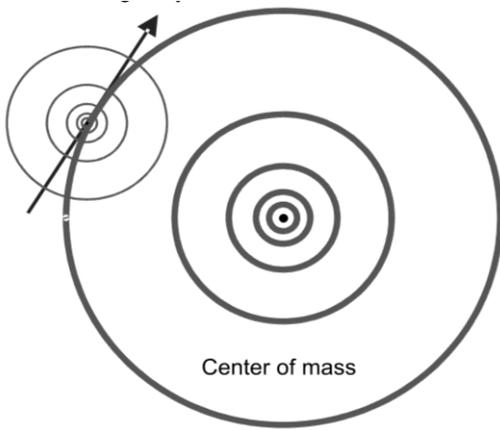


Figure 4. Perturbation in the movement of a mass.

Like the refraction rate  $n_0$  of (Equation 3),  $n_0$  of (Equation 13) also corresponds to the fixed speed of light in (Equation 1) (Figure 4). Furthermore, regarding the correlation between the momentum of light ( $P$ ) and the energy ( $E$ ), there is no equivalence between the gravitational mass and the inertial mass ( $M = m$ ); however, an equivalent based on the momentum of light ( $P = MC = mw$ ) can be obtained:

$$P = MC = m\sqrt{C^2 - v^2 - 2\phi} = mw \tag{14}$$

$$E = PC = MC^2 = M(v^2 + w^2 + 2\phi) = mwC \tag{15}$$

When stationary under gravity ( $v = 0$ ), we can obtain an approximation of the kinetic energy ( $mv^2/2$ ) on the basis of Newtonian mechanics:

$$E = M_2C^2 = M_2(w^2 + 2GM_1/r) \tag{16}$$

$$GM_1M_2/r = M_2(C^2 - w^2)/2 = M_2v^2/2 = m_2v^2/2n_0 \tag{17}$$

If we use the progress of an atomic clock installed in a GPS satellite as an example (Ashby, 2007),

Light wave speed on Earth's surface

$$w_0 = 299,792,458 \text{ m/s}$$

Geocentric gravitational constant

$$GM = 3.986 \times 10^{14} \text{ m}^3/\text{s}^2$$

Radius of Earth  $r = 6,378,000 \text{ m}$

$$\text{Speed of light in free space } C = \sqrt{w_0^2 + 2GM/r} \tag{18}$$

Altitude of GPS satellite	$h = 20,200,000 \text{ m}$
Orbital velocity of GPS satellite	$v = 3,874 \text{ m/s}$
Wave speed of GPS satellite	$w' = \sqrt{C^2 - v^2 - 2GM/(r+h)}$
	(19)
Progress of clock	$w'/w_0 = 1 + (4.45 \times 10^{-10})$
	(20)

### SPEED AND MASS OF PHOTONS UNDER GRAVITY

The following can be derived from Einstein's photon hypothesis and the electromagnetic wave energy of (Equation 2):

$$E = PC = hf \tag{21}$$

where  $h$  is Planck's constant and  $f$ , the frequency. Furthermore, the following can be derived from Equations 7, 15 and 21, although the frequency of the photon hypothesis is not given by  $f = C/\lambda$  but by  $f = w/\lambda$ .

$$hf = mwC = mCf\lambda \tag{22}$$

$$\lambda = \frac{h}{mC} \tag{23}$$

This is also understandable from the Compton Effect (Greiner, 2001a). When a material is exposed to X-rays, some of the rays are scattered and become secondary X-rays. In the Compton Effect, the wavelengths of the secondary X-rays are larger than those of the incident ones.

$$\lambda_s - \lambda_i = \frac{h}{mC} (1 - \cos\theta) = \frac{w}{f_s} - \frac{w}{f_i} \tag{24}$$

where  $\lambda_s$  is the wavelength of the secondary X-rays;  $\lambda_i$ , the wavelength of the incident X-rays;  $\theta$ , the scattering angle;  $f_s$ , the frequency of the secondary X-rays; and  $f_i$ , the frequency of the incident X-rays. In this case, energy is lost under scattering, resulting in a reduced frequency ( $f_s < f_i$ ). Moreover, because the light wave speed ( $w$ ) remains unchanged, the wavelength increases ( $\lambda_s > \lambda_i$ ) (Figure 5).

$$E = MC^2 = M(w_0^2 + 2\phi_0) = m_0w_0C = m_0Cf\lambda_0 = hf \tag{25}$$

$$E = MC^2 = M(w'^2 + 2\phi') = m'w'C = m'Cf'\lambda' = hf' \tag{26}$$

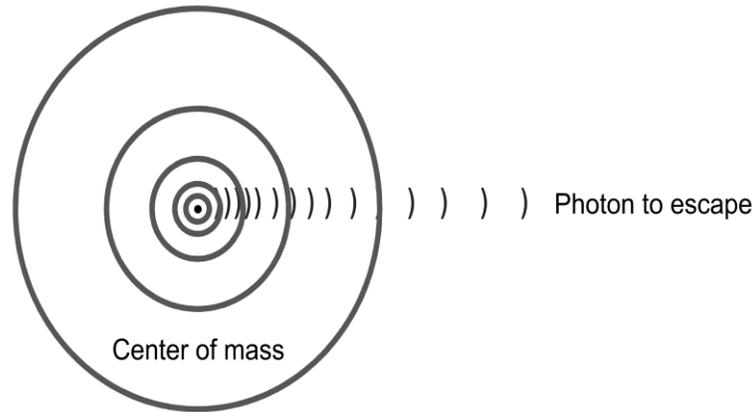


Figure 5. Photon escaping from a gravitational field.

However, with regard to the energy of the photons that escape from the gravitational field (that is, gravitational redshift), the frequency (f) remains unchanged and the gravitational potential ( $\varphi_0 > \varphi'$ ) decreases, resulting in an increase in the light wave speed ( $w_0 < w'$ ). Hence the wavelength increases ( $\lambda_0 < \lambda'$ ) and the energy Equation 21 is maintained, even with a slight decrease in the inertial mass ( $m_0 > m'$ ).

This is similar to Equation 20, where the progress of the atomic clock installed in the GPS satellite is faster than that of a clock on Earth's surface, owing to the weakened gravitational pull of the GPS satellite. This indicates that the propagated photon should be handled in the same manner as a material with mass.

**SPEED OF LIGHT AND WAVE SPEED ARE DIFFERENT PHYSICAL QUANTITIES**

The Shapiro Delay (Shapiro, 1964) is a combined effect of gravitational blueshift and redshift. In this phenomenon, the distance and frequency do not change, and the electromagnetic wave speed is only reduced by the expansion and contraction of the wavelength under changes in the gravitational field. This results in a delay in the arrival time (Hosokawa, 2004).

With regard to the relationships between the positions of several bodies, the Doppler Effect and gravitational blueshift and redshift are different phenomena. It has also been suspected that the speed of an electromagnetic wave does not change with frequency and energy (Abdo et al., 2009).

For gravitational blueshift,

$$E = MC^2 = M((w \downarrow)^2 + (2\varphi \uparrow)) = (m \uparrow)C(w \downarrow) = (m \uparrow)C(\lambda \downarrow)f = hf \quad (27)$$

For gravitational redshift,

$$E = MC^2 = M((w \uparrow)^2 + (2\varphi \downarrow)) = (m \downarrow)C(w \uparrow) = (m \downarrow)C(\lambda \uparrow)f = hf \quad (28)$$

For Doppler blueshift,

$$\uparrow E = (M \uparrow)C^2 = (M \uparrow)(w^2 + 2\varphi) = (m \uparrow)Cw = (m \uparrow)C(\lambda \downarrow)(f \uparrow) = h(f \uparrow) \quad (29)$$

For Doppler redshift,

$$\downarrow E = (M \downarrow)C^2 = (M \downarrow)(w^2 + 2\varphi) = (m \downarrow)Cw = (m \downarrow)C(\lambda \uparrow)(f \downarrow) = h(f \downarrow) \quad (30)$$

In the above equations, E is the energy; M, the gravitational mass; C, the speed of light; w, the electromagnetic wave speed;  $2\varphi$ , the gravitational potential; h, Plank's constant; f, the frequency; m, the inertial mass; and  $\lambda$ , the wavelength.

The ratio of the speed of light to the wave speed is the same as the ratios of the progress of the clock. The ratios of the progress of the clock and the wave propagation speed are constant relative to stationary observer under given conditions, and the speed of light is always the same.

$$C = (C/w) f\lambda$$

However, the propagation speed of the electromagnetic wave changes with the position of the observer and with the conditions.

$$C/w = n: \text{index of refraction}$$

This is similar to how the propagation speed of light waves changes in different mediums.

$$C/n = w = f\lambda$$

In a sense, this can be used to determine when a

**Table 1.** Comparison according to the theory of lens and redshift.

Theory	Gravitation	Weak field→				→Strong field→				→Weak field (observer)			
	Phenomenon	Wave length	Frequency	Velocity	Energy	wl	f	v	e	wl	f	v	e
General theory of relativity	Lens	—	—	C	—	↓	↑	C	↑	—	—	C	—
	Redshift	—	—	C	—	—	—	C	—	↑	↓	C	↓
This paper	Lens	—	—	w	—	↓	—	↓	—	—	—	w	—
	Redshift	—	—	w	—	—	—	w	—	↑	—	↑	—

gravitational field is not a perfect vacuum (free space) and contains matter. If the binding energy is generated by the perturbation of objects separated by a distance, the potential energy would be restored. The equivalence principle of the momentum of light precisely relates all energy forms in space-time.

$$C = d/t$$

In the above equation, d is the distance and t, the time. A random change in the local energy can therefore be distinguished from a cosmology-like space-time transformation.

**MATTER WAVE AND UNCERTAINTY RELATION**

It is not incorrect for the relation  $E = hf$  to be complete in a matter wave (Greiner, 2001b). It is incorrect to ignore both the light wave and the matter wave as a Lorentz invariance without the equivalence principle and correlation expecting relations, namely,  $v = f\lambda$  or  $C = f\lambda$ . The speed of the quantum, which changes to a different type, is expressed as given below. The energy speed of each quantum is the speed of light.

$$C^2 = v^2 + w_1^2 \rightarrow C = w_2 \tag{31}$$

In the matter wave, a wavelength (speed) change and an inertial mass change have an inverse relation in a manner similar to the gravitational red (blue) shift of the electromagnetic wave.

$$E = MC^2 = M((\Delta w)^2 + (\Delta 2\phi)) = (\Delta m)C(\Delta w) = (\Delta m)C(\Delta \lambda)f = nhf \tag{32}$$

The reference time that we set artificially does not fluctuate. Therefore, energy and time do not show uncertainty relations. An observable result of the energy fluctuates from the space-time of the established reference. Even if the wave speed of the quantum

fluctuates under gravity uncertainly, Planck’s constant remains, and is the reason called the Hertzian oscillator.

The inverse relation of a wavelength (speed) change and an inertial mass change is similar to the group speed of the mixed wave and the relations of the phase velocity, and photons, matter waves, neutrino oscillations (Ahn et al., 2006), and so on can all be explained by the same mechanism. In the energy expression of the wave speed, speed lower than that of light in the mass-energy equivalence of mass have some meaning (Table 1).

**CONCLUSION**

The principles of electromagnetism state are that the speed of light is constant in free (virtual) space and that it is expressed by variable wave speeds in different mediums. Through this paper, it has been shown that the gravitational field could be expressed by the variable wave speed in different mediums. If the speed of light is assumed constant in the mass-energy equivalence, the invariant mass corresponds to the speed of light in free space, and the variable mass and variable wave speed in the matter field and the inertial system should be combined. The rest mass measured on the surface of Earth, which is not a free space, is not invariant. The progress of the clock by the ratio of the speed of light and the wave speed and the equivalence of the two masses must be directly linked to electromagnetism for the original unification of mechanics in only one supposition ( $C/w = m/M$ ). The argument about whether a light wave is twisted or whether space-time is warped is not important. The inertia of light in the background has priority over the theory of relativity. Therefore, it is important how energy is observed from the view of free space. The equivalence principle of light’s momentum harmonizes observations from quantum theory to cosmology.

**Summary**

The speed and velocity, mass, momentum, energy, and

quantum resulting from the interrelation of the physical quantities based on the equivalence principle by the momentum of light are defined as follows:

**1. Speed and Velocity:** The propagation speed of the light waves observed on Earth's surface is ( $w$ ), and the gravitational potential ( $2\Phi$ ) on Earth's surface is given by Earth's mass plus the gravitational potential corresponding to the speed of light ( $C$ ) in free space.

$$(C^2 = w^2 + 2\phi)$$

In the case of an object that has a center of gravity, the light wave speed ( $w$ ) is decomposed into the particle velocity ( $v$ ) and the wave speed ( $w$ ).

$$(C^2 = v^2 + w^2 + 2\phi)$$

**2. Mass:** The invariable gravitational mass ( $M$ ) is proportional to the square of the speed of light ( $C^2$ ) and is the amount of matter that corresponds to the law of universal gravitation. The variable inertial mass ( $m$ ) is proportional to the light wave speed ( $w$ ) and is the amount of matter that corresponds to the law of conservation of momentum and Newtonian mechanics.

**3. Momentum:** The momentum of light ( $P$ ), momentum ( $MC$ ) obtained from the invariable gravitational mass ( $M$ ) and the speed of light ( $C$ ), and momentum ( $mw$ ) obtained from the variable inertial mass ( $m$ ) and the light wave speed ( $w$ ) are all equivalent (that is,  $P = MC = mw$ ) and correspond to the momentum of the material conforming to the conservation of momentum and the laws of motion of Newtonian mechanics.

**4. Energy:** For a substance that obeys the law of conservation of energy, the total energy ( $E$ ) is calculated using the square of the speed of light ( $C^2$ ) and the invariant mass ( $M$ ), whereas the particle energy is calculated using the velocity ( $v$ ) of the center of gravity, light wave speed ( $w$ ), gravitational potential ( $2\phi$ ), speed of light ( $C$ ), and momentum of light ( $P$ ).

$$E = PC = MC^2 = M(v^2 + w^2 + 2\phi) = mwC$$

**5. Quantum:** The digital measurement of energy ( $E$ ) is satisfied with the law of conservation of energy using the product of Planck's constant ( $h$ ), frequency ( $f = w/\lambda$ ), and quantum number ( $n$ ).

$$E = PC = MC^2 = mwC = nhf$$

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