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Full Length Research Paper

Direction of arrival estimation using modified orthogonal matching pursuit algorithm

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Direction of arrival (DOA) estimation is a sparse reconstruction problem. However, conventional orthogonal matching pursuit (OMP) may fail to identify the correct atoms since the redundant dictionary composed of the direction vectors is highly coherent. To mitigate the coherence problem, in this paper, we propose a modified OMP by constructing data dependent sensing dictionary for sparse reconstruction in the noisy case. Simulation results are presented to demonstrate the superior performance of the proposed algorithm for DOA estimation.

Key words: Direction of arrival (DOA) estimation, orthogonal matching pursuit (OMP), sparse reconstruction, redundant dictionary, sensing dictionary.

INTRODUCTION

Direction of arrival (DOA) estimation plays an important role in many fields involving wireless communication systems (Godara, 1997; Chandran, 2006), radar detections (Wan and yang, 2002; Li and Stoica, 2009) and underwater acoustics (UWA) communications (Lake, 1998; Tao et al., 2007). When a small number of signal sources exist in far-field space, the spatial spectrum is sparse and the DOA estimation can be a problem of sparse representation over a redundant dictionary. In view of this, the goal of DOA algorithm is to find the sparsest representation for observed data by using redundant dictionary composed of direction vectors.

In general, the problem of sparse signal representation over redundant dictionary is non-deterministic polynomial-time hard (Davis et al., 1997). Several suboptimal methods have been proposed for this problem. Among them is orthogonal matching pursuit (OMP) (Pati et al., 1993), which is an attractive algorithm since it is fast and easy to implement. It has been proved that a signal can be reconstructed by OMP algorithm if the redundant dictionary is incoherent and the signal is sparse enough (Tropp, 2004; Gribonval and Vandergheynst, 2006). Unfortunately, the redundant dictionaries for DOA estimation are highly coherent and the ordinary OMP may fail to reconstruct observed data in this case. In a recent literature, Schnass and Vandergheynst (2008) proposed a modified version of OMP by introducing a sensing dictionary. This method generalized the ordinary OMP to the case of coherent dictionary. Based on alternating projection (AP), a method for designing sensing dictionary was proposed in the noiseless case. However, the application of sparse representation algorithm for DOA estimation must be robust against additive noise since the observed data obtained from sensor arrays is generally contaminated by noise.

In this paper, we extend sensing dictionary to the noisy case and develop a modified OMP for DOA estimation by constructing data dependent sensing dictionary. Different from the iterative sensing dictionary design methods (Schnass and Vandergheynst, 2008), our proposed method can provide a sensing dictionary with closed-form by solving a simple optimization problem. Hence, our proposed method can easily be implemented in potential applications.

Notation: Throughout the paper, we denote vectors in C^N by boldface lowercase letter, e.g., $\mathbf{s}$ or by uppercase

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letters A and matrices by boldface uppercase letters, e.g. $\mathbf{A}$.

PROBLEM FORMULATION

Consider K narrowband far-field signals $s_k(t)\ (k=1,...,K)$ impinge on an M-element array from directions $\theta_k\ (k=1,...,K)$. The discrete time samples of the signals $x(t)$ received at the array are expressed using the vector form as (Krim and Viberg, 1996).

$$\mathbf{x}(t)=\mathbf{A}(\theta)\mathbf{s}(t)+\mathbf{v}(t), \tag{1}$$

where $\mathbf{x}(t)=[x_1(t),...,x_M(t)]^T$, $\mathbf{A}(\theta)$ is the array manifold matrix which is composed of direction vectors $A(\theta_k)\ (k=1,...,K)$, $\mathbf{s}(t)=[s_1(t),...,s_K(t)]$, $\mathbf{v}(t)$ is the unknown additive noise and $(\cdot)^T$ denotes the transpose operation. Given the observed data $\mathbf{x}(t)$, the goal of DOA estimation is to find the unknown directions of sources $\theta_k\ (k=1,...,K)$.

Let θ_n ($n=1,...,N$, in general, N>>M) be the sampling grids of all directions of interest. We construct a redundant dictionary $\mathbf{A}$ composed of direction vectors in terms of potential directions $\theta_n\ (n=1,...,N)$ as $\mathbf{A}=[A(\theta_1),...,A(\theta_N)]$, where the column vector $A(\theta_n)$ is called atom. The problem of DOA estimation can be shown as sparse representation problem by introducing the redundant dictionary $\mathbf{A}$. For a single snapshot, Equation 1 reduces to:

$$\mathbf{x}=\mathbf{A}\mathbf{s}+\mathbf{v}, \tag{2}$$

where the k-th element $\mathbf{s}$ is nonzero and equal to $s_k(t)$ if source comes from θ_k for some k and zeros otherwise. The received signal $\mathbf{x}$ is called K-sparsity since $\mathbf{s}$ only contains K nonzero entries. The ordinary OMP algorithm iteratively selects K atoms in dictionary $\mathbf{A}$ which have the strongest correlation with the residual $(\mathbf{x}-\mathbf{A}\mathbf{s})$. In step k ($k=1,2,\cdots,K$), the best matching atom A_{n_k} is selected through solving the simple optimization problem

$$n_k=\arg\max_{n\in\Omega}\hat{s}_n^{(k)}, \tag{3}$$

where $[\hat{s}_1^{(k)},\hat{s}_2^{(k)},\cdots,\hat{s}_N^{(k)}]^T=\hat{\mathbf{s}}^{(k)}=\left|\mathbf{A}^H\mathbf{r}_k\right|$, $\Omega=\{1,...,N\}$, and $(\cdot)^H$ represents complex conjugate transpose. We have $\mathbf{r}_1=\mathbf{x}$ for initialization, and $\mathbf{r}_{k+1}=\mathbf{P}_k\mathbf{x}$ for $k=1,2,\cdots,K-1$, where

$$\mathbf{P}_k=\mathbf{I}_k-\hat{\mathbf{A}}^{(k)}((\hat{\mathbf{A}}^{(k)})^H\hat{\mathbf{A}}^{(k)})^{-1}(\hat{\mathbf{A}}^{(k)})^H,$$

$\hat{\mathbf{A}}^{(k)}=[A_{n_1},A_{n_2},\cdots,A_{n_k}]$, and $\mathbf{I}_k$ is a $k\times k$ identity matrix.

MODIFIED OMP ALGORITHM

In order to identify the correct atoms in highly coherent dictionary, we exploit a sensing dictionary $\mathbf{B}$ ($\mathbf{B}\in C^{M\times N}$) and relax the ordinary OMP to the more general case. In each step of the modified OMP, we solve the optimization to find $\hat{\mathbf{s}}^{(k)}=\left|\mathbf{B}^H\mathbf{r}_k\right|$ rather than $\hat{\mathbf{s}}^{(k)}=\left|\mathbf{A}^H\mathbf{r}_k\right|$. At first step, the new sensing is $\mathbf{B}^H(\mathbf{A}\mathbf{s}+\mathbf{v})$ since $\mathbf{r}_1=\mathbf{x}$ in this step. A good sensing dictionary $\mathbf{B}$ should satisfy the condition that the correlation between the atoms $B_n\ (n=1,...,N)$ and the optimal atoms $A_{\Lambda_{opt}}$ ($A_{\Lambda_{opt}}$ is a sub-dictionary whose columns are the optimal atoms with the index set Λ_{opt}) is as small as possible. Thus, we design the sensing dictionary $\mathbf{B}$ as the solution to the following optimization

$$\min_{B_n}\left\|A_{\Lambda_{opt}}^H B_n\right\|_2^2, \quad \text{s.t.}\quad B_n^H A_n=1,\ for\ \ n=1,...N. \tag{4}$$

Using Lagrangian function, we have:

$$B_n=R_nA(\theta_n) \tag{5}$$

for $n=1,...,N$, where

$$R_n=\frac{\left(A_{\Lambda_{opt}}A_{\Lambda_{opt}}^H+\alpha I_K\right)^{-1}}{A^H(\theta_n)\left(A_{\Lambda_{opt}}A_{\Lambda_{opt}}^H+\alpha I_K\right)^{-1}A(\theta_n)}, \tag{6}$$

and α is a positive regularization parameter, which is introduced to improve the robustness in the noisy case.

However, the ideal sensing dictionary (Equation 5) cannot be obtained since $A_{\Lambda_{opt}}$ is unknown in practical applications. Note that the optimal atoms $A_{\Lambda_{opt}}$ are dependent on the observed data $\mathbf{x}$, we weighed the atoms of the ordinary dictionary $\mathbf{A}$ by introducing effective posteriori knowledge obtained from observed data to design the sensing dictionary. Accordingly, the data dependent sensing dictionary can be constructed as the solution to the following optimization.

$$\min_{B_n}\left\|(\mathbf{A}\mathbf{W})^H B_n\right\|_2^2, \quad \text{s.t.}\quad B_n^H A_n=1,\ \ for\ \ n=1,...,N, \tag{7}$$

where $\mathbf{W}=diag\{w_1,...,w_N\}\ (w_i\in[0,1])$ is the weighting

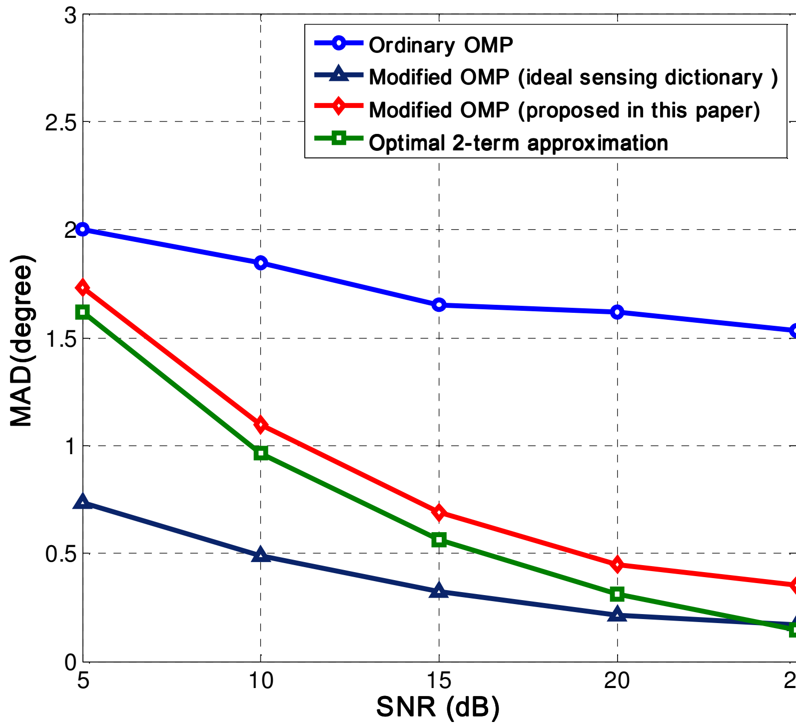

Figure 1. MAD via SNR for estimation of DOA1=8°.

matrix. It is easy to get each atom of the data dependent sensing dictionary as:

$$B_n = U_n A(\theta_n), \quad (8)$$

where

$$U_n = \frac{\left(AW^2A^H + \beta I_M\right)^{-1}}{A^H(\theta_n)\left(AW^2A^H + \beta I_M\right)^{-1} A(\theta_n)}, \quad (9)$$

for $n = 1, \ldots, N$ and β is a positive regularization parameter. Note that the correlation between each atom $A(\theta_n)$ and the observed signal $\mathbf{x}$ suggests the probability for the corresponding atom to appear in the optimal atoms (Divorra et al., 2006). One possible choice for the weighting matrix is $\mathbf{W} = \mathrm{diag}(|\mathbf{A}^H\mathbf{x}|)$.

SIMULATION RESULTS

Consider two narrowband far-field signal sources impinge from DOA1=8° and DOA2=17° on a uniform linear array of 12 elements separated by half a wavelength. DOA dictionary is composed of steering vectors with uniform grid $\Delta\theta = 0.2^\circ$. According to our preliminary simulations, we found that the performance of the proposed method is not sensitive to regularization parameter and hence its value is set to $\beta \in [0.01, 0.3]$. Wan and Yang (2002) defined an optimal 2-term approximation as the solution to the following optimization problem:

$$\min_{\substack{A(\theta_i), A(\theta_j), s_i, s_j \\ \theta_i, \theta_j \in \{\theta_1, \ldots, \theta_N\}}} \left\| \mathbf{x} - A(\theta_i)s_i - A(\theta_j)s_j \right\|_2 . \quad (10)$$

We compare the performance of the proposed method with that of the ordinary OMP algorithm, the modified OMP with ideal sensing dictionary obtained by Equation 5 and the exhaustive 2-term approximation method in Equation 10. Simulation results are obtained over independent 1000 Monte-Carlo trials. Figures 1 and 2 present the mean absolute deviation (MAD) via signal noise ratio (SNR) with a single snapshot. MAD is calculated as $1/P\sum_{p=1}^{P}\left|\hat{\theta}_{ip} - \theta_i\right|$, where θ_i $(i = 1, 2)$ is the true DOA, $\hat{\theta}_{ip}$ is the estimate value and P is the number of independent trials. These results show that ordinary OMP algorithm fails to estimate DOAs because the DOA dictionary is highly coherent. The performance of the proposed method is close to that of exhaustive optimal 2-term approximation. The modified OMP algorithm based on ideal sensing dictionary outperforms all the other methods because it takes the correct atoms

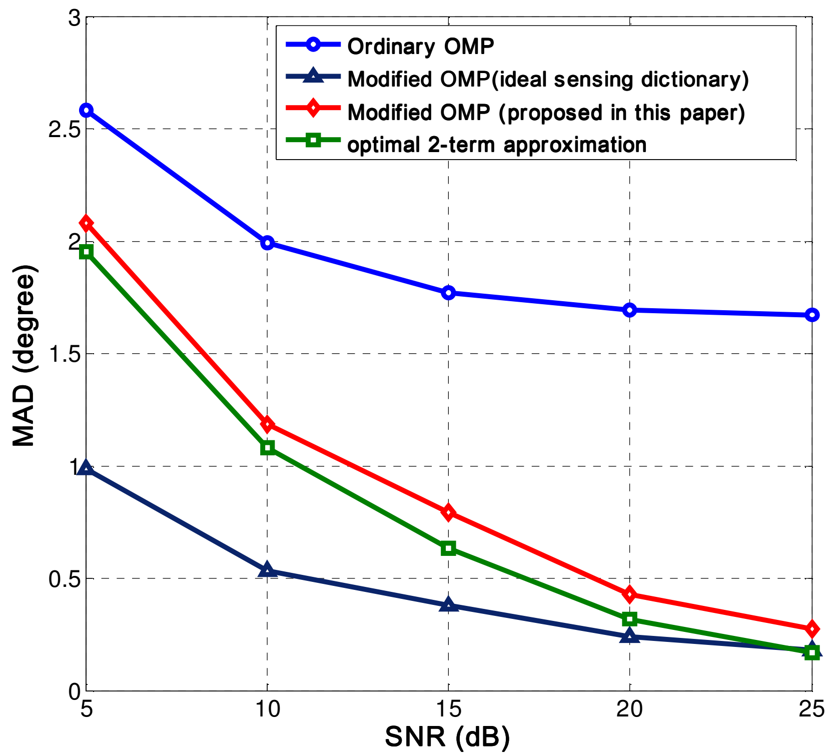

Figure 2. MAD via SNR for estimation of DOA2=17°.

as a priori information.

Conclusions

In this paper, we proposed a method to construct data dependent sensing dictionary in noisy case and developed a modified OMP based on sensing dictionary for DOA estimation. Simulation results were provided to illustrate the improved performance of the proposed algorithm. Thus, our proposed method has a potential application for developing DOA estimation. Our proposed method can also be utilized on sparse channel estimation problem.

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