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Full Length Research Paper

Application of Benjamin-Feir equations to Tornadoes' rogue waves modulational instability in oceans

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Pilots put their plane in the ideal conditions for the atmosphere to suck it and allow a successful launch; similarly some portions of the ocean surface put a Troposphere column placed above them in ideal temperature and humidity training tornadoes and primary gravity waves that accompany them. To study the behavior of rogue waves triggered by tornadoes in terms of their space and time evolution, that is, their motion and also in terms of mechanical transformations that these systems may suffer in their dealings with other systems, we use Benjamin-Feir equations on gravity waves modulational instability. Spectacular images available on tornadoes when impacting on land allow each of us to have an opinion on the magnitude of the pressure forces deployed by that space weather phenomenon. This study on atmosphere-oceans interactions is unique and based on the effectiveness of Mbanes' fluid dynamic balance model which provides relevant knowledge on the vertical profile of winds triggered by tornadoes.

Key words: Tornadoes' gravity waves, modulational instability, Benjamin-Feir equations.

INTRODUCTION

Wherever observations come from, they can lead to the understanding of the effects but not to the knowledge of the causes which produce these effects. To understand the physics of a natural phenomenon, it is essential to clearly identify (beyond the observed facts), all elementary processes at the origin of the phenomenon. Mathematical theories on physics principles, subsequently leads to the development of models. Questions raised by observations lead to physics' laws while observations can in no way serve as physics' laws. The adverse effects of climate change modify considerably earth's ecosystem and reveal human beans vulnerability. In fact, each year the occupants of the Earth suffer extreme events, often deadly, due to climate change and related phenomena as floods, landslides, or destruction of unsuitable habitats, etc. ... often causing irreversible damage like the eradication of entire coastal cities. To fully understand the physical processes responsible for the formation of tornadoes and which in turn responsible for the appearance of rogue waves, it is essential to assimilate the concept of Troposphere Dynamic Balance and the vertical profiles of wind thereto, observed within cyclones and hurricanes (Mbane, 2012). Additional studies or experiments to identify the values of temperature and humidity, training the outbreak (or birth) of tornadoes over the oceans (including land) should be seriously considered. Space technology could then be used effectively, as decision tools in the prediction of tornadoes related rogue waves. We continue our quest for knowledge about "tornadoes' rogue waves" by making use of Benjamin-Feir instability equations on gravity waves to simulate the behavior of tornadoes' rogue waves that are generally born very far from the coast and yet unfortunately, sometimes close to boats or ships. Given the fact that mechanical transformations that occur on rogue waves, during their space and time evolution, also depend on the propagation milieu; our simulations will allow prospects that space technology observation unfortunately cannot allow (until now rogue waves, as

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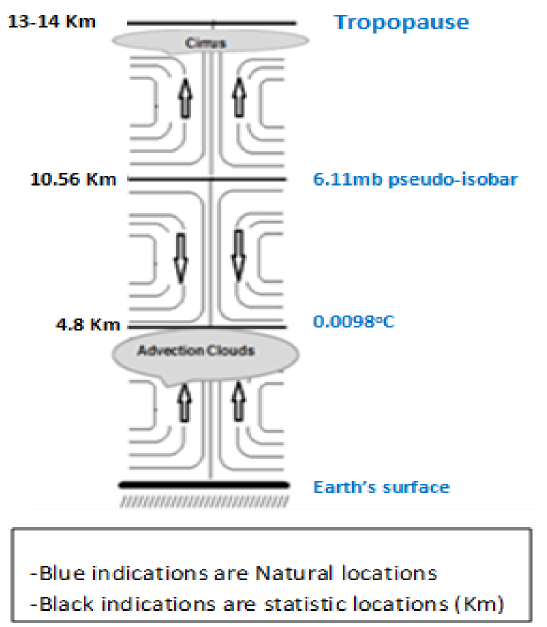


Figure 1. Tornadoes are triggered by passive deep convection generated by a heat hot source located at the surface of the earth. They appear as very high towers (12 to 14 Km) consisting of three floors: warm updrafts occupy floors 1 and 3 while warm downdrafts occupy floors 2.

tornadoes that trigger them, are unpredictable).

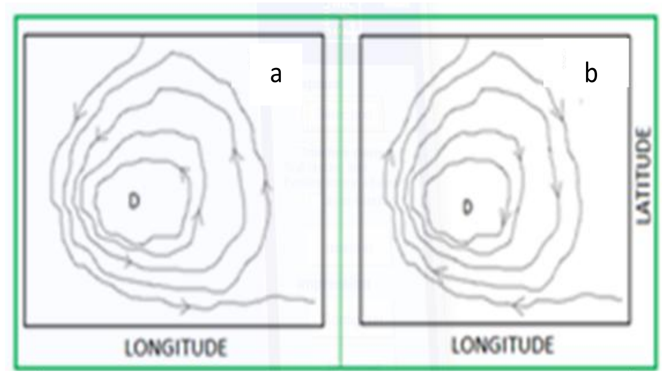
BASIC KINEMATICS AND THERMODYNAMICS OF TORNADOES

Tornadoes result according to Mbane's atmosphere dynamic balance model (Mbane, 2012), from combination *of* very deep and passive convection (Figure 1) with geostrophic winds (Figure 2) inside troposphere moister and warmer columns. These space weather phenomena are triggered by passive deep convection generated by a hot source located at the surface of the earth and appear (Figure 1) as very high towers (12 to 14 Km) consisting of three floors: warm updrafts occupy Floors 1 and 3 while warm downdrafts occupy Floors 2. According to photographs of Figure 3, over-land tornadoes trigger dust clouds whose base is thin compared to the peak which is very broad. Tornadoes can also electrify (Mbane, 2012) the troposphere column in which it is formed (Figure 3c). The broadest peaks of the dark clouds indicate the presence of hot downdrafts that prevent the progression of warm updrafts.

FORMULATION OF BENJAMIN-FEIR EQUATIONS

a) Physics behind Benjamin-Feir equations

The general fluid continuity equation is given by:



Fiure 2. Streamlines of geostrophic winds triggered by tornadoes around their low pressure groove. a) Rotative in the Northern hemisphere, b) Contra-rotative in the Southern hemisphere.

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}+\frac{\partial \rho w}{\partial z}=0 \qquad (1)$$

This leads to the continuity equation for an incompressible fluid

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (2)$$

The velocity perpendicular to the surface of the water and to the impermeable bottom is zero:

$\vec{V}.\vec{n}=0$, at z = -H or z = η (η is sea surface level) (3)

Here $\vec{n}$ is the unit vector normal to the surface.
When the bottom is parallel to the undisturbed surface

w = 0, z = -H (4)

and the kinematic boundary condition at the surface becomes:

$$\left(u,v;w-\frac{\partial \eta}{\partial t}\right).\left(-\frac{\partial \eta}{\partial x},\frac{\partial \eta}{\partial y},1\right)=0 \text{ , } z=\eta \Rightarrow w=\frac{\partial \eta}{\partial t}+u\frac{\partial \eta}{\partial x}+v\frac{\partial \eta}{\partial y} \qquad (5)$$

where the surface of the water is allowed to change with time.

The last condition comes from the Newtonian force on a moving fluid element

$$\frac{\partial \vec{V}}{\partial t}+(\vec{V}.\vec{\nabla})\vec{V}=\nu((\vec{\nabla}.\vec{\nabla})\vec{V})-\frac{1}{\rho}\vec{\nabla}P+\vec{g} \qquad (6)$$

For an inviscid fluid this simplifies to

Figure 3. Tornadoes trigger dust clouds whose base is thin compared to the peak which is very broad. Tornadoes can also electrify (Mbane, 2012) the troposphere column in which it is formed (Figure 3c). The broadest peak indicates the presence of hot downdrafts that prevent the progression of warm updrafts.

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V} = -\frac{1}{\rho}\vec{\nabla}P + \vec{g} \quad (7)$$

When the flow is irrotational

$$\vec{\nabla}.\vec{V} = 0 \quad (8)$$

The velocity potential is also given by

$$\vec{V} = (u, v, w) = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right) = \vec{\nabla}\phi \quad (9)$$

Given the continuity equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \Delta\phi = 0 \quad (10)$$

The kinematic boundary condition at the bottom

$$\vec{\nabla}\phi.\vec{n} = 0, \ z = -H \quad (11)$$

The kinematic boundary condition at the surface

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} + \nabla_{\perp}\phi.\nabla_{\perp}\eta, \ z = \eta \quad (12)$$

Integrating Equation (7) with respect to x, y, z, one can get the Bernoulli equation, the arbitrary functions of integration $C_1(y,z,t)$, $C_2(x,z,t)$, $C_3(x,y,t)$ must be the same function C(t), which can be absorbed by the velocity potential, yielding exactly the same flow

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\vec{\nabla}_Z\phi)^2 + g\eta = -\frac{1}{\rho}(P_{AS} - P_0), \ z = \eta \quad (13)$$

Here we have made the assumption that $\vec{g}$ is constant $\vec{g} = (0,0,-g)$ making the gravitational force conservative and making it possible to define a potential energy.

Furthermore we have made the assumption that the surface tension can be neglected. At the sea surface, z = η when tornadoes occur: P_{AS} represents the atmosphere pressure at sea surface below tornado eye and P_0 the atmosphere pressure at level (Z = 0). Taking ϕ constant and equal to 0 at sea surface, the boundary condition (13) becomes,

$$\eta\ (x, y, T_S, t) = -\frac{1}{\rho(T_S).g}(P_{AS} - P_0) \quad (14)$$

T_S is the sea surface temperature

According to Equation (14), Tornadoes primary gravity wave height depends on both deepest of depression (that is, P_{AS} tends to zero) and sea surface temperature T_S.

Equations (10, 11, 12 and 14) are basis for all the following calculations. By introducing the stream function $\psi(x,y,t)$, defined by $\psi(x,y,t)=\phi(x,y,\eta,t)$, Equations 12 and 13 become:

$$\eta_t + (\vec{\nabla}\ \eta).(\vec{\nabla}_\perp\psi) - \phi_z\left(1+(\vec{\nabla}_\perp\eta)^2\right) = 0 \quad (15)$$

$$\psi_t + g\eta + \frac{1}{\rho}(P_A - P_O) + \frac{1}{2}(\vec{\nabla}_\perp\psi)^2 - \frac{1}{2}\phi^2(1+(\vec{\nabla}_\perp\eta)^2) = 0 \quad (16)$$

b) Materialization of the interface between Euler and Benjamin-Feir equations

Benjamin-Feir equations were obtained for the first time in 1967 (Benjamin, 1967) for ultra-deep waters and for deep waters. Benjamin-Feir equations were gotten and Dyachenko and Zakharov (2005). The use of kinematic boundary conditions (Equations 11 and 12) and dynamic (Equation 14), and the Fourier transform of the Dirac δ function and the development in Taylor series of hyperbolic functions yields the Fourier transform of the stream function $\psi(x,y,t)$ at the free surface of water:

$$\overline{\psi} = \overline{\phi} + \frac{1}{2\pi}\iint \overline{\phi}_1 \tanh(k_1H)\vec{k}_1\overline{\eta}_2\delta(\vec{k}_1+\vec{k}_2-\vec{k})d\vec{k}_1d\vec{k}_2 + \frac{1}{(2\pi)^2}\iiint \overline{\phi}_1\frac{k_1^2}{2}\overline{\eta}_2\overline{\eta}_3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k})d\vec{k}_1d\vec{k}_2d\vec{k}_3 + \ldots \quad (17)$$

Equation (17) is inverted iteratively. It is natural to choose the starting guess as $\overline{\phi}^{(1)} = \overline{\psi}$. Then,

$$\begin{aligned}\overline{\phi} = \overline{\psi} - \frac{1}{2}\iint k_1\tanh(\vec{k}_1H)\overline{\psi}_1\overline{\eta}_2\delta(\vec{k}_1+\vec{k}_2-\vec{k})d\vec{k}_1d\vec{k}_2d\vec{k}_1d\vec{k}_2 - \\ \frac{1}{(2\pi)^2}\iiint \frac{1}{4}k_1\overline{\psi}_1\overline{\eta}_2\overline{\eta}_3(2k_1 - \tanh(k_1H) \\ \left(|\vec{k}_1+\vec{k}_2|\tanh(|\vec{k}_1+\vec{k}_2|H) + |\vec{k}_1+\vec{k}_2|\tanh(|\vec{k}_1+\vec{k}_2|H) + \right. \\ \left.|\vec{k}-\vec{k}_2|\tanh(|\vec{k}-\vec{k}_2|H) + |\vec{k}-\vec{k}_3|\tanh(|\vec{k}-\vec{k}_3|H)\ \right)) \\ \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k})d\vec{k}_1d\vec{k}_2d\vec{k}_3 - \ldots\end{aligned} \quad (18)$$

The Fourier transform of the velocity vector at the water surface is given by (19):

$$\begin{aligned}\overline{v} = \overline{\psi}k\tanh(kH) - \frac{1}{2\pi}\iint\left(k\tanh(kH)k_1\tanh(k_1H) - k_1^2\right)\overline{\psi}_1\overline{\eta}_2\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k})d\vec{k}_1d\vec{k}_2 - \\ \frac{1}{(2\pi)^2}\iiint\frac{1}{4}\overline{\psi}_1\overline{\eta}_2\overline{\eta}_3k_1\left(k\tanh(kH)\right)(2k_1 - \tanh(k_1H) \\ \left(|\vec{k}_1+\vec{k}_2|\tanh(|\vec{k}_1+\vec{k}_2|H) + |\vec{k}_1+\vec{k}_3|H)\tanh(|\vec{k}_1+\vec{k}_3|H) + \right. \\ \left.|\vec{k}-\vec{k}_2|\tanh(|\vec{k}-\vec{k}_2H|) + |\vec{k}-\vec{k}_3|\tanh(|\vec{k}-\vec{k}_3|H)\ \right)) + \\ \tanh(k_1H)(2k^2 - 4\vec{k}_2.\vec{k}_3)\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k})d\vec{k}_1d\vec{k}_2d\vec{k}_3 - \ldots\end{aligned} \quad (19)$$

Equations 15 and 16 are modified using relations 17-18-19, to obtain the Fourier transforms of the dynamic and kinematics boundary conditions.

$$\begin{aligned}\overline{\eta}_t - \overline{\psi}\frac{\omega^2}{g} + \frac{1}{2\pi}\iint\left(\frac{\omega^2}{g}\frac{\omega_1^2}{g} - \vec{k}.\vec{k}_1\right)\overline{\psi}_1\overline{\eta}_2\delta(\vec{k}_1+\vec{k}_2-\vec{k})d\vec{k}_1d\vec{k}_2 \\ \frac{1}{16\pi^2}\iiint\left(2k^2\frac{\omega_1^2}{g} + 2k_1^2\frac{\omega^2}{g} - \frac{\omega^2}{g}\frac{\omega_1^2}{g}\left(\frac{\omega^2_{\vec{k}_1+\vec{k}_2}}{g} + \frac{\omega^2_{\vec{k}_1+\vec{k}_3}}{g} + \right.\right. \\ \left.\left.\frac{\omega^2_{\vec{k}-\vec{k}_2}}{g}\frac{\omega^2_{\vec{k}-\vec{k}_3}}{g}\right)\right)\overline{\psi}\overline{\eta}_2\overline{\eta}_3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k})d\vec{k}_1d\vec{k}_2d\vec{k}_3 + \ldots = 0\end{aligned} \quad (20)$$

$$\begin{aligned}\overline{\psi} + g\overline{\eta} - \frac{1}{4\pi}\iint\left(\frac{\omega_1^2}{g}\frac{\omega_2^2}{g} + \vec{k}_1.\vec{k}_2\right)\overline{\psi}_1\overline{\psi}_2\delta(\vec{k}_1+\vec{k}_2-\vec{k})d\vec{k}d\vec{k} - \\ \frac{1}{16\pi^2}\iiint\left(2k_1^2\frac{\omega_2^2}{g} + 2k_2^2\frac{\omega_1^2}{g} - \frac{\omega_1^2}{g}\frac{\omega_2^2}{g}\left(\frac{\omega^2_{\vec{k}_1+\vec{k}_3}}{g} + \frac{\omega^2_{\vec{k}_2+\vec{k}_3}}{g} + \right.\right. \\ \left.\left.\frac{\omega^2_{\vec{k}-\vec{k}_2}}{g} + \frac{\omega^2_{\vec{k}-\vec{k}_1}}{g}\right)\right)\overline{\psi}_1\overline{\psi}_2\overline{\eta}_3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3-\vec{k})d\vec{k}_1d\vec{k}_2d\vec{k}_3 - \ldots = 0\end{aligned} \quad (21)$$

Equations 20 and 21 are combined into a single equation by introducing the complex function b given by (22):

$$b(\vec{k},t)=\sqrt{\frac{g}{2\omega}}\overline{\eta}(\vec{k},t)+i\sqrt{\frac{\omega}{2g}}\overline{\psi}(\vec{k},t) \quad (22)$$

From that moment, we define the Fourier transforms of η and $\psi(x,y,t)$ as function of b and its conjugate b *.

$$\overline{\eta}(\vec{k},t)=\sqrt{\frac{\omega}{2g}}\left(b(\vec{k},t)\right)+b^{*}(-\vec{k},t) \quad (23)$$

$$\overline{\psi}(\vec{k},t)=i\sqrt{\frac{g}{2\omega}}\left(b(\vec{k},t)+b^{*}(\vec{k},t)\right) \quad (24)$$

Then multiplied [eq. 20] by $(g/2\omega)^{1/2}$ and [eq. 21] by $i\,(\omega/2g)^{1/2}$. Sums of the terms are given by (25).

$$\begin{aligned}
&b_t+i\omega b+i\iint V^1(\vec{k},\vec{k}_1,\vec{k}_2)b_1b_2\delta(\vec{k}-\vec{k}_1-\vec{k}_2)d\vec{k}_1d\vec{k}_2+\\
&\quad i\iint V^2(\vec{k},\vec{k}_1,\vec{k}_2)b_1^*b_2^*\delta(\vec{k}+\vec{k}_1-\vec{k}_2)d\vec{k}_1d\vec{k}_2+\\
&\quad i\iint V^3(\vec{k},\vec{k}_1,\vec{k}_2)b_1^*b_2^*\delta(\vec{k}+\vec{k}_1+\vec{k}_2)d\vec{k}_1d\vec{k}_2+\\
&\quad i\iiint W^1(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3)b_1b_2b_3\delta(\vec{k}-\vec{k}_1-\vec{k}_2-\vec{k}_3)d\vec{k}_1d\vec{k}_2d\vec{k}_3+\\
&\quad i\iiint W^2(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3)b_1^*b_2b_3\delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3)d\vec{k}_1d\vec{k}_2d\vec{k}_3+\\
&\quad i\iiint W^3(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3)b_1^*b_2^*b_3\delta(\vec{k}+\vec{k}_1+\vec{k}_2-\vec{k}_3)d\vec{k}_1d\vec{k}_2d\vec{k}_3+\\
&\quad i\iiint W^4(\vec{k},\vec{k}_1,\vec{k}_2,\vec{k}_3)b_1^*b_2^*b_3^*\delta(\vec{k}+\vec{k}_1+\vec{k}_2+\vec{k}_3)d\vec{k}_1d\vec{k}_2d\vec{k}_3+\ldots=0
\end{aligned} \quad (25)$$

In Equation (26), the surface wave $b(\vec{k},t)$ is decomposed into a principal component B and two minor components B' and B'': these components are all functions of t, $t_1=\varepsilon.t$ and $t_2=\varepsilon^2 t$.

$$b(\vec{k},t)=\left(\varepsilon B(\vec{k};t,t_1,t_2)+\varepsilon^2B^{'}(\vec{k},t,t_1,t_2)+\varepsilon^3B^{''}(\vec{k},t,t_1,t_2)\right)e^{(-i\omega_k t)} \quad (26)$$

We derive the surface wave (26) with respect to time and substituted in (25). Then taking $\beta=\varepsilon.B$, we obtain what we call the Benjamin-Feir integral Equation (Equation 27).

$$i\beta_t=\iiint T_{0,1,2,3}\beta_1^*\beta_2\beta_3\delta(\vec{k}+\vec{k}_1-\vec{k}_2-\vec{k}_3)e^{i(\omega_0+\omega_1-\omega_2-\omega_3)t}d\vec{k}_1d\vec{k}_2d\vec{k}_3 \quad (27)$$

METHODOLOGY AND APPROACH

Determination of the coupled nonlinear Schrödinger equations (CNLSE)

According to the nonlinear Schrödinger equation (NLSE), the evolution of an unstable wave group generates a single wave that can reach up to 3 times the amplitude of the initial carrier wave (that is, the wave energy is basically concentrated in a single wave number). Considering a surface wave whose main component β is of the form $\beta=a(k,t).e^{-i\omega.t}$, then Benjamin-Feir integral equation for this type of wave has the form:

$$\frac{\partial a_1}{\partial t}+i\omega a_1=-i\int T_{1,2,3,4}\delta(\vec{k}_1+\vec{k}_2-\vec{k}_3-\vec{k}_4)d\vec{k}_1d\vec{k}_2d\vec{k}_3 \quad (28)$$

We consider the case of energy concentrated mainly around two-wave numbers

$$a(\vec{k})=A(\vec{k}-\vec{k}_{(a)})e^{-i\omega_{(a)}t}+B(\vec{k}-\vec{k}_{(b)})e^{-i\omega_{(b)}t} \quad (29)$$

Where A and B satisfy the CNLSE (Bespalov and Talanov, 1966; Karsten and Igor, 2000; Shener, 2010; Socquet et al., 2005; Zakharov and Kharitonov, 1970; Hasimoto and Ono, 1972; Feir, 1967; Benjamin and Feir, 1967; Kharif and Pelinovsky, 2003; White and Fornberg, 1998; Wu and Yao, 2004; Yuen and Lake, 1980):

$$\frac{\partial A}{\partial t}=-C_x\frac{\partial A}{\partial x}-C_y\frac{\partial A}{\partial y}+i\left(\tau\frac{\partial^2A}{\partial x^2}+\varsigma\frac{\partial^2A}{\partial y^2}+\vartheta\frac{\partial^2A}{\partial x\partial y}\right)-i\left(\xi|A|^2A+2\zeta|B|^2A\right) \quad (30)$$

$$\frac{\partial B}{\partial t}=-C_x\frac{\partial B}{\partial x}-C_y\frac{\partial B}{\partial y}+i\left(\tau\frac{\partial^2B}{\partial x^2}+\varsigma\frac{\partial^2B}{\partial y^2}+\vartheta\frac{\partial^2B}{\partial x\partial y}\right)-i\left(\xi|B|^2B+2\zeta|A|^2B\right) \quad (31)$$

We now consider the following plane wave solution of CNLSE:

$A=A_0.\,(1+a).e^{-i(\omega.t+\Phi_a)}$, $B=B_0.\,(1+b).e^{-i(\omega.t+\Phi_b)}$ (32)

Where a, b, Φ_a,Φ_b are small perturbations in amplitude and of the wave solution. We substitute (32) in (30) and (29), then linearize the resulting equations and use the normal mode approach, with the wave number, $\vec{K}(K,L)$, and the angular frequency, Ω, of the perturbation, to obtain the following dispersion equation:

$$\Omega=\pm\sqrt{\tau K^2\left[(\xi(A_0^2+B_0^2)+\tau K^2)\pm\sqrt{\xi^2(A_0^2-B_0^2)^2+16\zeta^2A_0^2B_0^2}\right]} \quad (33)$$

Approach

Calculations are performed by a MATLAB program. The curves are shown in 3D and can be rotated according to our desire. We choose 300 iterations that produce figure with relevant color coding.

RESULTS AND DISCUSSION

In the following, we numerically solve our nonlinear dispersion relation (33) and quest for evolution of tornadoes' primary waves that generally occur very far from the coast and yet unfortunately, sometimes close to big boats. Figure 4 presents tsunami gravity wave and

Figure 4. Tsunami gravity wave occurs like uninvited guest at a part.

Figures 5(a-c) present various configurations of tornadoes' primary waves. Full dynamics of modulational instability primary rogue waves subjected to their demodulation or filamentation (Figure 6). Simulations of phase's modulations of sinusoidal waves are shown on Figures 7(a-d) for each value of wave number. Instead the growth rates "gain" or imaginary part of Ω (Atock et al., 2011) depend on different angles θ between interacting tornadoes' primary waves directions. Obviously, the rogue waves are generated by filamentation or interference of tornadoes' primary waves whose directions of propagation are nearly parallel (that is, $0 < \theta < \pi / 6$).

Due to high thermal inertia of liquid water, tornadoes over the oceans do not trigger stratocumuliform dark clouds that announce storms but only gravity waves as visual manifestation. For this reason, ocean's tornadoes seem to come from nowhere as stated by many authors. All that ensures their unexpected character (expressed by the quasi-spontaneous passage from a calm situation to a sea greatly agitated). The results will have relevance to the nonlinear instability of colliding water waves, which may interact nonlinearly in a constructive way to produce large amplitude freak waves in the oceans.

Conclusion

The existence of rogue waves is universally recognized and images on the extent of the damage caused by these monsters of the ocean are available. However, the critical values of temperature and humidity training, the formation of ocean tornadoes as well as their prediction are not completely known. This paper shows that rogue waves are a combination of complex processes that occur under accuracy conditions like interference between primary waves whose directions of propagation are nearly parallel. Rogue waves are not waves that appear from nowhere as stated by some authors: According to the nonlinear Schrödinger equation (NLSE), the evolution of unstable tornadoes' wave group can generates a single wave that can reach up to 3 times the amplitude of the initial carrier wave (that is, the wave energy is basically concentrated in a single wave number).

This paper presents the application of Benjamin-Feir equation to deep water gravity waves, which is an important tool for acquiring information on the scientifically conceivable reasons for the formation of rogue waves. In this regard, additional assumptions are implemented to make the transition from hydrodynamic Euler equations to Benjamin-Feir modulation instability. Considering only deep rivers (H tends to infinity), we try to avoid interactions between the bottom and the surface of the rivers (where the waves are located): stories describing the rogue waves, do not mention the appearance of ocean volcanoes.

The same precautions recommended considering only

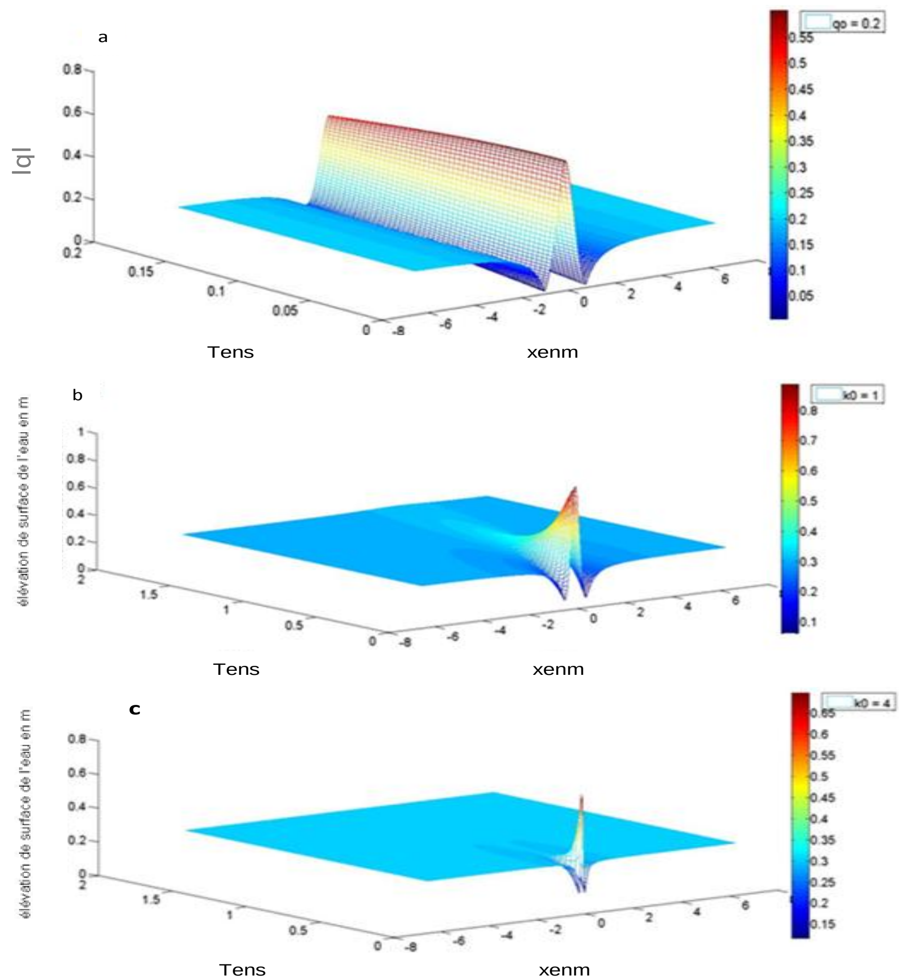


Figure 5(a-c). Simulations of tornadoes primary waves for different wave number (K0) a) (K0=0.2); b) (K0=1.0); c) (K0=4.0).

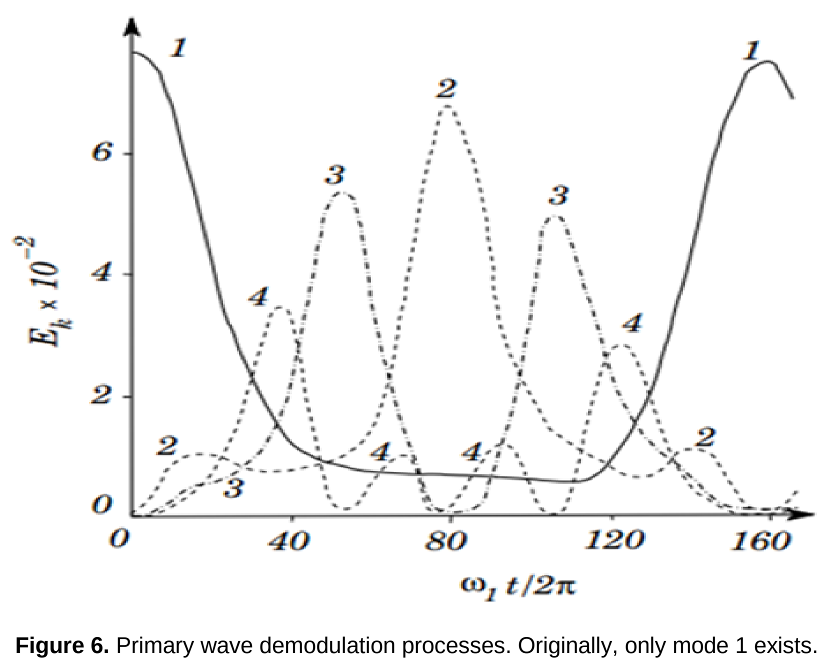


Figure 6. Primary wave demodulation processes. Originally, only mode 1 exists.

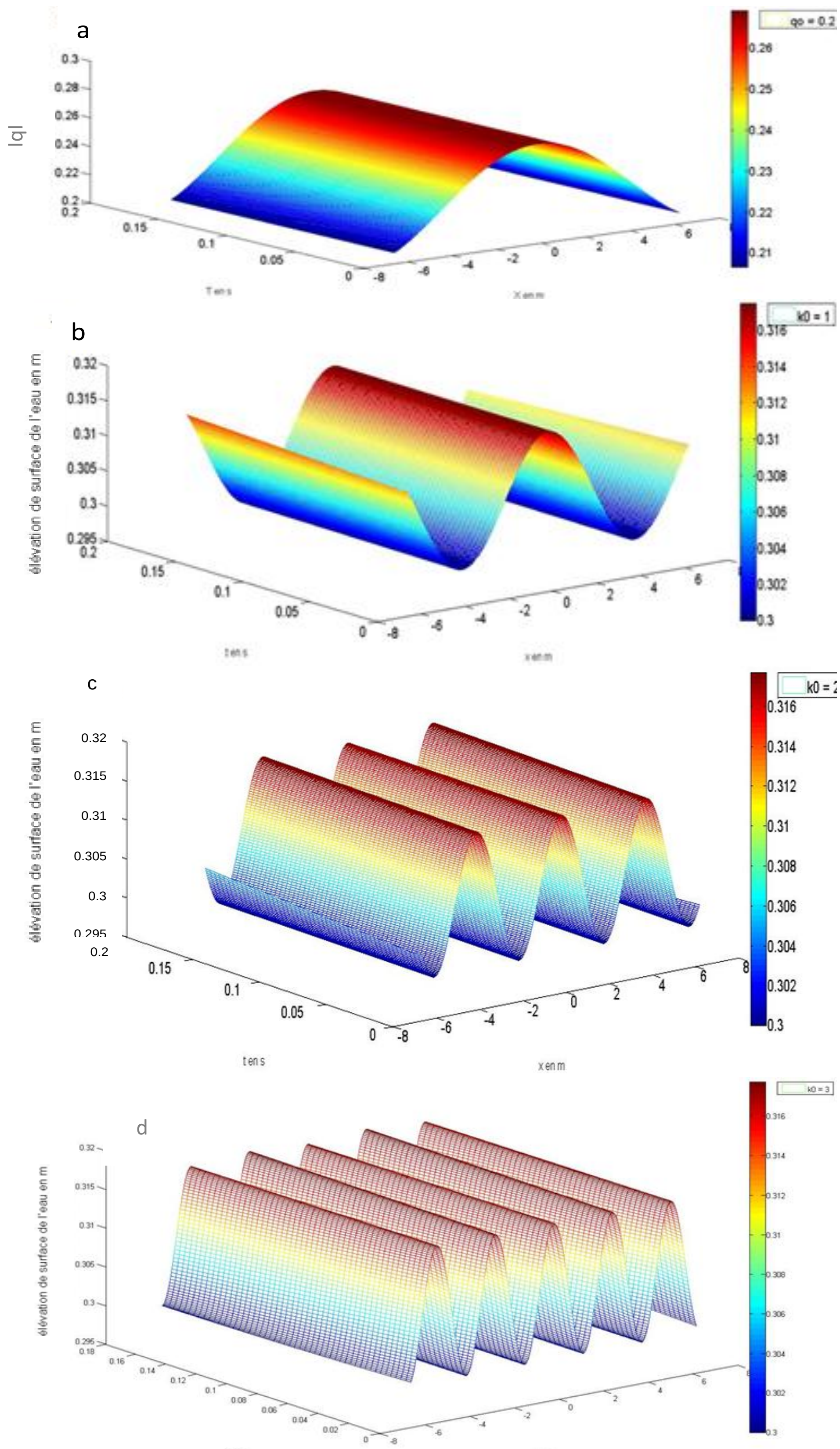


Figure 7(a-d). simulation of sinusoidal primary wave propagation for different wave number (K0), a) (K0=0.2); b) (K0=1.0); c) (K0=2.0); d) (K0=3.0).

very large rivers. This prevents interference between the primary waves and those produced by their reflection on the shores. The results presented in this paper are new, clear and neat. Tornadoes impacts on sea surface are now well known.

REFERENCES

Atock AS, Mbane BC, Essimbi ZB, Nkoa NT (2011). Application of Zakharov equation in three dimension to deep water gravity waves. IJPS 6(33):7447-7455.

Benjamin TB (1967). Instability of periodic wave trains in nonlinear dispersive systems. Proceeding of Roy. So. London, series A. 299:59-75.

Benjamin TB, Feir JE (1967). The disintegration of wave trains on deep water. J. Fluid Mech. 27(3):417-430.

Bespalov VI, Talanov (1966). Filamentary structure of light beams in nonlinear liquids. J.E.T.P. Lett. 3:307-310.

Dyachenko AI, Zakharov VE (2005). Modulational instability of Stokes wave – freak wave. J. Exp. Theor. Phys. 81(6):318-322.

Feir JE (1967). Discussion: some results from wave pulse experiments. Proceeding of the Royal Society of London. Series A. 299:54-58.

Hasimoto H, Ono H (1972). Nonlinear modulation of gravity waves. J. Phys. Soc. Jpn. 33:805-811.

Karsten T, Igor K (2000). On weakly nonlinear modulation of waves on deep water. Phys. Fluids 12(10):24-32.

Kharif C, Pelinovsky E (2003). Physical mechanism of the rogue wave phenomenon. Eur. J. Mech. B-Fluid. 22(6):603-634.

Mbane BC (2012). Physics of Atmosphere Dynamic or Electric Balance Processes Such as Thunderclouds and Related Lightning Flashes. Geosciences. 2(1):6-10. DOI: 10.5923/j.geo.20120201.02.

Shener L (2010). On Benjamin – Feir instability and evolution of non – linear wave with finite – amplitude side bands. Nat. Hazards Earth Syst. Sci. 10:2421-2427.

Socquet H, Juglar A, Dysthge K, Trulsen K, Krogstad HE, Liu J (2005). Probability distributions of surface gravity waves during spectral change. J. Fluid Mech. 542:195.

White BS, Fornberg B (1998). On the chance of freak waves at sea. J. Fluid Mech. 355:113-138.

Wu CH, Yao A (2004). Laboratory measurements of limiting freak waves on current. J. Geophys. Res. 109(C 12002):1 -18.

Yuen HC, Lake BM (1980). Instability of Waves on Deep Water. Ann. Rev. Fluid Mech. 12:303-334.

Zakharov VE, Kharitonov (1970). Instability of Periodic Waves of Finite Amplitude on the Surface of a Deep Fluid. Zh. Prikl. Mekh.1Tekhn.Fiz. 9:45-49.