Vol. 14(12), pp. 139-143, August, 2019
DOI: 10.5897/IJPS2019.4818
Article Number: D4AF7DB61772
ISSN: 1992-1950

http://www.academicjournals.org/IJPS

International Journal of Physical Sciences

Full Length Research Paper

Gravitational waves as a test of general relativity

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Received 16 April, 2019; Accepted 24 July, 2019

General relativity has been very successful as a model for space and gravity. It is supported from a range of experimental results. With the advent of the detection of gravitational waves, another platform exists to trumpet its triumph. The merger of black holes has provided evidence in support of the theory of Einstein. Furthermore, the one neutron star merger so far detected provides much more information because of the multi-messenger discovery of the spectrum of electromagnetic radiation in conjunction with gravitational waves.

Key words: Gravitational waves, general relativity, black hole mergers, neutron star merger

INTRODUCTION

The declaration on 11 February 2016 (Abbott et. al., 2016a) that gravitational waves had been detected met with praise and wonder from physicists and many members of the public. Why had scientists embarked on this quest, which has taken over three decades, and how did they convince governments to fund such a venture? The endeavor rested on confidence in the history of General Relativity espoused by Albert Einstein (1879-1955).

PREVIOUS SUCCESSES OF GENERAL RELATIVITY

From Einstein's mind came the prediction that light could be bent in the presence of mass (Einstein, 1907), that this deviation could be measured experimentally near the Sun (Einstein, 1911), and that the amount of divergence may be ascertained (Einstein, 1916a). This account may be followed through the 1919 and 1922 total solar eclipses and confirmation in 1928 (Treschman, 2014a). The gravitational deflection of light became known as one of the three classical tests of general relativity. The other two tests were the anomalous advance in the perihelion of Mercury (Einstein, 1916) and gravitational redshift (Einstein, 1907). The developmental evidence for these latter two forecasts may be pursued elsewhere (Treschman, 2014b). In time, general relativity was subjected to a battery of experimental investigations. These include the equivalence of inertial and gravitational mass, gravitational redshift (other than for the Sun), relativistic perihelion advance of the planets (additional to Mercury), relativistic periastron advance of binary pulsars, geodetic precession, the Lens-Thirring effect, gravitational optical light deflection (extra to total solar eclipses), gravitational radio deflection due to the Sun, time dilation, atomic clocks and the Nordtved effect. A summary of these components, year of publication and % difference from predictions from relativity are presented in a table (Treschman, 2015a).

One remaining aspect of general relativity was the existence of gravitational waves.

GRAVITATIONAL WAVES

Einstein is often credited with the first suggestion of

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gravitational waves, but this is cautioned (Darrigol, 2006). James Clerk Maxwell (1831-1879) treated electricity and magnetism as fields, had connected them and proposed that electromagnetic fields can propagate as waves at the speed of light (Maxwell, 1865). Building on this, Hendrik Antoon Lorentz (1853-1928) described the propagation of light in moving reference frames where the speed of light was invariant (Lorentz, 1892). His transformations mirror the symmetry of Maxwell's equations. Oliver Heaviside (1850-1925) remarked on the similarity of the inverse square law for gravitation and electricity and suggested the possibility of gravitational waves (Heaviside, 1893). More definite was (Jules) Henri Poincaré (1854-1912) who stated that, as required by the Lorentz transformations, gravitation must generate waves traveling at the speed of light, just as electromagnetism does (Poincaré, 1905).

In the same year as his final publication of general relativity, Einstein predicted gravitational waves (Einstein, 1916b). Realizing that he made an error in his belief that gravitational waves could arise from a spherically symmetric system, he corrected this two years later (Einstein, 1918). "In 1937, Einstein briefly thought that gravitational waves do not exist." Pais (1982), a reviewer pointed out his error, and he affirmed his belief in a paper written jointly with Nathan Rosen (1909-1995) (Einstein and Rosen, 1937).

BINARY PULSAR EVIDENCE FOR GRAVITATIONAL WAVES

The first system of a pulsar and a companion in mutual orbit was discovered in 1974 (Hulse and Taylor, 1975). Expected to lose gravitational wave energy according to Einstein, the system would result in a slowing of the orbital period that could be measured via the pulse frequency of the pulsar. After 30 years of analysis, the orbital decay result of $-2.410 \pm 0.009 \times 10^{-12}$ s s^{-1} has a 0% departure from general relativity (Will, 1995).

While this pair consisted of a pulsar and likely a neutron star, in 2003 a double pulsar arrangement (Treschman, 2015b) emerged from a capture by the 64 m Parkes Australia radio telescope (Lyne et al., 2004). Properties of both bodies were thus able to be measured, and this allowed a number of constraints to be placed on components of the system. The orbital decay figure of -1.252×10^{-12} s s^{-1} was within 0.3% of that predicted by general relativity (Kramer et al., 2006).

MERGER OF BLACK HOLES AS EVIDENCE FOR GRAVITATIONAL WAVES

Data from the binary pulsars relied on the mathematics of general relativity. This was not direct evidence for the existence of gravitational waves, but an interpretation of the slowing rate of the system. The search for gravitational waves was initiated in the 1960s when Joseph Weber (1919-2000) constructed what became known as Weber bars (Weber, 1960). The apparatus consisted of multiple aluminium cylinders 2 m in length and 1 m in diameter, with piezoelectric sensors. The resonant frequency of vibration was 1660 Hz and a passing wave was expected to alter the length of the cylinders by 10^{-16} m.

The underpinning of the next attempt was from Rainer Weiss who was the inventor of the laser interferometric technique in 1967 (Weiss, 1967). He led a team at the Massachusetts Institute of Technology (MIT) in further research and in the development of a prototype for detecting gravitational waves. Meanwhile, Kip Thorne at the California Institute of Technology (Caltech) pursued a similar line. In 1979, the National Science Foundation (NSF) funded research and development on prototypes at MIT and Caltech. Weiss and Thorne joined forces in 1984 and approval to fund their project was obtained in 1990. The NSF selected two sites in 1992, Hanford Washington and Livingstone Louisiana in what was named the Laser Interferometer Gravitational-Wave Observatory (LIGO). Construction began at Hanford in 1994 and Livingstone in 1995. Completion occurred in 1999 with the cost eventually reaching $1.1 billion.

Each facility consists of two L-shaped arms 4 km long. The spacing between the sites is 3002 km, designed to provide a long baseline for the interferometric technique. The light in each system starts from a laser diode producing 4 W at 808 nm. Several of these are shone into neodymium (Nd^{+3}) doped to 1% replacement in yttrium (Y^{3+}) aluminium garnet $Y_3Al_5O_{12}$ to stimulate a 2 W beam of 1064 nm until it is amplified to 200 W. Each arm is made a resonant optical cavity where the laser is reflected 400 times, so the effective length is 1600 km. The system is most sensitive at 150 Hz (optimum signal-to-noise ratio), and this corresponds to a wavelength of 2000 km, which is approximately one half of the expected gravitational waves. The strain of 10^{-21} being searched for means a 10^{-21} m change per 1 m. In a 1 km arm, this represents a change in length of 10^{-18} m which is just detectable by LIGO.

LIGO collected data 2002-2010 with zero detection. The sensitivity was increased by further noise reduction to become Advanced LIGO. Collaboration began with other world facilities, and the number of people involved surpassed 1000. The operation recommenced in 2015 with the first run 12 September 2015 to 19 January 2016, and the second 30 November 2016 to 25 August 2017.

Within a few days, the two detectors of LIGO received a signal within the 10 ms intersite travel time of a transient gravitational signal. In just over 0.2 s for eight cycles, there was an upward frequency change from 35 to 250 Hz. "It matches the waveform predicted by general relativity for the inspiral and merge of a pair of black holes and the ringdown of the resulting single black hole. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger" (Abbott et. al., 2016a). The collaborators followed a strict protocol of checking results before their announcement five months later.

A month following this first merger event, there was a trigger for a second one that required more extensive work before it was accepted much later (Abbott et. al., 2018). In the first run, a third black hole coalescence was realized (Abbott et. al., 2016b). In the second run, a further seven black hole mergers were claimed, as well as one binary neutron star collision (Abbott et. al., 2017a,b,c,d, 2018). The fourth detection was the first to include a third observatory, Advanced Virgo operated by six European countries stationed in Italy. The 11 gravitational wave detections are listed in Table 1 in chronological order. Some of the data originally reported was reanalyzed and the later figures are shown here. Positive and negative uncertainties are listed in the papers but are not recorded in the table. The individual masses are displayed in terms of the mass of the Sun. After merger, M_f represents the final mass, and the difference (within experimental uncertainty) from the addition of the two objects is radiated as gravitational wave energy. From $E = mc^2$, this mass may be multiplied by the mass of the Sun and the speed of light squared to give the energy in J. The peak luminosity is in the next column, except it was reported in erg s^{-1}, and this has been altered by 1 erg = 10^{-7} J, and 1 J s^{-1} = 1 W. The final column of figures for the luminosity distance to the event in light years was given in Mpc and the conversion 1 Mpc = 3.262×10^6 ly has been applied. This was measured by relation of an inverse proportionality to the signal's amplitude (Abbott et. al., 2018). The ninth entry (highlighted) is the neutron star collision, which is treated in the next section.

From Table 1, the largest combined mass is from number 6 at 80 times the mass of the Sun. "Only a small fraction (0.02-0.07) of the binary's total mass is radiated away in GWs." (Abbott et. al., 2018) Hence, the larger the total mass, the stronger the signal for comparable distances. This particular event allowed detection from as far away as nearly 9×10^9 ly. The mass of an individual black hole is 5 solar masses and above. In contrast, that for a neutron star is in the region of 2-3 solar masses, and this fits with the data from number 9. Among the 11 mergers, there is not a black hole-neutron star collision.

MERGER OF NEUTRON STARS AS EVIDENCE FOR GRAVITATIONAL WAVES

In general terms, detection of a collision is more likely with a larger total mass and closer proximity to Earth. The downside of black hole-black hole collisions is that electromagnetic radiation cannot escape the vicinity of such large masses. In contrast, while a neutron star-neutron star merger may produce a weaker signal unless it is much closer, the neutron star coalescence was able to provide a lot more information from a production of the spectrum of light since the escape velocity for the lower mass of the system is less than the speed of light. The first detection of light was by the NASA Fermi Gamma-ray Space Telescope from 550 km altitude. "The association with the ɣ-ray burst GRB 170817A, detected by Fermi-GBM 1.7 s after the coalescence, corroborates the hypothesis of a neutron star merger and provides the first direct evidence of a link between the mergers and the ɣ-ray bursts" (Abbott et. al., 2017d). The first GRB was detected in 1967. In bursts recorded since then, the energy may be 100 times that from a supernova. The collision of neutron stars had been proposed as the source of shorter bursts, believed to be a relativistic jet produced by the central remnant after merger (Hotokezaka et. al., 2018).

The chance that the short delay time between the gravitational wave and the gamma ray burst of 1.74 ±0.05 s is not simultaneous has been analyzed as 5.0×10^{-8} (Abbott et. al., 2019), and so these events confirm that gravitational waves travel at the speed of light.

From its name, a neutron star is composed almost entirely of neutrons. It is theorized to be the collapsed core resulting from a supernova, which forced the protons and electrons to combine to form neutrons. A kilonova had been predicted to be associated with a neutron star merger with expected ultraviolet, optical and infrared emissions (Abbott et. al., 2017e). This is proposed because of the ejecta resulting from the collision. These energetic neutrons moving at 0.2 times the speed of light (Pian, 2017) combine to produce larger nuclei by the process of r-process (rapid) nucleosynthesis. As the unstable nuclei decay, energy in the mentioned realm of electromagnetic radiation was expected.

Hence, a worldwide multi-messenger search from ground- and space-based observatories was initiated. The two LIGO instruments had detected the gravitational wave, and Advanced Virgo was operational at the time, but reported a null result. However, from a limited area of blind spots for this instrument, it was still possible to triangulate a small area of sky for the origin of the signal. As this incorporated the short gamma ray burst position, this information was sent to the astronomical community. In less than 11 hours, success was confirmed. The ultraviolet-blue transient faded within 48 hours, a bright optical transient was associated with NGC 4993, and this component along with the infrared radiation evolved towards the red over 10 days.

There were still further parts of the electromagnetic spectrum to explore. A delayed radio emission was anticipated as the ejecta interacted with the ambient medium (Abbott et. al., 2017e). There were no initial detections in the X-ray or radio regions, but these were received 9 days and 16 days later respectively.

Still more from this collision is being investigated. The abundance of elements heavier than iron intrigued and confused scientists. They were suggested to arise from supernova and neutron star mergers. Now, the contribution of this event will be matched to the abundances already determined (Abbott et. al., 2017f). It

Table 1. The 11 gravitational wave detections listed chronologically with individual masses before collision, final combined mass, mass radiated as gravitational energy, the peak intensity of the signal, the distance to the collision and the reference to each event.

No.	Date	m_1 M_{Sun}	m_2 M_{Sun}	M_f M_{Sun}	$E_{radiated}$ $M_{Sun}\,c^2$ in J	I_{peak} $\times 10^{49}$ W	Distance $\times 10^9$ ly	Reference
1	15/09/14	35.6	30.6	63.1	3.1	3.6	1.40	Abbott et. al. (2016a)
2	15/10/12	23.3	13.6	35.7	1.5	3.2	3.46	Abbott et. al. (2018)
3	15/12/26	13.7	7.7	20.5	1.0	3.4	1.44	Abbott et. al. (2016b)
4	17/01/04	31.0	20.1	49.1	2.2	3.3	3.13	Abbott et. al. (2017a)
5	17/06/08	10.9	7.6	17.8	0.9	3.5	1.04	Abbott et. al. (2017b)
6	17/07/29	50.6	34.3	80.3	4.8	4.2	8.98	Abbott et. al. (2018)
7	17/08/09	35.2	23.8	56.4	2.7	3.5	3.23	Abbott et. al. (2018)
8	17/08/14	30.7	25.3	53.4	2.7	3.7	1.89	Abbott et. al. (2017c)
9	17/08/17	1.46	1.27	≤ 2.8	≥ 0.04	≥ 0.1	0.13	Abbott et. al. (2017d)
10	17/08/18	35.5	26.8	59.8	2.7	3.4	3.33	Abbott et. al. (2018)
11	17/08/23	39.6	29.4	65.6	3.4	3.6	6.03	Abbott et. al. (2018)

is estimated that this particular union resulted in 1-5 Earth masses of europium and 3-13 Earth masses of gold (Côté et. al., 2018). It is thought that this avenue likely synthesizes the entire abundance of the two most neutron-rich stable isotopes of each heavier element.

GRAVITATIONAL WAVES AS A FUTURE PROBE

Gravitational waves can penetrate regions of space that electromagnetic waves cannot. This will continue to allow the signals to be used as a tool for combinations of black hole and neutron star mergers, but it may also be extended to investigate the possibility of other exotic compact objects. These could be bodies composed of something other than protons, neutrons, electrons or muons. Some candidates are quark stars (where neutron have released their up and down quarks under greater pressure), strange stars (the strange quark is present additionally), boson stars, gravastars (alternatives to black holes), preon stars (the proposed building blocks of dark matter), axion stars (dark matter bound as an Bose-Einstein condensate) (Braaten and Zhang, 2018).

The signal from a coalescing binary has three phases: inspiral, merger and ringdown (Sennett et. al., 2017). In particular, the ringdown phase incorporates a series of damped oscillation modes that enable precise computation (Cardoso et al , 2016). Discrimination is also possible in the inspiral period when tidal deformity may be measured. Here, any effect is reported in Love numbers, which are ratios relating to the elastic response of bodies (Hinderer, 2008). For low compact stars, there is a smooth transition between the first two phases, but an abrupt change occurs for high compact stars where the merger results in a black hole (Bezares and Palenzuela, 2018).

Of particular interest will be research based on strong field regimes. The source of gravitational waves gives rise to polarization. For circular orbits of a pair of masses, the planes of polarization are 45° apart and rotate at twice the orbital rate. Comparisons may be made between Einstein's work and modified gravity theories, such as those by Brans-Dicke, Rosen and Lightman-Lee (Chatziioannou et al., 2017).

Attempts to match the abundances of heavier nuclei in the Milky Way and in the universe should lead to constraints on r-process nucleosynthesis.

Since the sensitivity to gravitational waves is proportional to the separation between stations, a massive boost to detection could become a reality in 2034. This is when the European Space Agency aims to launch the Laser Interferometer Space Antenna (LISA). Three spacecraft will be positioned at the vertices of an equilateral triangle following Earth in its orbit by 20°. Lasers will monitor the distances between the spacecraft which will have each arm length 2.5×10^6 km long. The concept has been scrutinized on two test masses in gravitational freefall on LISA Pathfinder launched in 2005.

CONCLUSION

The arrival of the first gravitational wave imaged in 2015 is a monumental celebration of the technology, perseverance and ingenuity of a body of scientists, and the preparedness of governments to fund such an enterprise. The detection has added another chapter in the support of general relativity. It is now known that these waves exist and that they travel across space at the speed of light. An association between neutron star mergers and gamma ray bursts has been established. A further connection is that this collision is responsible for a kilonova, which produces energy across much of the electromagnetic spectrum. Scientists have now embarked on connecting the nuclear abundances of heavier elements and their origin. Finally, the exercise provides a tribute to the cooperation of the world community of astronomers who turned their instruments to gather data across the electromagnetic spectrum.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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