Full Length Research Paper

Investigation of discrepancies of some geoids determined using various methods for Turkey

Nazan Yilmaz* and Celalettin Karaali

Department of Geomatics Engineering, Engineering Faculty, Karadeniz Technical University, 61080 Trabzon, Turkey.

Accepted 8 August, 2011

In this study, the causes and size of discrepancies between various geoid models were investigated. Local geoid models (Turkish Geoid-1991, Turkish GPS/Leveling Geoid, Updated Turkish Geoid-1999, Turkish Geoid-2003, Turkish Hybrid Geoid Model-2009) and global geoid models (EIGEN-GL04C, EIGEN-GL04S1, EIGEN-5C) were used in Turkey. Thirty different points were selected in Turkey. The geoid heights of the points in different geoid models were calculated. These models were compared in terms of the criteria based on their developments and geoid height differences calculated using these models at the test points. The differences of geoid heights calculated from various models at the points were compared and some discrepancies were observed among the models. The reasons behind these discrepancies were discussed. It is disclosed that TG-99A and EIGEN-5C geoid models are the best fitting to Turkish GPS/Leveling geoid.

Key words. Gravity, Turkish geoids, global geopotential models, GPS/leveling.

INTRODUCTION

Ellipsoidal heights obtained with GPS do not reflect natural situation. So, they can not meet precision practical needs related heights. However, orthometric heights are more compatible with physical event and so, they are used succesfully in solving many problems related to heights in practice. But obtaining orthometric heights with traditional measurements is a very difficult process. Therefore, ellipsoidal heights obtained with GPS easily must be converted to orthometric heights. For this conversion, geoid heights with certain accuracy must be known.

In order to convert the high-precision ellipsoidal heights to orthometric heights, as is required for engineering purposes, the determination of the geoid is necessary. Because the relation between ellipsoidal and orthometric heights involves the geoid undulation (Corchete et al., 2005).

Common problem is to select the best model, e.g. for engineering applications regional gravimetric geoid models. A related problem is that in order to improve the local geoid models, the selection of the best global geopotential model (GGM) model for the region is essential, to be used in a combined solution from GGM and local gravimetric data (Kiamehr and Sjöberg, 2005).

In this article, we used Turkish Geoid-1991, Turkish GPS/Leveling Geoid, Updated Turkish Geoid-1999, Turkish Geoid-2003, Turkish Hybrid Geoid Model-2009, EIGEN-GL04C, EIGEN-GL04S1, and EIGEN-5C geoid models. Firstly, 30 points were selected from Turkish National Fundamental GPS Network. Later, geoid heights of all geoid models were calculated in the selected points. These models were compared in terms of the criteria based on their developments and in terms of different geoid heights calculated using these models at the test points. The geoid height comparisons were made between geoid models and Turkish GPS/leveling geoid, because GPS/Leveling geoid gives geoid heights directly (N = h - H) and the geoid is directly related to Vertical Control Network.

The differences of geoid heights calculated from various models at the points were evaluated and some findings were obtained. Taking these findings and the aforementioned criteria into account, the causes of discrepancies between various geoid models were examined and the size of discrepancies between geoid

^{*}Corresponding author. E-mail: n_berber@ktu.edu.tr. Tel: +90 462 377 3768. Fax: +90 462 328 0918.

Table 1. The criteria based on developments of Turkey local geoid models.

Model	Datum	Ellipsoid	Data	Method	Reference
Turkish Geoid-1991 (TG-91)	Approximate geocentric	GRS-80 ellipsoid	GPM2-T1 earth potential model, gravity and topographic heights	RCR technique and EKKK method	Ayhan (1992)
Turkish GPS/leveling geoid	ITRF 96	GRS-80 ellipsoid	Ellipsoidal heights regarding Turkish National Fundamental GPS Network-1999 and orthometric heights regarding Turkish National Vertical Control Network-1999	GPS/Leveling method (h = H+N)	Ayhan et al. (2002)
Updated Turkish Geoid-1999 (TG- 99A)	ITRF 96	GRS-80 ellipsoid	TG-91 geoid heights, trend value in common points between TG-91 and GPS/leveling geoid (t), residual measurements (dN)	N _{TG99A} =N _{TG91} -t-dN	Ayhan et al. (2002) and Kılıçoğlu (2002)
Turkish Geoid-2003 (TG-03)	ITRF 96	GRS-80 ellipsoid	EGM96 earth potential model, land gravity anomalies, sea gravity anomalies, digital terrestrial model, GPS/leveling geoid heights	RCR technique and EKKK method	Kılıçoğlu et al. (2005)
Turkish Hybrid Geoid Model-2009 (THG-09)	ITRF 96	GRS-80 ellipsoid	EGM08 geoid model, DNSC08 gravity anomalies from satellite altimeter measurements from sea, land gravity measurements, digital terrestrial model and GPS/Leveling geoid heights	RCR technique and FFT method	HGK (2010)

models were determined.

THE CRITERIA BASED ON DEVELOPMENTS OF DIFFERENT GEOID MODELS

Geoid models are different from each other in terms of some criteria which are datum, ellipsoid, data and method. These differences were summarized in Tables 1 and 2.

Working area and selection of test points

In the paper, 30 points of Turkish National Fundamental GPS Network were used. The test points are shown in accordance with latitude and longitude in Figure 1. They are distributed as homogenously as possible.

Data of the test points

Geographical coordinates (φ, λ) of the selected 30 test points, ellipsoidal heights (h), orthometric heights (H), height anomalies from TG-09 and d ζ correction values which are used to convert gravimetric height anomalies (ζ) to geoid heights (N) are given in Table 3.

RESULTS AND DISCUSSION

Firstly, geoid heights were calculated in test points in different geoid models. Then, local and global geoid models were compared with Turkey Local GPS/leveling geoid models. Some statistical data were calculated from differences of geoid heights.

Calculation of the geoid heights of test points in different Turkey local geoid models

Geoid undulation values of selected 30 test points were calculated in TG-91, GPS/leveling, TG-99A, TG-03 and THG-09 Turkey local geoid models.

Geoid heights of the selected points in TG-91, TG-99A and TG-03 geoid models were calculated using the program harmp.exe (Tscherning et al., 1994) of GRAVSOFT software package by using grid file of these models. The grid files were supplied by General Command Of Mapping in Turkey.

Geoid heights of the selected points in GPS/leveling models were calculated with N = h - H general equation. In the equation, symbols are shown that N is geoid height, h is ellipsoidal height and H is orthometric height. Geoid heights of the test points in THG-09 geoid were obtained from General Command of Mapping in Turkey. Geoid heights determined according to different Turkey local geoid models in test points are given in Table 4.

Calculation of the geoid heights of test points in different global geoid models

Geoid undulation values of 30 selected test points were calculated in EIGEN-GL04C, EIGEN-GL04S1 and EIGEN-5C global geoid models. To obtain the geoid heights firstly, the height anomaly values of points were

Model	Datum	Ellipsoid	Data	Method	Max. resolution (Degree)	Reference
EIGEN- GL04C	WGS84	WGS-84 ellipsoid	S(Grace,Lageos),G,A	Geoid determination from geopotential coefficients	360	Förste et al. (2006)
]EIGEN- GL04S1	WGS84	WGS-84 ellipsoid	S(Grace,Lageos)	Geoid determination from geopotential coefficients	150	Förste et al. (2006)
EIGEN-5C	WGS84	WGS-84 ellipsoid	S(Grace,Lageos),G,A	Geoid determination from geopotential coefficients	360	Förste et al. (2008)

Table 2. The criteria based on developments of global geoid models.

S, Satellite tracking data, G, gravity data; A, altimetry data.



Figure 1. Positions of test points.

interpolated according to each model with the program harmp.exe (Tscherning et al., 1994) of GRAVSOFT software package by using the known latitude, longitude and heights of points. After then, having applied the $d\zeta = \zeta$ -N correction to the determined height anomalies, geoid heights (N) were calculated. N geoid heights determined according to the different specified models are shown in Table 5.

Comparison of geoid heights obtained from different geoid models

Local and global geoid models were compared with Turkey local GPS/leveling geoid models. Because GPS/leveling geoid gives geoid heights directly (N = h - H) and the geoid is directly related to vertical control network. Geoid heights are compared by taking their differences in test points. These differences between GPS/leveling and other geoid models are shown numeric and equal difference maps. The differences between GPS/Leveling and other geoid models are shown in Table 6; units in centimeters.

Findings related to differences of geoid heights calculated from various geoid models

Differences of geoid heights (ΔN_{ij}) calculated from various geoid models in k = 30 test points for Turkey were shown in Table 6 and these differences were subjected to evaluation. The evaluations were made on the basis of the smallest, the biggest and root mean square values of different geoid models' deviations from each other. Root mean square values were computed from $_{RMS} = \pm \left[\left(\sum_{i=1}^{k} \Delta N_{ij}^{2} \right) / k \right]^{1/2}$ equation in 30 test

points. The statistical values are shown in Tables 7 and 8 for Turkey local and global geoid models.

Table 3. Data of the test points.

Point No.	Latitude (Degree) φ	Longitude (degree) λ	Orthometric height, H (m)	Ellipsoidal height, h (m)	Correction, dζ (m)	Height anomalies from TG-09 ζ (m)
1	41.52647	26.85857	143.8827	182.9942	-0.0014	39.0451
2	40.68223	30.13313	510.6212	548.1760	0.0031	37.5287
3	41.31073	32.69392	909.6400	945.5467	-0.0017	35.8900
4	40.94944	35.11299	854.1896	888.1105	0.0303	33.8910
5	41.22045	36.66915	15.1787	43.1707	-0.0001	27.9324
6	40.90173	38.42659	216.1831	243.6590	-0.0026	27.4210
7	40.83748	39.61471	832.0689	860.4165	0.0238	28.2876
8	41.37056	41.33871	7.3858	30.1790	-0.0002	22.9070
9	41.26802	41.77701	288.3085	313.6850	0.0228	25.3657
10	40.68534	43.16953	1846.2478	1871.1180	0.3083	24.7611
11	39.55561	44.14494	1555.0286	1577.8730	0.2413	23.0468
12	38.75943	42.53173	1691.4762	1716.7800	0.2377	25.4594
13	37.43420	41.34487	976.6762	998.8345	0.0764	22.0583
14	37.16559	38.84852	507.6026	532.3930	0.0235	24.7250
15	36.78796	36.52351	404.1737	432.2256	0.0135	27.9635
16	37.20670	34.81055	785.5074	815.6803	0.0765	30.1371
17	36.43085	32.15983	60.0010	87.4360	-0.0012	27.3365
18	37.04295	30.17824	1085.8402	1116.2550	0.0814	30.3653
19	37.39535	27.66037	100.2777	134.8520	-0.0045	34.5086
20	39.31131	26.70002	59.1133	98.2089	-0.0021	39.0109
21	39.72167	27.90643	409.0186	447.5946	-0.0014	38.5147
22	38.76888	30.64437	1182.1537	1219.9000	0.0936	37.7184
23	38.90111	33.00963	1087.2560	1123.2500	0.0708	35.8851
24	39.18273	36.05526	1271.3797	1305.3060	0.1617	33.9347
25	38.3585	38.39381	1110.0351	1140.0810	0.0834	29.9584
26	39.59078	39.85349	1501.7747	1532.4540	0.2323	30.5639
27	39.98204	41.68113	1735.3093	1762.9800	0.3382	27.6414
28	37.86887	32.39391	1375.8953	1411.9050	0.1182	35.9829
29	37.77106	35.89635	707.8182	739.6496	0.0625	31.7868
30	38.02062	28.86042	673.3114	708.6318	0.0292	35.2800

Table 4. TG-91, GPS/leveling, TG-99A, TG-03 and THG-09 geoid heights.

Point No.	TG-91 geoid heights (m)	GPS/leveling geoid heights (m)	TG-99A geoid heights (m)	TG-03 geoid heights (m)	THG-09 geoid heights (m)
1	39.022	39.112	39.117	39.054	39.054
2	39.244	37.555	37.494	37.006	37.373
3	37.127	35.907	35.914	35.848	35.892
4	33.544	33.921	33.926	33.827	33.865
5	27.274	27.992	27.878	27.975	27.913
6	28.691	27.476	27.406	27.483	27.428
7	29.342	28.348	28.177	28.258	28.306
8	21.932	22.793	22.665	22.806	22.873
9	23.461	25.377	25.184	25.317	25.409
10	22.415	24.870	24.834	24.374	24.841
11	20.078	22.845	22.830	22.624	23.014
12	25.473	25.304	25.334	25.018	25.493
13	22.860	22.158	22.170	22.101	22.055

14	24.501	24.790	24.800	24.774	24.690
15	27.342	28.052	28.062	28.013	27.978
16	30.089	30.173	30.215	30.116	30.160
17	28.694	27.435	27.538	27.442	27.353
18	31.705	30.414	30.495	30.313	30.388
19	35.381	34.574	34.595	34.574	34.505
20	39.671	39.096	39.090	39.080	38.996
21	39.434	38.576	38.567	38.628	38.500
22	38.822	37.746	37.736	37.617	37.723
23	36.394	35.994	36.022	35.843	35.866
24	33.644	33.926	33.902	33.770	33.875
25	29.639	30.046	30.018	29.912	29.972
26	30.323	30.680	30.667	30.513	30.545
27	27.121	27.671	27.655	27.357	27.751
28	36.834	36.010	36.008	35.824	35.897
29	31.587	31.831	31.887	31.772	31.790
30	36.206	35.320	35.313	35.399	35.254

Table 5. Geoid heights of EIGEN-GL04C, EIGEN-GL04S1 and EIGEN-5C.

Point no.	EIGEN-GL04C (m)	EIGEN-GL04S1 (m)	EIGEN-5C (m)
1	40.700	40.870	39.810
2	37.850	38.060	37.570
3	35.720	35.720	36.010
4	34.420	34.630	34.130
5	29.120	29.130	28.800
6	27.870	28.310	27.500
7	28.590	28.100	28.050
8	21.830	22.680	21.930
9	23.890	24.500	24.270
10	26.600	26.760	26.460
11	22.740	24.790	22.110
12	27.870	26.380	27.920
13	22.920	22.530	22.730
14	25.730	26.020	25.860
15	28.040	28.430	27.950
16	32.460	30.840	32.400
17	28.740	27.930	28.620
18	30.870	30.730	31.270
19	35.680	34.940	35.040
20	40.380	40.420	40.400
21	39.740	39.470	39.640
22	39.180	38.280	39.570
23	36.650	37.040	36.640
24	35.080	35.990	35.330
25	31.620	30.420	31.580
26	32.430	32.730	32.800
27	29.900	30.400	29.780
28	36.880	37.430	36.410
29	32.600	33.590	32.660
30	36.200	37.120	36.670

Point	GPS/Lev-	GPS/Lev-	GPS/Lev-	GPS/Lev-	GPS/Lev-	GPS/Lev-	GPS/Lev-
no.	TG91 (cm)	TG99A (cm)	TG03 (cm)	THG09 (cm)	EIGENGL04C (cm)	EIGENGL04S1 (cm)	EIGEN5C (cm)
1	9	-1	6	6	-159	-176	-70
2	-169	6	55	18	-30	-51	-2
3	-122	-1	6	1	19	19	-10
4	38	-1	9	6	-50	-71	-21
5	72	11	2	8	-113	-114	-81
6	-122	7	-1	5	-39	-83	-2
7	-99	17	9	4	-24	25	30
8	86	13	-1	-8	96	11	86
9	192	19	6	-3	149	88	111
10	245	4	50	3	-173	-189	-159
11	277	1	22	-17	10	-195	73
12	-17	-3	29	-19	-257	-108	-262
13	-70	-1	6	10	-76	-37	-57
14	29	-1	2	10	-94	-123	-107
15	71	-1	4	7	1	-38	10
16	8	-4	6	1	-229	-67	-223
17	-126	-10	-1	8	-131	-50	-119
18	-129	-8	10	3	-46	-32	-86
19	-81	-2	0	7	-111	-37	-47
20	-58	1	2	10	-128	-132	-130
21	-86	1	-5	8	-116	-89	-106
22	-108	1	13	2	-143	-53	-182
23	-40	-3	15	13	-66	-105	-65
24	28	2	16	5	-115	-206	-140
25	41	3	13	7	-157	-37	-153
26	36	1	17	13	-175	-205	-212
27	55	2	31	-8	-223	-273	-211
28	-82	0	19	11	-87	-142	-40
29	24	-6	6	4	-77	-176	-83
30	-89	1	-8	7	-88	-180	-135

Table 6. Differences of geoid heights between GPS/leveling and other geoid models ($_{\Delta N_{ij}}).$

Table 7. Statistical values regarding to geoid height differences calculated from different Turkey local geoid models .

Compared geoid models	The smallest value of deviations from each other (m)	The biggest value of deviations from each other (m)	Root mean square of deviations from each other \pm RMS (m)
GPS/Lev-TG91	-1.689	2.767	1.083
GPS/Lev-TG99A	-0.103	0.193	0.066
GPS/Lev-TG03	-0.079	0.549	0.181
GPS/Lev-THG09	-0.182	0.189	0.090

Table 8. Statistical values regarding to differences of geoid heights calculated from global geoid models and Turkey local GPS/Leveling geoid.

Compared geoid models	The smallest value of deviations from each other (m)	The biggest value of deviations from each other (m)	Root mean square of deviations from each other \pm RMS (m)
GPS/Lev- IGENGL04C	1.487	-2.566	1.244
GPS/Lev-EIGENGL04S1	0.877	-2.729	1.242
GPS/Lev-EIGEN5C	1.107	-2.616	1.218

Compared geoid models	The smallest value of deviations from each other (m)	The biggest value of deviations from each other (m)	Root mean square of deviations from each other \pm RMS (m)
GPS/Lev-TG91	-1.627	2.829	1.060
GPS/Lev-TG99A	-0.119	0.176	0.064
GPS/Lev-TG03	-0.190	0.437	0.142
GPS/Lev-THG09	0.141	-0.230	0.081

Table 9. Statistical values regarding to geoid height differences calculated from different Turkey local geoid models after applying trend surfaces

Table 10. Statistical values regarding differences of geoid heights calculated from global geoid models and Turkey local GPS/leveling geoid after applying trend surfaces.

Compared geoid models	The smallest value of deviations from each other (m)	The biggest value of deviations from each other (m)	Root mean square of deviations from each other \pm RMS (m)
GPS/ Lev-EIGENGL04C	-1.689	2.364	0.882
GPS/ Lev-EIGENGL04S1	-1.788	1.818	0.810
GPS/ Lev-EIGEN5C	-1.819	1.904	0.920

To investigate the causes of discrepancies between different geoid models, plane trend surfaces were applied by taking the averages of geoid heights differences (ΔN_{ij}) in Table 6 separately. Each of values (average

value) regarding to trend surfaces was subtracted from geoid height differences and remaining differences were evaluated. Statistical values regarding to geoid height differences calculated from different Turkey local and global geoid models after applying trend surfaces are shown in Tables 9 and 10.

Conclusions

The differences of geoid heights calculated from various geoid models for Turkey in test points were examined by taking into account statistical values, figures related to geoid height differences and criteria based on developments of geoid models. According to this;

1. The deviation of GPS/Nivelman-TG91 geoid models from each other is approximately ± 1 m in Table 7. Ellipsoids used in the calculation of both models are the same, but their datum, data and calculation method are different in Table 1. Therefore, there is no discrepancy related to ellipsoid selection. To see datum discrepancies, plane trend surface was applied. After applying trend surface, it was seen that differences between GPS/leveling and TG-91 geoid models did not change in Table 9. In this case, it was shown that there was no discrepancy related to datum between the models. There are data and methods discrepancies among these models. When the distribution of differences in the country from Figure 2 was examined, it was seen that there were bigger differences in the coast of the Eastern Black Sea and Eastern Anatolia regions.

2. The deviation of GPS/leveling-TG99A geoid models from each other is approximately \pm 7 cm (Table 7). The models are very consistent with each other, because datum and ellipsoid of these geoid models are the same from Table 1. When the distribution of differences in the country from Figure 3 was examined, it was seen that there were bigger differences in the coast of the Eastern Black Sea and Mediterranean regions.

3. The deviation of GPS/leveling-TG03 geoid models from each other is approximately \pm 18 cm in Table 7. The models are very consistent with each other, because datum and ellipsoid of these geoid models are the same from Table 1. When the distribution of differences in the country from Figure 4 was examined, it was seen that there were bigger differences in the coast of the Western Black Sea region and in the Eastern Anatolia region.

3. The deviation of GPS/leveling-THG09 geoid models from each other is approximately \pm 9 cm in Table 7. The models are very consistent with each other, because datum and ellipsoid of these geoid models are the same from Table 1. When the distribution of differences in the country from Figure 5 was examined, it was seen that there were bigger differences in the coast of the Southeastern Anatolia region and in the Eastern Anatolia region.

4. The most suitable global geoid model for Turkey is EIGEN-5C from Table 8. The deviation of EIGEN-5C global geoid from Turkey local GPS/leveling geoid model is \pm 1.218 m. The value is smaller than root mean square values related to other global geoid models. It is thought that the cause of the situation is development of technology and better quality of data. When the distribution of differences in the country from Figure 8 were examined, it was seen that there were bigger differences in the coast of the Eastern Black Sea and Eastern Anatolia regions. There were bigger differences



Figure 2. Equal difference map of GPS/leveling-TG91 geoid height differences (units in cm).



Figure 3. Equal difference map of GPS/leveling-TG99A geoid height differences (units in cm).



Figure 4. Equal difference map of GPS/leveling-TG03 geoid height differences (units in cm).



Figure 5. Equal difference map of GPS/leveling-THG09 geoid height differences (units in cm).



Figure 6. Equal difference map of GPS/leveling- EIGENGL04C geoid height differences (units in cm).

in the coast of the Eastern Black Sea and Eastern Anatolia regions in Figures 6 and 7, too. There is datum,

ellipsoid, data and method discrepancies between GPS/leveling and EIGEN-5C models in Figure 8, Tables



Figure 7. Equal difference map of GPS/leveling- EIGENGL04S1 geoid height differences (units in cm).



Figure 8. Equal difference map of GPS/leveling- EIGEN5C geoid height differences (units in cm).

1 and 2.

REFERENCES

- Ayhan ME (1992). Turkey Geoid-1991 (TG-91). Map J., 108: 1-17 (In Turkish).
- Ayhan MÉ, Demir C, Lenk O, Kılıçoğlu A, Aktuğ B, Açıkgöz M, Fırat O, Şengün YS, Cingöz A, Gürdal MA, Kurt Aİ, Ocak M, Türkezer A, Yıldız H, Bayazıt N, Ata M, Çağlar Y, Özerkan A (2002). Turkish National Fundamental GPS Network-1999A (TFGN99A). Map J. Spec. Edition, 16: 1-69 (in Turkish).

Corchete V, Chourak M, Khattach D (2005). The high-resolution gravimetric geoid of Iberia: IGG2005. Geophys. J. Int., 162: 676-684.

- Förste C, Flechtner F, Schmidt R, König R, Meyer U, Stubenvoll R, Rothacher M, Barthelmes F, Neumayer KH, Biancale R, Bruinsma S, Lemoine JM (2006). A mean global gravity field model from the combination of satellite mission and altimetry/gravimetry surface gravity data. Geophys. Res. Abstr., 8: 03462.
- Förste Ć, Flechtner F, Schmidt R, Stubenvoll R, Rothacher M, Kusche J, Neumayer KH, Biancale R, Lemoine JM, Barthelmes F, Bruinsma S, König R, Meyer U (2008). EIGEN-GL05C-A new global combined high-resolution GRACE-based gravity field model of the GFZ-GRGS cooperation. Geophys. Res. Abstr., 10: 03426.

- HGK (2010). General command of mapping official website. http://www.hgk.msb.gov.tr/uyekurulus/THG/thg.html.
- Kılıçoğlu A (2002). Updated Turkish Geoid-1999 (TG-99A). Tectonics and Geodetic Networks Workshop. October, Iznik, Proceedings, pp. 153-166 (In Turkish).
- Kılıçoğlu A, Fırat O, Demir C (2005). Data and methods used in computation of new Turkish Geoid (TG-03). Turkey National Geodesy Commison Scientific Meeting in 2005. Geoid and Vertical Datum Workshop, pp. 53-65 (In Turkish).
- Kiamehr R, Sjöberg LE (2005). Comparison of the Qualities of Recent Global and Local Gravimetric Geoid Models in Iran. Stud. Geophys. Geod., 49: 289-304.
- Tscherning CC, Knudsen P, Forsberg R (1994). Description of the GRAVSOFT Package. Geophysical Institute, University of Copenhagen.