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Full Length Research Paper

Fractal characterization of an after-sales spare parts supply chain in Telecom industry

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Data series generated by complex systems exhibit fluctuations on a wide range of time scales, which often follow a scaling relation over several orders of magnitude. Such scaling laws allow for a characterization of the data and the generating complex system by fractal scaling exponents, which can serve as characteristic fingerprints of the systems in comparisons with other systems and models. In this article was developed a fractal characterization of the data series generated by the closed loop supply chain that supports the availability of spare parts in Telecom industry since this complex system displays fluctuations in its processes caused by endogenous and exogenous variables that create a difficulty for matching the recovery process with the demand process.

Key words: Spare parts, closed loop supply chain, complex systems, fluctuations, data series, fractals.

INTRODUCTION

Hence the telecommunications service provider (carrier) has powerful backbone networks to carry up terabytes of traffic data with up to 99.999% reliability, it is necessary for the “carrier” to eliminate or reduce the impact of an outage in the network due to: congestive degradation, unanticipated peaks above the capacity of the network, software quality, network design, hardware fault, etc. Pushed by market demands for efficient services, “carrier” is using telecom equipment manufacturers (TEM) after-sales services, to minimize operational and capital expenditures, as well as the impact of an outage in their network (Cohen et al., 2006). The maintenance service that concerns in this article is related with spare parts of repairable circuit packs. Carriers and TEM established an advance and exchange (AE) spare part service scope through an agreement (Hartley, 2005), which is designed to give support to critical network elements that put on risk the availability of the network. The AE service is triggered when a critical network element fails, then TEM must send to the “carrier” a good circuit pack from its stock under a determined “service level agreement”; once received, the “carrier” must return the faulty unit back to TEM's warehouse, so this one can be repaired and returned back to the pool of good stock (circles 1, 2, 3 and 4 in Figure 1). The defective collect process plus the repair process is called the recovery process. Most research on supply chain fluctuations has focused on the amplification of upstream order variability, namely the “bullwhip effect" (Lee et al., 1997). The study of supply chain from the point of view of complex systems theory has started only recently (Helbing, 2008). Concepts from statistical physics and non-linear dynamics have recently been used for the investigation of supply (Radons and Neugebauer, 2004). In Helding (2003) it was generalized concepts from traffic flow to describe instabilities of supply chains (called stop-and-go traffic (Helding, 2003)). This work remarks how small changes in the supply network topology can have enormous impact on the dynamics and stability of supply chains. In order to stabilize the supply chain, some strategies are mention in Radons and Neugebauer (2004).

By simulating a supply chain model, in Larsen et al. (1999) it was showed a wide range of non-linear dynamic phenomena that produce an exceedingly complex behavior in the production-distribution chain model. In Makui and Madadi (2007) it was proposed to measure the bullwhip effect by using the Lyanupov exponent. In

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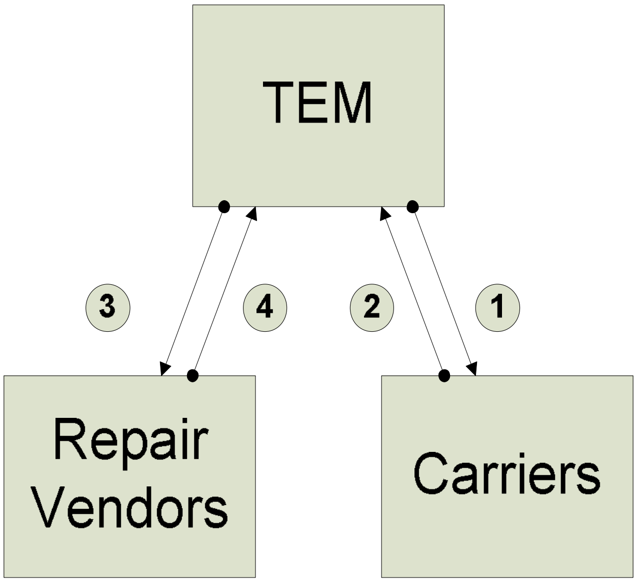

Figure 1. Closed loop supply chain of repairable items.

Hwarng and Xie (2008) it was used by chaos theory through the Lyanupov exponent across all levels of a specific supply chain, showing that chaotic behaviors in supply chain systems can be generated by deterministic exogenous and endogenous factors, and discovering the phenomenon "chaos-amplification": the inventory becomes more chaotic at the upper levels of the supply chain. This article analyzes the data series of the amplification of fluctuations in the queue of defective circuit packs waiting to be recovered by applying the fractal characterization that is, we calculated the scaling exponents of the fluctuations.

Fractals

The characterization and understanding of complex systems is a difficult task, since they cannot be split into simpler subsystems without tampering the dynamical properties. One approach in studying such systems is the recording of long time series of several selected variables (observables), which reflect the state of the system in a dimensionally reduced representation. Complex systems are characterized by periodic components that extend over a wide spectrum, and fluctuations on many time scales as well as broad distributions of the values are found. Often no specific lower frequency limit - or, equivalently, upper characteristic time scale - can be observed. In these cases, the dynamics can be characterized by scaling laws (that is, power-laws with scaling exponents) which are valid over a wide (possibly even unlimited) range of time scales or frequencies; at least over orders of magnitude. Such dynamics are usually denoted as fractal, that is, they are characterized by scaling exponents. Fractals can be seen as objects or phenomena under an invariant structure in different scales. Fractals are irregular shapes, in either mathematics or the real world, wherein each small part is very much like a reduced-size image of the whole (Mandelbrot, 2002). To identify fractals there are two central points: they should be objects with non-integer dimension, such as Hausdorff dimension, or they should be approximately (or statistically) self-affine (Mumford et al., 2002).

Let us assume that we have a function, $Y(t)$, of one variable only. Here t (usually time) is the horizontal variable, while Y is the vertical one. Self-affinity is defined through statistical invariance under the transformation:

$$t \longrightarrow \lambda t \quad (1)$$

$$Y \rightarrow \lambda^{H} Y \quad (2)$$

Where H is called the Hurst exponent. An alternative way of expressing this invariance is by the standard definition of self-affine that says that a process of continuous time $Y = \{Y(t), t > 0\}$ isself-affine (Embrechts and Maejima, 2002; Zhou and Taqqu, 2006) if:

$$Y(\lambda t) \triangleq \lambda^{H} Y(t) \quad (3)$$

Where the scaling exponent H measures the correlation persistence of data and $\triangleq$ denotes quality in distribution.

One of the most useful mathematical models for self-affine processes has been the fractional Brownian motion (fBm) which is an extension of the central concept of Brownian motion (Mandelbrot and Van Ness, 1968; Embrechts and Maejima, 2002). Self-affine processes such as fBm are currently used to model fractal phenomena of different nature. Let (Ω, F, P) be a complete probability space. The fBm $B_H(t)$ is a Gaussian process with mean 0, stationary self-affine increments (fractional Gaussian noise (fGn)), and variance $(B_H(t)^2) = t^{2H}$, that can be characterized by the scaling exponent $H \in (0,1)$. The special value $H = 1/2$ gives the familiar Brownian motion, then:

For $0 < H < 0.5$, the process is said to have antipersistent correlation.
For $0.5 < H < 1$, the process has persistence correlation and infinite variance. Because of this property, the time series is said to be long-range dependent.
For $H = 0.5$, the time series is said to be memoryless or short-range dependence.

In order to observe fractal scaling behavior in data series, several tools have been developed. In this article we use two different methods to estimate the scaling exponent H: the rescale range (R/S) analysis method and the visibility graph algorithm. These methods allow the calculation of the scaling exponent H (Lacasa et al., 2009; Gao et al., 2007; Beran, 1994; Taqqu et al., 1995). Once we determined the scaling exponent H, we map the data series into graphs; in so doing, methods of complex networks analysis are applied to have an insight into the emergence of fluctuation in the queues of the recovery process.

R/S analysis

The R/S analysis refers to a statistical technique to estimate the scaling exponent H (Mandelbrot, 2002). For a given set of observations $\{X_k, k = 1,2,\ldots,n\}$ with sample mean $\overline{X}(n)$ and sample variance $S^2(n)$ the R/S statistic is given by:

$$\frac{R(n)}{S(n)} = \frac{1}{S(n)}\left(max(0, W_1, W_2, \ldots, W_n) - min(0, W_1, W_2, \ldots, W_n)\right) \quad (4)$$

Where,

$$W_k = \sum_{i=1}^{k}[X_i - \overline{X}(n)] \quad (5)$$

Then $R(n)/S(n)$ characterizes the range of the process W_k.

One expects that the square of this extend scales with n as n^{2H}. We have:

$$E\left[\frac{R(n)}{S(n)}\right] \sim n^H \text{as } n \to \infty \quad (6)$$

Thus the value of the scaling exponent H can be obtained by running a simple linear regression over a sample of increasing time horizons:

$$\log\left(E\left[\frac{R(n)}{S(n)}\right]\right) = \log(c) + H\log(n) \quad (7)$$

Where $\log(c)$ is a constant that determines the point at which the line of Equation 7 crosses the axis $\log(E[R(n)/S(n)])$.

Visibility graph analysis

The visibility graph algorithm is a new method to estimate the scaling exponent H by mapping a fBm into a scale-free network according to the following criterion (Lacasa et al., 2009; Lacasa, 2008):

Two arbitrary data $(t_a; y_a)$ and $(t_b; t_b)$ in the data series have visibility, and consequently become two connected nodes in the associated graph, if any other data $(t_c; y_c)$ such that $t_a < t_c < t_b$ fulfills:

$$y_c < y_a + (y_b - y_a)\frac{t_c - t_a}{t_b - t_a} \quad (8)$$

In Lacasa et al. (2009) it was showed that the degree distribution of graphs derived from generic fBm follows a power-law $P(k) \sim k^{-\gamma}$ where k stands for the degree of a given node. A linear relation between the scaling exponent γ of the power-law degree distribution in the visibility graph and the scaling exponent H of the associated fBm series exists through:

$$\gamma(H) = 3 - 2H \quad (9)$$

And in order to estimate the scaling exponent γ we plotted the logarithm of the vertex degree k versus the logarithm of the number of vertices of degree k: m_k. The resulting curve should approximate a straight line and the points satisfy the equation (Clauset et al., 2009):

$$\log(m_k) = a - \gamma\log(k) \quad (10)$$

The range $1 < \gamma < 2$ is produced by a “partial duplication model” which is motivated on the duplication of information in the genome in biological networks (Chung et al., 2003).

SPARE PART SERVICE PROCESS

In this article we analyzed an advance and exchange (AE) spare part service process which happens between three stakeholders: the TEM, the “carrier” and the “repair vendor” (Figure 1). The AE service is triggered when a critical element of the telecom network from the “carrier” failed. At time t_1, a good unit is delivered [that is, delivery process (DP(t))] at “carrier” site. At time t_2 the “carrier” returns the defective unit to the TEM [that is, defective collect process (DCP(t))],

Where, $t_2 \geq t_1$.

At time t_3 the defective unit arrived at the “repair vendorsite” [that is, inbound repair process (IRP(t))], where $t_3 \geq t_2$. Finally at time t_4, once repaired, the unit out bounds the repair process [ORP(t)] and returns to the pool of good units,

Where, $t_4 \geq t_3$.

The dynamics of items that flow in the closed loop supply chain described earlier can be visualized by cumulative plots as shown in Figure 2. The vertical difference between two curves represents the queue $Q(t)$ of material pending to be processed and the horizontal separation between the curves represents the lead time L each item experiences between consecutive echelons. Within this system a conservation principle applies: what comes in must go out. In general, the queues can be represented as:

$$Q_{qr}(t) = Curve_q(t) - Curve_r(t) \geq 0 \quad (11)$$

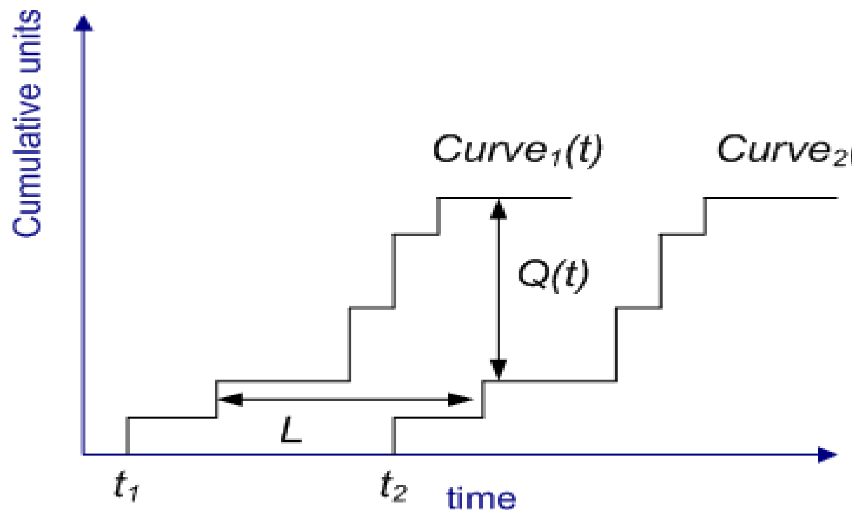

Figure 2. Cumulative data of each process in the closed loop supply chain.

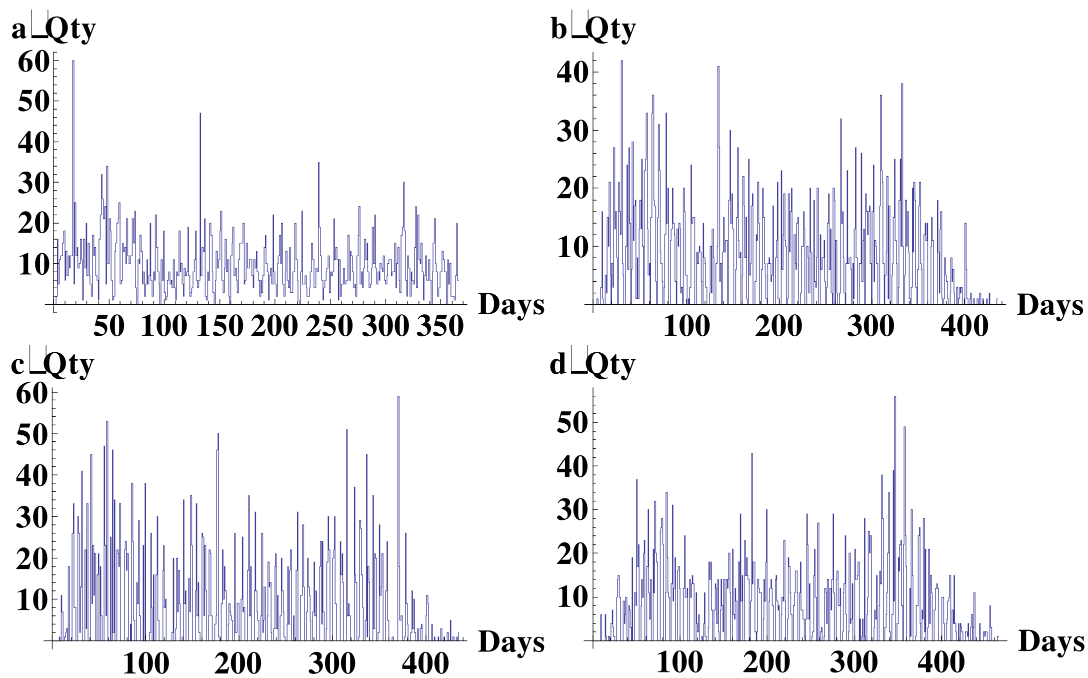

Figure 3. Data series of: (a) delivery process, (b) defective collect process, (c) inbound repair process, and (d) outbound repair process.

For $q > r$, where $q = 1, 2, 3, \dots\ n-1$ and $r = 2, 3, 4, \dots n$, and n is the number of echelons in the closed loop supply chain.

Similar with the measure of the bullwhip effect (Cachon et al., 2007), here the amplification of fluctuations in the queues is calculated as:

$$Amp.radio = \frac{v[queue\ echelon_{i+1}]}{v[queue\ echelon_i]} \quad \text{for} \quad i = 1, 2, 3, \dots, n \qquad (12)$$

EMPIRICAL FINDINGS

The data series encompassed one year of failures (demand of spare parts) of 4217 units. Unfortunately not all defective units were collected and/or repaired at the moment we began the analysis. Then, only 3617 units completed the entire process, that is, since they were demanded until repaired. Figure 3 shows the time series of each process of the supply chain, that is (a) delivery process, (b) defective collect process, (c) inbound repair

Table 1. Simple statistics of the data series of actual deliveries of the closed loop supply chain.

	DP	DCP	IRP	ORP
Average	9.9095	8.3341	8.3341	7.7952
Std. Dev.	7.0877	8.5929	11.468	9.1287
Variance	50.23	73.83	131.51	83.3338

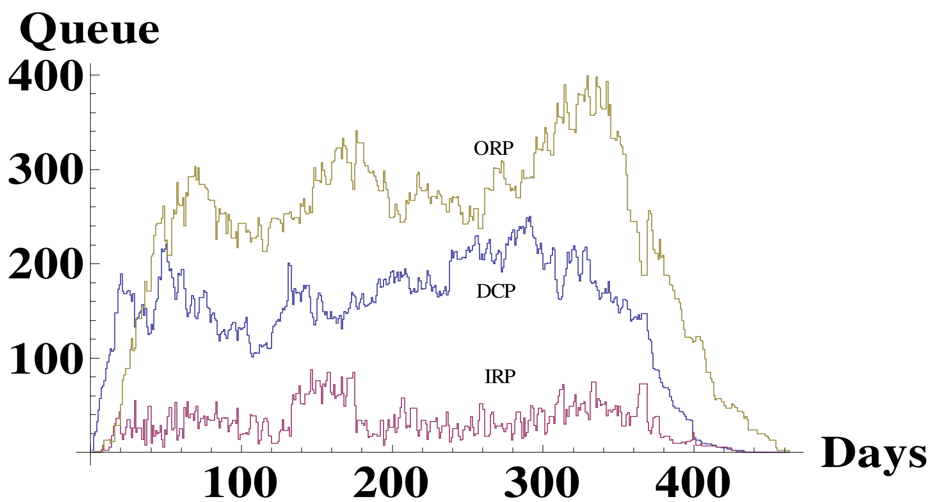

Figure 4. Circuit packs queues in the recovery process.

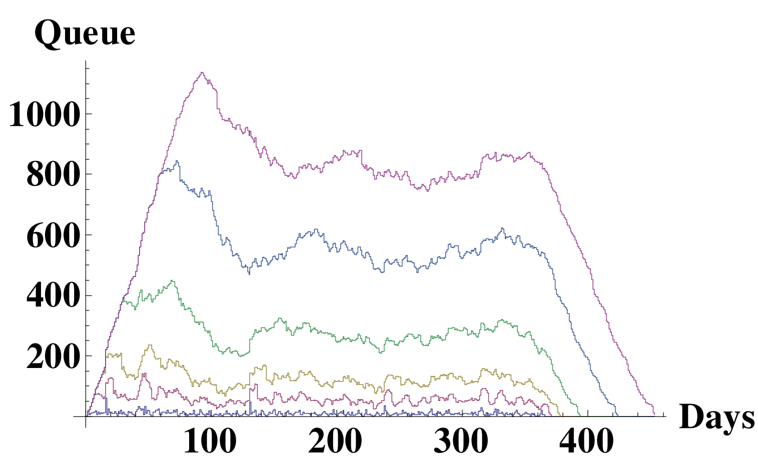

Figure 5. Circuit packs queues assuming different constant lead times.

process, and (d) outbound repair process. The demand of the 3617 units happened during 365 days, the defective collect process and the inbound repair process took 434 days, and finally the outbound repair process took 464 days. Although there is a constant amount of units processed in each echelon, the number of days of each process is required, increased going upstream in the supply chain. Table 1 shows simple statistics of the data series. It is notorious to see in this statistics the increment in fluctuations between DCP and DP, and between IRP and DCP which confirms the presence of the bullwhip effect. By applying Equation 11 in last four data series, Figure 4 plotted the three queues $Q(t)_{kl}$ involved in the recovery process. In order to verify the impact of lead time in the fluctuations of the queues, we also built different queues with constant lead time equals to: 1, 7, 14, 30, 60 and 90 days (Figure 5).

In Table 2 we can see simple statistics of the queue

Table 2. Simple statistics of the circuit packs queues.

	Average	Std. Dev.	Variance	CV
DCPqueue	147.70	60.91	3710.57	0.41
IRPqueue	32.48	19.49	379.94	0.60
ORPqueue	223.42	105.03	11032.17	0.47
L=1queue	9.90	7.08	50.23	0.71
L=7queue	58.65	20.61	425.14	0.35
L=14queue	124.72	37.12	1378.40	0.29
L=30queue	266.90	81.89	6707.14	0.30
L=60queue	504.49	178.05	31704.03	0.35
L=90queue	710.62	273.52	74817.45	0.38

Table 3. The scaling exponent$H$ calculated by queue.

	H (R/S)	H (Vis. Graph A.)
DCPqueue	1.00	0.85
IRPqueue	0.89	0.84
ORPqueue	0.95	0.96
L=1queue	0.74	0.78
L=7queue	0.81	0.81
L=14queue	0.89	0.88
L=30queue	0.94	0.90
L=60queue	0.89	0.96
L=90queue	0.90	0.91

Table 4. The scaling exponent γ of the queues of the recovery process.

	DC Queue	IR Queue	OR Queue
n	433	431	458
Edges	1795	1183	2220
Average degree	8.29	5.48	9.69
Isolated nodes	27	31	33
Density	0.0191	0.0127	0.0212
Average clustering coef. (from data)	0.4875	0.5484	0.5261
Average clustering coef. (random graph)	0.0202	0.0138	0.0202
Diameter	10	10	12
Average shortest path (from data)	4.3325	4.0373	4.9235
Average shortest path (random graph)	3.1567	3.8783	2.9662
γ	1.2854	1.3069	1.0703

data series. The coefficient of variation (CV) shows the lowest dispersion in the queue when $L = 1$, but it increased rapidly with greater lead times.

FRACTAL RESULTS

Table 3 shows the estimated values of the scaling exponent H. Both methods: R/S and "visibility graph algorithm" are consistent due to they yielded values with the scaling exponent higher than $1/2$. Therefore, the data series analyzed shows persistence correlation that is, long-range dependence. Also we mapped each data series into undirected networks by applying the visibility graph algorithm, and we characterize by applying the Network Workbench tool software (NWB Team, 2006) (Tables 4 and 5). The power-law property of these networks was confirmed also by the value of the scaling

Table 5. The scaling exponent γ of lead times.

	L= 1	L= 7	L= 14	L= 30	L= 60	L= 90
n	365	370	377	393	423	453
Edges	838	1265	1442	2026	2390	2467
Average degree	4.59	6.83	7.64	10.31	11.30	10.89
Isolated nodes	40	26	29	25	26	26
Density	0.0126	0.0185	0.0203	0.0263	0.0267	0.0241
Average clustering coef. (from data)	0.5695	0.4675	0.4898	0.4742	0.4258	0.4518
Average clustering coef. (random graph)	0.0177	0.0219	0.0238	0.0266	0.0253	0.0227
Diameter	7	9	10	19	9	9
Average shortest path (from data)	3.7439	4.0429	3.7606	4.7307	4.0064	2.9688
Average shortest path (random graph)	4.1832	3.3913	3.2262	2.8181	2.7629	2.8300
γ	1.4251	1.3627	1.2309	1.1883	1.0673	1.1720

exponent γ shown in Tables 4 and 5. As the value of the scaling exponent γ is always in the range $1 < \gamma < 2$, then the Partial Duplication model describes how the network emerged.

CONCLUSIONS

In this article, we focus our attention to analyze the fluctuations of defective circuit packs pending to be collected and repaired, that is, in the recovery process. Simple statistics in the data series showed the presence of the "bullwhip effect" between two echelons of the closed loop supply chain. Later, by analyzing the variability in the queues we found that the shorter of lead time, the lower fluctuation. In the data we analyzed, the "defective collect queue" as well as the "outbound repair queue", both experience greater lead times than the "inbound repair queue", and as a consequence, more fluctuations. In the fractal characterization we observed that the shorter of lead time, the lower persistence or long-range dependence that experiences the queues that is, an increase in the queue is likely to be followed by another increase, while decreases are likely to be followed by decreases. In all cases the data series show persistence due they have a scaling exponent between $0.5 < H < 1$. With the "visibility graph algorithm" we mapped the three data series of the queues into graphs. All graphs show the presence of the power-law property $P(k) \sim k^{-\gamma}k$ with a scaling exponent $1 < \gamma < 2$. According with the value of the scaling exponent γ found in this article, the graphs emerged through the "partial duplication model". Unfortunately, the collected data are too small compared with the internet (Albert et al., 2000), and it would be recommended to wait for more years of data to characterize with more accurate the values of the scaling exponents γ of the power-laws. In conclusion, the fluctuations in the queues of the recovery process increases when the scaling exponent H is closer to 1 and the scaling exponent γ is also closer to 1 as a result of an increment of the lead time. So lead time variable represents a key factor to mitigate the fluctuations in the closed loop supply chain analyzed in this article. If one finds that a complex system is characterized by fractal dynamics with particular scaling exponents, this finding will help in obtaining predictions on the future behavior of the system and on its reaction to external perturbations or changes in the boundary conditions. In the literature, the number of outstanding circuit packs in the recovery process are normally modeled by an$M/G/\infty$ queueing system (M stands for Poisson arrivals, G for a general lead time distribution, and ∞ for an unlimited number of servers, (Beran, 1994)).

According with Palm's theorem (Palm, 1938), the total ocupacy in the $M/G/\infty$ system is Poisson distributed with mean $\lambda E(L)$, (where λ represents the intensity of circuit packs failures). However, the present analysis shows that the queueing system resulted to be heavy-tailed. In general, many social, technological and economic phenomena have being approximated by Poisson processes. In contrast, there are evidences that many phenomena is Non-Poisson distributed (Smethurst and Williams, 2005; Monte et al., 2002; Barabási, 2005). Finally, we can conclude that the development of a spare parts mathematical model should include fractional Brownian motion in the supply chain processes.

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