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A four-point fully implicit method for the numerical integration of third-order ordinary differential equations

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In this paper, we derived a continuous linear multistep method (LMM) with step number k = 4 through collocation and interpolation techniques using power series as basis function for approximate solution. An order-seven scheme is developed which was used to solve the third-order initial value problems (IVPS) in ordinary differential equation without first reducing to a system first-order. Taylor's series algorithm of the same order was developed to implement our method. The result obtained was compared favourably with existing methods.

Key words: Continuous collocation, multistep methods, interpolation, third-order, power series.

INTRODUCTION

Linear multistep methods (LMMs) are very popular for solving first-order initial value problems (IVPS). Conventionally, they are used to solve higher order ordinary differential equations by first reducing them to a system of first-order. This approach has been extensively discussed in Bruguano and Trigiante (1998), Lambert (1973) and Fatunla (1988). However, the method of reducing to a system first-order has some serious drawback which includes wastage of human effort and computer time (Butcher, 2003).

The general k-step method or LMM of step number k is as given in Lambert (1973).

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} \tag{1}$$

Where α_j and β_j are uniquely determined and $\alpha_0 + \beta_0 \neq 0 \quad , \alpha_k = 1$

The LMM in Equation (1) generates schemes discrete schemes which are used to solve first-order ordinary differential equations. Various form of this LMM has been developed (Awoyemi, 2003; Henrici, 1962; Lambert, 1973; Fatunla, 1988). Other researchers have introduced the continuous LMM using the continuous collocation and interpolation approach. This has led to the development of continuous LMM of form

$$Y(x) = \sum_{j=0}^{k} \alpha_j(t) y_{n+j} + h \sum_{j=0}^{k} \beta_j(t) f_{n+j} \tag{2}$$

α_j and β_j are expressed as continuous functions of t and are at least differentiable once.

The introduction of continuous collocation methods as against the discrete schemes enhances better global error estimation and ability to approximate solution at all interior points. In this study, we shall develop continuous multistep collocation method for the solution of third-order ordinary differential equations using power series as the

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basis function.

Power series collocation

In Awoyemi (1999) and Onumanyi et al. (1993), some continuous LMM of the type in Equation (2) were developed using the collocation function of form:

$$Y(x) = \sum_{j=0}^{k} a_j x^j \tag{3}$$

However, Adeniyi and Alabi (2006) used Chebyshev polynomial function of the form

$$Y(x) = \sum_{j=0}^{M} a_j T_j(x)\left(\frac{x - x_k}{h}\right) \tag{4}$$

Where $T_j(x)$ are some Chebyshev function to develop continuous LMM.

Power series is proposed as basis function for deriving continuous LMM based on the property of analytic function that given the Taylor's polynomial of the form:

$$p(x) = \sum_{j=0}^{n} \frac{y^r(x_0)(x - x_n)^r}{r!} \tag{5}$$

The approximate function p(x) reduces to Y(x) as $n \to \infty$

In this study, we proposed the polynomial function of the form as follows (Okunuga and Ehijie, 2009):

$$Y(x) = \sum_{j=0}^{M} a_j (x - x_n)^j \tag{6}$$

which is of the type in Equation (3) to develop a continuous LMM for the solution of IVPS of the form:

$$y''' = f(x, y, y', y''),$$
$$y(a) = y_0\ , y'(x) = \partial_0\ ,\ \ y''(a) = \partial_N \tag{7}$$

DERIVATION OF THE METHOD

In this section, we approximate the exact solution $y(x)$ by a polynomial of degree M of the form:

$$Y(x) = \sum_{j=0}^{m} a_j (x - x_n)^j \ \ \cong y(x)\ , \tag{8}$$

$$x_n \leq x \leq x_{n+k}$$

$$Y(x) = a_0 + a_1(x - x_n) + a_2(x - x_n)^2 + a_3(x - x_n)^3 + a_4(x - x_n)^4 + \cdots + a_{m-1}(x - x_n)^{m-1}$$
$$+ a_m(x - x_n)^m \tag{9}$$

Differentiating Equation (9) up to the third-order to obtain

$$Y'''(x) =$$
$$6a_3 + 24a_4(x - x_n) + \cdots + (m-1)(m-2)(m-3)a_{m-1}(x - x_n)^{m-4} +$$
$$+ (m)\ (m-1)(m-2)a_m(x - x_n)^{m-3} = f(x, y, y', y'') \tag{10}$$

Interpolating Equation 9 at $x = x_n, x_{n+1}, \dots, x_{n+k-1}$ and collocating Equation 10 at $x = x_n, x_{n+1}, \dots, x_{n+k}$ we have

$$Y(x_n) = Y_n = a_0$$

$$\begin{array}{l} Y(x_{n+1}) = Y_{n+1} = a_0 + a_1 h + a_2 h^2 + a_3 h^3 + \cdots + a_{m-1} h^{m-1} + a_m h^m \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \end{array}$$

$$Y(x_{n+k-1}) = Y_{n+k-1} = a_0 + (k-1)a_1 h + (k-1)^2 h^2 a_2 + \cdots + a_m (k-1)^m h^m$$

Collocating Equation (10) at $x = x_n, x_{n+1}, \dots, x_{n+k}$, we have

$$6a_3 = f_n$$

$$6a_3 + 24a_4 h + \cdots + (m-1)(m-2)(m-3)a_{m-1}h^{m-4} + (m)(m-1)(m-2)a_m h^{m-3} = f_{n+1}$$

$$6a_3 + 2.24a_4 h + \cdots + 2^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-4} + 2^{m-2}(m)(m-1)(m-2)a_m h^{m-3} = f_{n+2}$$

$$6a_3 + k.24a_4 h + \cdots + k^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-4} + k^{m-3}(m)(m-1)(m-2)a_m h^{m-3} = f_{n+k}$$

Multiplying the collocation by h^3 yield

$$h^3 6a_3 = h^3 f_n$$

$$h^3 6a_3 + 24a_4 h^4 + \cdots + (m-1)(m-2)(m-3)a_{m-1}h^{m-1} + (m)(m-1)(m-2)a_m h^m = h^3 f_{n+1}$$

$$h^3 6a_3 + 2.24a_4 h^4 + \cdots + 2^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-1} + 2^{m-2}(m)(m-1)(m-2)a_m h^m = h^3 f_{n+2}$$

$$h^3 6a_3 + k.24a_4 h^4 + \cdots + k^{m-1}(m-1)(m-2)(m-3)a_{m-1}h^{m-1} + k^{m-3}(m)(m-1)(m-2)a_m h^m =$$
$$= h^3 f_{n+k}$$

Hence the matrix equation is given as:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & h & h^2 & h^3 & \ldots & h^{m-2} & h^{m-1} & h^m \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
1 & (k-1)h & (k-1)^2h^2 & (k-1)^3h^3 & ,,, & (k-1)^{m-2}h^{m-2} & (k-1)^{m-1}h^{m-1} & (k-1)^m h^m \\
0 & 0 & 0 & 6h^3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 6h^3 & 24h^4 & \ldots & (m-1)(m-2)(m-3)h^{m-1} & (m)(m-1)(m-2)h^m \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots. & \ldots. & \ldots \\
0 & 0 & 0 & 6h^3 & k.24h^4 & \ldots & \ldots & k^{m-3}m(m-1)(m-2)h^m
\end{bmatrix}
$$

$$
\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \ldots \\ \ldots \\ \ldots \\ a_m \end{bmatrix}
=
\begin{bmatrix} y_n \\ y_{n+1} \\ .. \\ y_{n+k-1} \\ h^3 f_n \\ h^3 f_{n+1} \\ \ldots \\ h^3 f_{n+k} \end{bmatrix} \tag{11}
$$

The matrix above is solved to obtain the values of $a_j's$ $, j = 0(1)m$, which are then substituted into Equation (6).

After some algebraic simplification, we obtained a continuous polynomial, which is then evaluated at $x = x_{n+k}$. The resulting k-step LMM is of the form:

$$
\sum_{j=0}^{k} \alpha_j(x) y_{n+j} = h^3 \sum_{j=0}^{k} \beta_j(x) f_{n+j} \tag{12a}
$$

SPECIFICATION OF THE METHOD

Let

$$
Y(x) = \sum_{j=0}^{m} a_j (x - x_n)^j \tag{12b}
$$

as in Equation (6). By letting M = 2K, then

$$
Y(x) = \sum_{j=0}^{2K} a_j (x - x_n)^j \quad , h = x - x_n \quad , k = 4 \tag{13}
$$

$$
= a_0 + a_1(x - x_n) + a_2(x - x_n)^2 + a_3(x - x_n)^3 + a_4(x - x_n)^4 + a_5(x - x_n)^5 + a_6(x - x_n)^6 + a_7(x - x_n)^7 + a_8(x - x_n)^8 \tag{14}
$$

Differentiating Equation (14) up to the third order, we have

$$
Y'''(x) = 6a_3 + 24a_4(x - x_n) + 60a_5(x - x_n)^2 + 120a_6(x - x_n)^3 + 210a_7(x - x_n)^4 + 336a_8(x - x_n)^5 \tag{15}
$$

Interpolating Equation (13) at $x = x_n, x_{n+1}, x_{n+2}, x_{n+3}$ and collocating Equation (15) at $x = x_n, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}$ to give the system of equation

$$
\sum_{j=0}^{2K} a_j (x - x_n)^j = y_{n+j} \tag{16a}
$$

$$
\sum_{j=3}^{2K} j(j-1)(j-2) a_j (x - x_n)^{j-3} = f_{n+j} \tag{16b}
$$

The matrix equation arising from this is given as:

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & h & h^2 & h^3 & h^4 & h^5 & h^6 & h^7 & h^8 \\
1 & 2h & 4h^2 & 8h^3 & 16h^4 & 32h^5 & 64h^6 & 128h^7 & 256h^8 \\
1 & 3h & 9h^2 & 27h^3 & 81h^4 & 243h^5 & 729h^6 & 2187h^7 & 6561h^8 \\
0 & 0 & 0 & 6h^3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 6h^3 & 24h^4 & 60h^5 & 120h^6 & 210h^7 & 336h^8 \\
0 & 0 & 0 & 6h^3 & 48h^4 & 240h^5 & 960h^6 & 3360h^7 & 10710h^8 \\
0 & 0 & 0 & 6h^3 & 72h^4 & 540h^5 & 3240h^6 & 17010h^7 & 81648h^8 \\
0 & 0 & 0 & 6h^3 & 96h^4 & 960h^5 & 7680h^6 & 53760h^7 & 334064h^8
\end{bmatrix}
\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix}
=
\begin{bmatrix} y_n \\ y_{n+1} \\ y_{n+2} \\ y_{n+3} \\ h^3 f_n \\ h^3 f_{n+1} \\ h^3 f_{n+2} \\ h^3 f_{n+3} \\ h^3 f_{n+4} \end{bmatrix} \tag{17}
$$

Equation (17) is solved by Gaussian elimination or matrix inversion to obtain the $a_j's$ $, j = 0(1)$2k which when substituted in Equation (13), we have the continuous polynomial given by:

$$
Y(x) = \frac{1}{20160h^8}[20160h^8 + 10560h^7(x - x_n) - 64080h^6(x - x_n)^2 + 80640h^4(x - x_n)^4 - 67200h \; 23520h^2(x - x_n)^6 - 3840h(x - x_n)^7 + 240(x - x_n)^8]y_n
$$

$$
+\frac{1}{20160h^8}[-82080h^7(x - x_n) + 202320h^6(x - x_n)^2 + 241920h^4(x - x_n)^4 + 201600h^3(x - x_n)^5 - 70560h^2(x - x_n)^6 + 11520h(x - x_n)^7 - 720(x - x_n)^8]y_{n+1}
$$

$$
+\frac{1}{20160h^8}[11232h^7(x - x_n) - 212400h^6(x - x_n)^2 + 241920h^4(x - x_n)^4 - 201600h^3(x - x_n)^5 + 70560h^2(x - x_n)^6 - 11520h(x - x_n)^7 + 720(x - x_n)^8]y_{n+2}
$$

$$
+\frac{1}{20160h^8}[-40800h^7(x - x_n) + 74160h^6(x - x_n)^2 - 80640h^4(x - x_n)^4 + 67200h^3(x - x_n)^5 - 23520h^2(x - x_n)^6 + 3840h(x - x_n)^7 - 240(x - x_n)^8]y_{n+3}
$$

$$
+\frac{1}{20160h^8}[1398h^{10}(x - x_n) + 4200h^9(x - x_n)^2 - 3360h^8(x - x_n)^3 - 1414h^7(x - x_n)^4 + +210h^6(x - x_n)^5 + 28h^5(x - x_n)^6 - 12h^4(x - x_n)^7 + h^3(x - x_n)^8]f_n
$$

$$
+\frac{1}{20160h^8}[26064h^{10}(x - x_n) - 44328h^9(x - x_n)^2 + 42336h^7(x - x_n)^4 + -33936h^6(x - x_n)^5 + 11620h^5(x - x_n)^6 - 1872h^4(x - x_n)^7 + 116h^3(x - x_n)^8]f_{n+1}
$$

$$+\frac{1}{20160h^8}[20052h^{10}(x-x_n)-36330h^9(x-x_n)^2+39816h^7(x-x_n)^4+$$
$$-33684h^6(x-x_n)^5+12012h^5(x-x_n)^6-1992h^4(x-x_n)^7+126h^3(x-x_n)^8]f_{n+2}$$
$$+\frac{1}{20160h^8}[-48h^{10}(x-x_n)+88h^9(x-x_n)^2-224h^7(x-x_n)^4+$$
$$336h^6(x-x_n)^5-196h^5(x-x_n)^6+48h^4(x-x_n)^7-4h^3(x-x_n)^8]f_{n+3}$$
$$+\frac{1}{20160h^8}[54h^{10}(x-x_n)-99h^9(x-x_n)^2+126h^7(x-x_n)^4+$$
$$-126h^6(x-x_n)^5+56h^5(x-x_n)^6-12h^4(x-x_n)^7+h^3(x-x_n)^8]f_{n+4} \quad (18)$$

Evaluating Equation (18) at $x = x_{n+4}$ yield the following discrete:

$$y_{n+4}-2y_{n+3}+2y_{n+1}-y_n = \frac{h^3}{120}(f_{n+4}+56f_{n+3}+126f_{n+2}+56f_{n+1}+f_n) \quad (19)$$

ANALYSIS AND IMPLEMENTATION OF THE METHOD

The method Equation (19) is a specific member of the conventional LMM which can be expressed as

$$\sum_{j=0}^{k}\alpha_j y_{n+j} = h^3\sum_{j=0}^{k}\beta_j y'''_{n+j} \quad (20)$$

This can be written symbolically as:

$$\rho(E)y_n - h^n\sigma(E) = 0, \quad f_n = f(x_n, y_n) \quad (21)$$

E is the shift operator defined as $E^{i}y_n = y_{n+j}$ and $\rho(E)$ and $\sigma(E)$ are respectively the first and second characteristics polynomial of the LMM defined as:

$$\rho(E) = \sum_{j=0}^{k}\alpha_j E^j, \quad \alpha_k \neq 0 \quad , \sigma(E) = \sum_{j=0}^{k}\beta_j E^j \quad (22)$$

Following Fatunla (1988) and Lambert (1973), we define the local truncation error associated with Equation (20) by the difference operator

$$L[y(x);h] = \sum_{j=0}^{k}[\alpha_j y(x_n+jh) - h^3\beta_j y'''(x_n+jh)] \quad (23)$$

where $y(x)$ is assumed to have continuous derivatives of sufficiently high order. Therefore, expanding Equation (23) in Taylor series about the point x to obtain the expression

$$L[y(x);h] = c_0 y(x) + c_1 hy'(x) + c_2 h^2 y''(x) + \cdots + c_{p+2}h^{p+2}y^{p+2}(x) \quad (24)$$

where the $c_0, c_1, c_2, c_3, \ldots, c_p, \ldots, c_{p+2}$ are defined as

$$C_0 = \sum_{j=0}^{k}\alpha_j$$

$$C_1 = \sum_{j=1}^{k} j\alpha_j$$

$$C_2 = \frac{1}{2!}\sum_{j=1}^{k} j^2\alpha_j$$

$$C_q = \frac{1}{q!}\left[\sum_{j=1}^{k} j^q\alpha_j - q(q-1)(q-2)\sum_{j=1}^{k}\beta_j j^{q-3}\right]$$

In the sense of Lambert (1973), we say that the method of Equation (20) is of order p and error constant C_{p+3} if

$$C_0 = C_1 = C_2 = \cdots = C_p = C_{p+1} = C_{p+2} = 0, \quad C_{p+3} \neq 0$$

Using the concept above, the method Equation (19) has order $p = 7$ and error constant given by:

$$c_{p+2} = \frac{12}{362880}$$

In order to analyze the method for zero stability we write:

$$\rho(r) = r^4 - 2r^3 + 2r - 1,$$

Where $r-1$ is a factor

Therefore we have

$$(r-1)(r^3 - r^2 - r + 1) = (r-1)(r-1)(r^2-1)$$

And $r = 1,1,1$ and 1 so that $|r| \leq 1$. Thus the method is zero stable

IMPLEMENTATTION

Single step method can be used to solve higher order ordinary differential equations directly without the need to first reducing it to an equivalent system of first order.

Consider the IVPS in Equation (7). For our method of order p = 7, Taylor series expansion is used to calculate

$y_{n+1}, y_{n+2}, y_{n+3}$, and their first, second, third derivatives up to order p = 7

$$y_{n+j} \equiv y(x_n+jh) \cong y(x_n) + jhy'(x_n) + \frac{(jh)^2}{2!}y''(x_n) + \frac{(jh)^3}{3!}f_n + \frac{(jh)^4}{4!}f'_n + \frac{(jh)^5}{5!}f''_n + \frac{(jh)^6}{6!}f'''_n + \frac{(jh)^7}{7!}f_n^{iv} \ldots$$

Table 1. Results of test Problem 1.

X-value	Exact solution	New result (p = 7)	Error in Adesanya (2011) Block method (P = 7)	Error in our new method (P = 7)
0.1	4.98752E-003	4.98752E-003	1.189947E-11	1.1899E-11
0.2	1.98011E-002	1.98011E-002	3.042207E-09	3.0422E-09
0.3	4.39996E-002	4.39997E-002	7.779556E-08	7.7796E-08
0.4	7.68675E-002	7.68676E-002	7.746692E-07	1.5559E-07
0.5	1.174433E-00	1.174437E-00	4.59901E-06	3.0541E-07
0.6	1.645579E-001	1.64558E-001	6.478349E-06	4.6102E-07
0.7	2.16881E-001	2.16881E-001	5.783963-06	3.138E-07
0.8	2.72975E-001	2.72976E-001	2.354715E-06	7.0374E-07
0.9	3.31350E-001	3.31352E-001	3.766592E-06	1.0177E-06
1.0	3.905280E-001	3.90531E-001	1.23312E-05	1.6528E-06

$$y_{n+j}^{(1)} \equiv y'(x_n+jh) \cong y'(x_n) + jhy''(x_n) + \frac{(jh)^2}{2!}f_n + \frac{(jh)^3}{3!}f'_n + \frac{(jh)^4}{4!}f''_n + \frac{(jh)^5}{5!}f'''_n + \frac{(jh)^6}{6!}f_n^{iv} + \frac{(jh)^7}{7!}f_n^{v} \ldots$$

$$y_{n+j}^{(2)} \equiv y''(x_n+jh) \cong y''(x_n) + jhf_n + \frac{(jh)^2}{2!}f'_n + \frac{(jh)^3}{3!}f''_n + \frac{(jh)^4}{4!}f'''_n + \frac{(jh)^5}{5!}f_n^{iv} + \frac{(jh)^6}{6!}f_n^{v} + \frac{(jh)^7}{7!}f_n^{vi} \ldots$$

$$y_{n+j}^{(3)} \equiv y'''(x_n+jh) \cong f_n + jhf'_n + \frac{(jh)^2}{2!}f''_n + \frac{(jh)^3}{3}f'''_n + \frac{(jh)^4}{4!}f_n^{iv} + \frac{(jh)^5}{5!}f_n^{v} + \frac{(jh)^6}{6!}f_n^{vi} + \frac{(jh)^7}{7!}f_n^{vii} \ldots$$

Then the known values of x_n and y_n are substituted into the differential equations. Next the differential equation is differentiated to obtain the expression for higher derivatives using partial differentiation as follows:

$$y''' = f(x, y, y', y'') \qquad = f_j$$

$$y^{iv} = f_x + y'f_y + y''f_{y'} + ff_{y''} = \left(\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} + y''\frac{\partial}{\partial y'} + f\frac{\partial}{\partial y''}\right)f_j = Df_j$$

$$y^{v} = f_{xx} + (y')^2 f_{yy} + (y'')^2 f_{y'y'} + f^2 f_{y''y''} + 2y'f_{xy} + 2y''f_{xy'} + 2ff_{xy''} + 2y'y''f_{yy'} + 2y'ff_{yy''} + 2y''ff_{y'y''} + Df_j(f_{y''}) + f_j(y'' + f_{y'})$$

$$= D^2 f_j + (f_{y''})Df_j + f_j(y'' + f_{y'})_j,$$

where

$$D = \frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} + y''\frac{\partial}{\partial y'} + f\frac{\partial}{\partial y''}, \text{ and } D^2 = D(D)$$

$$\vdots$$

$D^p f_j$ where p is the order of the method.

Numerical experiments

Our methods of order p = 7 were used to solve some IVPS of both general and special nature using Taylor's series. Our results were compared with the results of other researchers in this area.

The following IVPS were used as test problems:

1) $y''' + 4y' = x$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$, $h = 0.1$, $0 \le x \le 1$

Exact solution: $y(x) = \frac{3}{16}(1 - cos2x)$,

2) $y''' = -y$, $y(0) = 1$, $y'(0) = -1$, $y''(0) = 1$, $0 \le x \le 1$, $h = 0.1$

Exact solution: $y(x) = e^{-x}$,

3) $y''' + y'' + 3y' - 5y = 2 + 6x - 5x^2$, $y(0) = -1$, $y'(0) = 1$, $y''(0) = -3$, $0 \le x \le 1$

Exact solution: $y(x) = x^2 - e^x + e^{-x}\sin(2x)$.

The results of these problems were compared with that of Adesanya (2011), Awoyemi (2003) and Olabode (2007) which were implemented in predictor corrector mode and block method, respectively.

The error in this experiment is defined as:

$$Error = |y(x) - y_n(x)|,$$

Where y(x) is the exact solution and $y_n(x)$ is the computed result.

Conclusion

We have developed a k-step LMM using power series collocation method. A new scheme with continuous coefficient is obtained which was applied to solve some special and general third-order IVPS in ordinary differential equation. Evidence of the better accuracy of our method over existing methods are mentioned in Tables 1, 2 and 3.

Table 2. The result of test Problem 2.

X-value	Exact solution	New result (P =7)	Error in Olabode (2007) (P = 7)	Error in our new method (P = 7)
0.1	0.904837	0.940837	1.36929E-09	2.4525E-13
0.2	0.818731	0.818731	3.12272E-08	6.2109E-11
0.3	0.740818	0.740818	1.27694E-07	1.5746E-10
0.4	0.670320	0.670320	3.25196E-07	3.1477E-09
0.5	0.606531	0.606531	6.54297E-07	6.1617E-09
0.6	0.548812	0.548812	1.14406E-06	9.1732E-09
0.7	0.496585	0.496585	1.81784E-06	1.3329E-08
0.8	0.449329	0.449329	2.69774E-06	1.6378E-08
0.9	0.406570	0.406570	3.80241E-06	1.7134E-08
1.0	0.367879	0.367879	5.14755E-06	7.4405E-09

Table 3. The result of test Problem 3.

X-value	Exact solution	New result (P =7)	Error in Olabode (2007) (P = 7)	Error in our new method (P = 7)
0.1	0.915407E+00	0.915407E+00	6.408641E-07	8.54782E-11
0.2	0.862574E+00	0.862574E+00	1.511330E-05	2.232510E-09
0.3	0.841561E+00	0.841561E+00	6.364443E-05	5.824412E-08
0.4	0.850967E+00	0.850967E+00	1.675667E-04	1.226405E-06
0.5	0.888343E+00	0.888341E+00	3.507709E-04	2.811820E-06
0.6	0.950606E+00	0.950599E+00	6.410875E-04	6.295841E-06
0.7	0.103439E+01	0.103438E+01	1.071642E-03	1.695782E-05
0.8	0.113640E+01	0.113636E+01	1.682213E-03	4.765221E-05
0.9	0.125367E+01	0.125353E+01	2.520603E-03	1.316541E-04
1.0	0.138377E+0	0.138343E+0	3.644014E-03	3.417856E-04

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