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Full Length Research Paper

An attempt to solve neutron transport equation inside supercritical water nuclear reactors using the Boubaker Polynomials Expansion Scheme

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In this paper, we use a polynomial scheme for solving the system of nonlinear boundary value problems associated with nuclear fission inside supercritical water nuclear reactors. The results are calculated in terms of series with easily computable components. The suggested method is applied without any discretization or transformation, but under some restrictive assumptions. To illustrate the implementation and efficiency of the proposed method, plots of the wavelength-dependent neutron flux profile are provided.

Key words: Supercritical water nuclear reactors (SCWNR), neutron transport equation, distribution function, Boubaker Polynomial Expansion Scheme (BPES), neutron angular flux.

INTRODUCTION

Several nuclear reactor types have been used in the last decades. Very high temperature (VHT), supercritical-water-cooled (SWC) and molten-salt (MS) reactors are among the most known (Gallaway et al., 2008; Yoshikawa and Wakabayashi, 1970; Yousif et al., 2010; Corradini, 2009).

Very high temperature (VHT) offer both the possibility of burning actinides to further reduce waste and significant advances in sustainability, safety physical protection and reliability. Such reactors use a graphite-moderated core with a once-through uranium fuel cycle, using helium or molten salt as the coolant at about 960° value which allows hydrogen production. Supercritical-water-cooled reactors (SCWR) use supercritical water as working fluid while molten-salt reactors (MSR) nuclear fuel is dissolved in the molten fluoride salt as ThF_4 or UF_4, under low pressures and high temperatures.

Neutron transport equation, or neutron Boltzmann equation, is an equation which characterizes a relatively small number of neutrons colliding in a vast sea of nuclei inside such reactors (Kulikowska, 2000; Lewis and Miller, 1993). In this context, a statistical mechanics formulation, first attempted by Boltzmann for interacting gases, provides appropriate description of this phenomenon inside supercritical water nuclear reactors. Boltzmann's equation based on physical arguments, such as finite particle and quantum theory, gives a more physically precise picture of particle-particle interaction, as presented by Bell and Glasstone (1970) and Stammler and Abbate (1983).

PRESENTATION AND PROBLEM FORMALIZATION

Supercritical water reactors (SCWR) are designed according to the *Generation IV* reactor concept that uses supercritical water as working fluid. Such reactors usually operate at high pressure and temperature (Figure 1).

Supercritical water *Generation IV* reactor concept has been appreciated for the ability to consume existing nuclear waste in the production of electricity and the relative efficiency against classical nuclear fuel devices. Nevertheless, some of their drawbacks are safety risks which may be greater due to little experience with new designs and specific risks due to the use of mixtures of metallic Sodium and Argon as a coolant. These materials either explosively reacts with water or act as phyxiants.

The neutron transport equation inside supercritical water reactors (SCWR) has been solved using several techniques and protocols (Mohammadpour and Shamshirband, 2011; Alnour et al., 2011; Kebwaro et al., 2011; Omeje et al., 2011). In this study, several assumptions have been taken into account:

1) Each neutron is considered as a subatomic particle having the characteristics strong force of the standard model.
2) A quantum mechanical description is adopted, so that an involved system of Schrodinger equations describes neutron motion between and within nuclei.
3) For high speed, neutrons are considered as relativistic

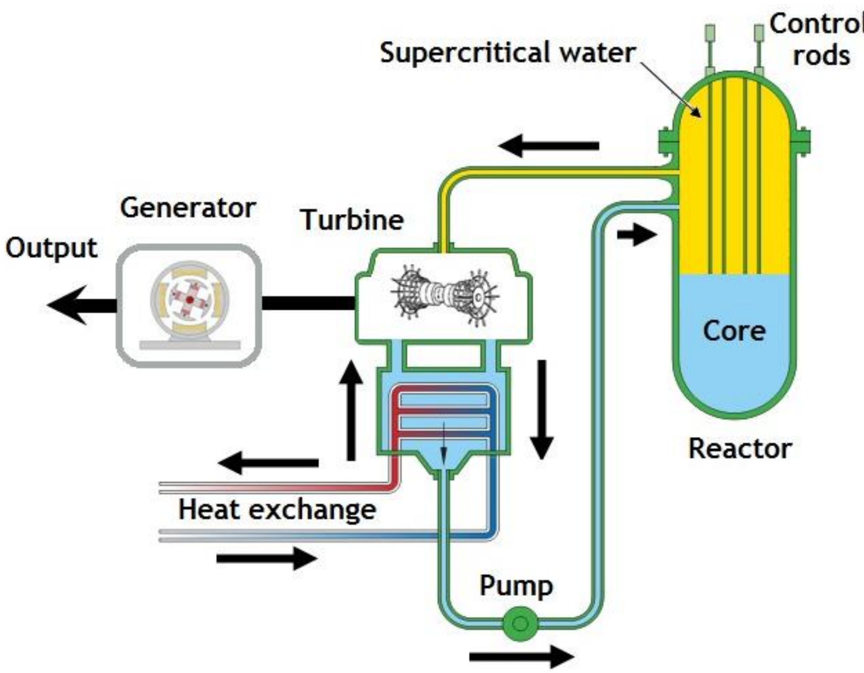

Figure 1. Supercritical water reactor (SCWR) scheme.

particles with variation of its mass over time.

4) Under moderate speed, neutron motion is governed by a complete set of Maxwell's equations.

Under these presumptions, integro-differential formulation, according to the most common neutron transport and reactor physics theory (Huber et al. 1998; Moshfegh and Modarres, 2005) is given as follows:

$$\begin{cases} \Gamma(\tilde{\psi}) = \int_0^\infty dE' \int_{4\pi} d\Psi' \Sigma_s(r, \Psi'.\Psi, E')\hat{\psi} + \dfrac{\chi(E)}{4\pi} \int_0^\infty dE' \int_{4\pi} d\Psi' V_E(E') \Sigma_f(r, E', t)\hat{\psi} = \hat{Q} \\ \text{with:} \begin{cases} \Gamma = \left\{ \dfrac{1}{v}\dfrac{\partial}{\partial t} + \Psi.\nabla + \Sigma(r, E, t) \right\} \\ \hat{\psi} = \psi(r, \Psi', E', t) \\ \tilde{\psi} = \psi(r, \Psi, E, t) \\ \hat{Q} = Q(r, \Psi, E, t) \end{cases} \end{cases} \tag{1}$$

where χ(E) is the distribution function, V_E is the neutron speed, Σ and Σ_f are macroscopic cross-sections, Σ_s is the scattering cross-section, ψ is the neutron angular flux, E and E' are energies, Ψ and Ψ' are inverse neutron directions while $\hat{Q}$ is the source function (Weber and Weigel, 1988; Zeng-Hua et al., 2004).

Employing the method of variable separation and for simplification purposes, it may be written:

$$A(r,t) = \Sigma_1 \Sigma_2 \Sigma_3 \psi_2 - \psi'_2 \Sigma_{s1}(r) \int_0^\infty dE' \int_{4\pi} d\Psi' \Sigma_{s2} - \frac{\chi(E)}{4\pi} .\psi'_2 \Sigma_{f1}(r) \Sigma_{f3}(t) \int_0^\infty V_E(E') \Sigma_{f2}(E') dE' \int_{4\pi} d\Psi' \tag{2}$$

If the function ψ_2 is separable such that:

$$\begin{cases} \psi_1(r,t) = U(r)V(t) \\ A(r,t) = A_1(r) + A_2(t) \\ Q_1(r,t) = \psi_2 Q_0 \\ Q_0 = Q_0(r) + V_0(t) \end{cases}$$

Hence:

$$\begin{cases} \dfrac{dG}{dt} = \dfrac{Q_2 \nu}{\psi_2}\left\{V_0(t) - \dfrac{A_2(t)}{Q_2} + \varepsilon^2\right\}V(t) \\ \dfrac{dR}{dr} = \dfrac{Q_2}{\Omega\psi_2}\left\{U_0(r) + \varepsilon^2 - \dfrac{A_1(r)}{Q_2}\right\}U(r) \end{cases}$$

Using the method of integrating factor or by direct integration, the solution to Equations 3 and 4 are as follows:

(3)

$$\begin{cases} V(t) = \theta_0 e^{\frac{Q_2 \nu}{\psi_2}\int\left\{V_0(t) - \frac{A_2(t)}{Q_2} + \varepsilon^2\right\}dt} \\ U(r) = \theta_1 e^{\Psi\frac{Q_2}{\psi_2}\int\left\{U_0(r) + \varepsilon^2 - \frac{A_1(r)}{Q_2}\right\}dr} \end{cases} \quad (5)$$

where θ_0 and θ_1 are constants.

This form of solution is very interesting because it allows different choices for the constants. Consequently, the reactor nature, the intended use and different applications may be easily imposed on the analytical expression for the neutron flux.
From Equation 2, the expression for the flux is given as:

$$\psi(r,\Psi,E,t) = \theta_0\theta_1\psi_2 e^{\frac{Q_2}{\psi_2}\left[\nu\int\left\{G_0(t) - \frac{A_2(t)}{Q_2} + \varepsilon^2\right\}dt + \Psi\frac{Q_2}{\psi_2}\int\left\{U_0(r) + \varepsilon^2 - \frac{A_1(r)}{Q_2}\right\}dr\right]} \quad (6)$$

SOLUTIONS AND DISCUSSION

The neutron transport equation without delayed neutrons is given as (Weber and Weigel, 1988):

$$\Gamma(\tilde{\psi}) = \int_0^\infty dE' \int_{4\pi} d\Psi' \Sigma_s(r,\Psi'.\Psi,E')\hat{\psi} + \frac{\chi(E)}{4\pi}\int_0^\infty dE' \int_{4\pi} d\Psi' V_E(E')\Sigma_f(r,E',t)\hat{\psi} = \hat{Q}_{ext.} \quad (7)$$

where $\hat{Q}_{ext.}$ is the external sources of neutrons, and V_E is the average number of neutrons per fission.

The equation assumes that all neutrons are emitted instantaneously at the time of fission. In fact, small fraction of neutrons is emitted later due to certain fission products (Zeng-Hua et al. 2004). Now, if we seek as asymptotic solutions to Equation 7 in the form:

$$\begin{cases} \psi(\vec{r},\Psi,E,t) = \psi_a(\vec{r},\Psi,E)e^{\alpha t} \\ \Psi\dfrac{d\psi_{a1}}{dr} = B(\vec{r})\psi_{a1} \end{cases} \quad (8)$$

where the solution satisfies the boundary conditions. If we suppose that the integral $\int_0^\infty \Psi B(\vec{r})d\vec{r}$ is convergent, and taking into account the characteristics of a given nuclear reactor with spherical symmetry ($B(\vec{r}) = \|B(\vec{r})\|\dfrac{\vec{r}}{\|\vec{r}\|} = \|B(\vec{r})\|\vec{u}_r$), within radial range $[0,R]$:

$$\begin{cases} B(\vec{r})\big|_{\vec{r}=\vec{0}} = k_1 \\ \dfrac{d\|B(\vec{r})\|}{dr}\bigg|_{\vec{r}=\vec{0}} = 0 \\ B(\vec{r})\big|_{\vec{r}=R\vec{u}_r} = 0 \\ \dfrac{d\|B(\vec{r})\|}{dr}\bigg|_{\vec{r}=R\vec{u}_r} = k_2 \end{cases} \quad (9)$$

where k_1 and k_2 are core reactor characteristic constants.
For solving Equation 8, the Boubaker Polynomials Expansion Scheme (BPES) (Ghanouchi et al., 2008; Awojoyogbe and Boubaker, 2009; Labiadh and Boubaker, 2007; Slama et al., 2009, 2008; Tabatabaei et al., 2009; Fridjine and Amlouk, 2009; Belhadj et al., 2009a; 2009b, Barry and Hennessy, 2010; Yildirim et al., 2010; Kumar, 2010; Milgram, 2011) is proposed. This scheme is applied through setting the expression:

$$\|B(\vec{r})\| = \frac{1}{2N_0}\sum_{k=1}^{N_0}\lambda_k \times B_{4k}\left(\frac{r}{R}\mu_k\right) \quad (10)$$

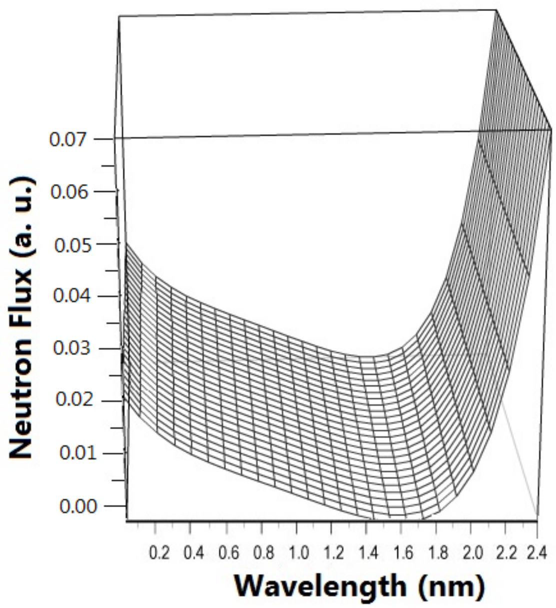

Figure 2. Neutron flux profile for different wavelengths.

where B_{4k} are the 4k-order Boubaker polynomials, r is the radius ($r \in [0, R]$), μ_k are B_{4k} minimal positive roots, N_0 is a prefixed integer, and $\lambda_k\big|_{k=1..N_0}$ are unknown pondering real coefficients.

The main advantage of this step lies in Equation 10 which ensures verifying the four boundary conditions in Equation 9, at the earliest stage of resolution protocol. In fact, due to the properties of the Boubaker polynomials (Weber and Weigel, 1988; Yildirim et al., 2010; Yoshikawa and Wakabayashi, 1970; Yousif et al., 2010; Zeng-Hua et al., 2004), and since $\mu_k\big|_{k=1..N_0}$ are roots of $B_{4k}\big|_{k=1..N_0}$, Equation 7 is reduced to:

$$\begin{cases} \left.\sum_{k=1}^{N_0} \lambda_k \times B_{4k}(\frac{r}{R}\mu_k)\right|_{\vec r=\vec 0} = \sum_{k=1}^{N_0} \lambda_k \times (-2) = 2k_1 N_0 \\ \left.\sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(\frac{r}{R}\mu_k)}{dr}\right|_{\vec r=\vec 0} = \sum_{k=1}^{N_0} \lambda_k \times 0 = 0 \\ \left.\sum_{k=1}^{N_0} \lambda_k \times B_{4k}(\frac{r}{R}\mu_k)\right|_{\vec r=R\vec u_r} = \sum_{k=1}^{N_0} \lambda_k \times B_{4k}(\mu_k)\big| = 0 \\ \left.\sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(\frac{r}{R}\mu_k)}{dr}\right|_{\vec r=R\vec u_r} = \sum_{k=1}^{N_0} \lambda_k \times \frac{dB_{4k}(\mu_k)}{dr} = \sum_{k=1}^{N_0} \lambda_k \times H_k = 2k_2 N_0 \end{cases} \quad (11)$$

$$with: H_k = \left.\frac{dB_{4k}(\frac{r}{R}\mu_k)}{dr}\right|_{\vec r=R\vec u_r} = \left(\frac{4\mu_k[2-\mu_k^2] \times \sum_{j=1}^{k} B_{4j}^2(\mu_k)}{B_{4(k+1)}(\mu_k)} + 4\mu_k^3 \right)$$

The solution is then assigned to the set of pondering real coefficients $\tilde{\xi}_j\big|_{j=1..N_0}$ which minimizes the amount Δ:

$$\Delta = \left(\sum_{j=1}^{N_0} \tilde{\xi}_j \times (-2) - 2k_1 N_0 \right)^2 + \left(\sum_{j=1}^{N_0} \tilde{\xi}_j \times H_j - 2k_2 N_0 \right)^2 \quad (12)$$

which gives the following solution to Equation 7:

$$\psi_a(\vec r) = \theta_3 e^{\int \Psi \frac{1}{2N_0} \sum_{k=1}^{N_0} \tilde{\lambda}_k \times B_{4k}(\frac{r}{R}\mu_k) d\vec r} \quad (13)$$

with θ_3 constant.

From Equation 13 and earlier assumptions, it becomes:

$$\psi(\vec r, \Psi, E, t) = \theta_3 \psi_{a2}(\Psi) \psi_{a3}(E) e^{\alpha t + \int \Psi \frac{1}{2N_0} \sum_{k=1}^{N_0} \tilde{\lambda}_k \times B_{4k}(\frac{r}{R}\mu_k) d\vec r} \quad (14)$$

Equation 14 is the neutron flux which defined the distribution neutron inside supercritical water nuclear reactors. Plots of wavelength-dependent flux profile are presented in Figure 2.

CONCLUSION

In this paper, we have used a polynomial scheme for solving the system of nonlinear boundary value problems associated with nuclear fission inside the supercritical water nuclear reactors. It is worth mentioning that we have solved a nonlinear system of boundary value problem by our proposed technique which consists of ensuring boundary conditions validity before solving the main equations. We give an example of solution for systems which are highly nonlinear, with compound Newmann-Dirichlet boundary conditions. After applying our proposed technique we obtained series solution as well as its graphical representation over the whole wavelength domain. We remark that our proposed method is well suited for such physical problems as it provides solution in less number of iterations. It is worth mentioning that the method is capable of reducing the volume of the computational work as compared to the classical protocols. The use of natural and experimental restrictions allows better understanding on how each parameters involved in nuclear reactor modeling interact with one another and further in-depth understanding of Physics better, giving us the opportunity to manipulate the parameters within experiment. The advantages of the performed method will be explored one at a time in our next investigation by introduction of Gauss-Cauchy-type boundary conditions. .

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