

Full Length Research Paper

Unsteady magnetohydrodynamic (MHD) thin film flow of a third grade fluid with heat transfer and no slip boundary condition down an inclined plane

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An investigation is made for unsteady magnetohydrodynamic (MHD) thin film flow of a third grade fluid down an inclined plane. The non-linear partial differential equation governing the flow and heat transfer are reduced to a system of non-linear algebraic equations using implicit finite difference approximation to obtain velocity and temperature profile. The effect of various physical parameter on both velocity and temperature profile obtained are studied through several graphs. It is noticed that the velocity and temperature profile decreases due to increase in third grade parameter and magnetic parameter.

Key word: Non-Newtonian fluid, magnetohydrodynamic (MHD) flow, third grade fluid, implicit finite difference approximation, heat transfer.

INTRODUCTION

Recently, many authors have carried out investigation on the flows of electrical conducting non-Newtonian fluid down an inclined plane. These fluids also known as Rivlin- Ericksen third grade fluid are molten plastics, emulsion, pulps, ink-jet printing; micro fluidics, hemodynamic, the flow of synovial fluid in joints and large variety of industrial products that have viscoelastic behaviour in their motion. These viscoelastic fluids phenomena are responsible for normal stress effects, contrary to what the Newtonian viscous effect does. The normal stress effects shows fluid elasticity, which is added to the viscous effects when the fluid is in motion. Since there is no single constitute equation that can be use to analyze all the non-Newtonian fluids, various linear and non linear equations have been proposed. A third grade fluid is a subclass of non-Newtonian fluid and its governing non-linear equation has successfully studied and treated in many literatures. Hayat et al. (2008)

investigate the influence of a magnetic field on the unsteady flow of an incompressible third grade electrically conducting fluid bounded by a rigid plate. They obtain the solution of the equations of conservation of mass and momentum in balance analytically using Lie symmetry analysis. Chakraborty (2001), studied the laminar convention flow of an incompressible electrically conducting second order viscoelastic stratified fluid in porous medium down an inclined channel. He was able to obtain the expression for viscous drag, the rate of heat transfer at the plate and flow flux for fluid and particle. Mohyuddin (2007), investigate an exact solution of unsteady flow of a viscous fluid due to a sudden pull with constant velocities of non-coaxial rotations between two porous infinite disks. He uses two different methods to obtain the solution at large time and the solution at small times. Nayak et al. (2012) studied the unsteady convective flow of a third grade fluid past an infinite

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vertical porous plate with uniform suction applied at the plate using an implicit finite difference method. They were able to show the effect of some physical parameter on the velocity field through several graphs.

Gamal (2011) examined the effects of magnetic field on thin film of unsteady micropolar fluid through a porous medium. He considered these thin films for three different geometries. Aiyesimi et al. (2012) investigate the steady magnetohydrodynamic (MHD) flow of a third grade fluid down an inclined plane with ohmic heating. They analysed the effect of magnetic parameter and Brinkman number on the velocity and temperature profile obtained.

In this work a detailed investigation of the unsteady MHD thin film flow of a third grade fluid down an inclined plane in the presence of uniform magnetic field applied externally transverse to the direction of flow is presented. The heat transfer analysis is also carried out. The effect of physical parameter such as magnetic parameter, third grade parameter, gravitational parameter, Prandtl number and Eckert number on both velocity and temperature distribution are discussed and shown graphically.

BASIC EQUATION AND FORMULATION OF THE PROBLEM

The basic equations governing the unsteady MHD flow of an incompressible electrically conducting fluid are the field equation.

$$\nabla \cdot v \tag{1}$$

and

$$\rho \frac{Dv}{Dt} = -\nabla p + \text{div } \tau + J \times B + \rho f \tag{2}$$

where ρ is the density of the fluid, v is the fluid velocity B is the magnetic induction. So that $B = B_0 + b$ (B_0 and b are applied and induced magnetic field respectively) and

$$J = \sigma (E + V \times B) \tag{3}$$

is the current density, σ is the electrical conductivity, E is the electrical field which is not considered that is ($E = 0$), $\frac{D}{Dt}$ denote the material derivative, P is the pressure, f is the external body force and τ is the Cauchy stress tensor which for a third grade fluid satisfies the constitution equation.

$$\tau = -PI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (\text{tr} A_1^2) A_1 \tag{4}$$

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1} \nabla V + (\nabla V)^{\perp} A_{n-1}, \quad n \geq 1 \tag{5}$$

where pI is the isotropic stress due to constraint

incompressibility, μ is the dynamics viscosity, $\alpha_0 (i = 1, 2), \beta_i (i = 1-3)$ are the material constants, \perp indicates the matrix transpose, $A_i (i = 1-3)$ are the first three Rivlin-Ericksen tensors. In the absence of modified pressure gradient, Equations (1) to (3) along with Equations (4) and (5) yield

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} + 6(\beta_2 + \beta_3) \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 + \rho g \sin \theta \tag{7}$$

with relevant boundary and initial condition

$$u(y, t) = 0 \quad \text{at} \quad y = 0 \quad t > 0 \tag{8}$$

$$u(y, t) = 0 \quad \text{at} \quad y = 1 \quad t > 0 \tag{9}$$

$$u(y, 0) = 0 \quad 0 < y < 1 \tag{10}$$

The energy equation for the thermodynamically compatible third grade fluid with viscous dissipation, work done due to deformation and Joule heating is given as:

$$\rho c_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \alpha_1 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y \partial t} + 2(\beta_2 + \beta_3) \left(\frac{\partial u}{\partial y} \right)^4 + \sigma B_0^2 u^2 \tag{11}$$

with boundary and initial condition

$$T(y, t) = T_w \quad \text{at} \quad y = 0 \quad t > 0 \tag{12}$$

$$T(y, t) = T_\delta \quad \text{at} \quad y = \delta \quad t > 0 \tag{13}$$

$$T(y, 0) = 0 \quad 0 < y < 1 \tag{14}$$

where c_p and k are respectively, specific heat capacity and thermal conductivity of the fluid. T is the temperature and T_δ is the temperature of the ambient fluid. Equations (7) and (11) are simplified together with boundary and initial condition (8)-(10) and (10)-(14) by writing them in the non-dimensional form. We define the following non-dimensional quantities:

$$u = \bar{\mu} \mu u, \quad y = \bar{y} \delta, \quad \eta = \frac{\mu y}{\delta}, \quad \bar{t} = \frac{\mu^2 t}{\delta}, \quad T = \bar{T} (T_\delta - T_w)$$

$$\alpha = \frac{\alpha_1 \mu^2}{\rho \delta^2}, \quad \text{is the second grade parameter}$$

$$\beta = \frac{(\beta_2 + \beta_3) \mu^4}{\rho \delta^3}, \quad \text{is the third grade parameter}$$

$$M = \frac{\delta \sigma B_0^2}{\rho \mu^2}, \quad \text{is the magnetic parameter}$$

$$K = \frac{\delta g \sin \theta}{\mu^3},$$

is the gravitational parameter

$$P_r = \frac{\mu \rho c_p}{K},$$

is the Prandth number

$$E_c = \frac{\mu^4}{c_p (T_\infty - T_w)},$$

is the Eckert number

$$\lambda = \frac{\alpha}{\beta},$$

is the fluid grade ratio

In terms of these above non-dimensional variables and parameters, Equations (7) and (11) together with Equations (8)–(10) and (10)–(14) are written as (we dropped “hats” for convenience)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial \eta^2} + \beta \left[\lambda \frac{\partial^3 u}{\partial \eta^2 \partial t} + 6 \left(\frac{\partial u}{\partial \eta} \right)^2 \frac{\partial^2 u}{\partial \eta^2} \right] - Mu + K = 0 \tag{15}$$

with the boundary and initial condition

$$u(0, t) = 0 \quad t > 0 \tag{16}$$

$$\frac{\partial u(1, t)}{\partial \eta} = 0 \quad t > 0 \tag{17}$$

$$u(\eta, 0) = 0 \quad 0 < \eta < 1 \tag{18}$$

For energy equation, we have

$$\frac{\partial T}{\partial t} = \frac{1}{P_r} \frac{\partial^2 T}{\partial \eta^2} + E_c \left(\frac{\partial u}{\partial \eta} \right)^2 + \beta \left[\lambda E_c \frac{\partial u}{\partial \eta} \frac{\partial^2 u}{\partial \eta \partial t} + 2E_c \left(\frac{\partial u}{\partial \eta} \right)^4 \right] + ME_c u^2 \tag{19}$$

With the boundary and initial condition

$$T(0, t) = 0 \quad t > 0 \tag{20}$$

$$T(1, t) = 1 \quad t > 0 \tag{21}$$

$$T(\eta, 0) = 0 \quad 0 < \eta < 1 \tag{22}$$

NUMERICAL RESULTS AND DISCUSSION

The momentum Equation (15) and energy Equation (19) represent a couple system of non-linear partial differential equations, which are solved numerically under the boundary and initial conditions (16) to (18) and (20) to (22) using finite difference approximation. Figures 1 to 3

illustrate the time development of the velocity profile u and temperature profile T for various values of time (t) and third grade parameter $\beta = 0, 0.5$ and 1 when $K=1, M=1$ and $\lambda=0.1$. It is noticed from the Figure 1 that increases in β increases u at all time. However, with non-zero values of β, u reach the steady state monotonically with time. Also, Figure 2 reveals that the velocity profile attains its steady state at higher values of magnetic parameter for various values of time (t) when $\beta = 0, 0.5$ and 1 . But Figure 3 shows that the temperature profile remains steady for various value of β at all time.

Figures 4 and 5 show the time evolution of velocity profile u and temperature profile T at $\eta = 0.5$ respectively for various values of the magnetic parameter M and third grade parameter $\beta = 0, 0.5$ and 1 . It is found in the Figure 4 that increase in parameter M decreases u and its steady state time for all values of β . This is due to the magnetic damping force on u which is increases with increase in M . Also the rate of transport is considerably reduced with increase in magnetic field parameter. It shows that the transverse magnetic field opposes the transverse phenomena.

From this figure, we noticed that the parameter β has a small effect on the steady state time of u and this effect is clearly shown when the value of M become smaller and smaller. Figure 5 illustrate the effect of magnetic parameter M on T depends on time. Temperature profile T slightly decreases as magnetic parameter M increases largely at all times. And as the values of β increases the temperature profile approach to its steady state. The reason for this is that for small times, an increase in M increase the Joule dissipation which is proportional to M . For large time, increasing in M decreases u largely and in turn decrease the Joule and viscous dissipation.

Figures 6 and 7 show the time evolution of u and T at $\eta = 0.5$ respectively for various values of gravitational parameter K and for $\beta = 0, 0.5$ and 1 . In this figure $M=1$ and $\lambda=0.5$. Figure 6 shows that increasing in value of gravitational parameter K increases u for all values of β . This is because increase in value of K correspond to increasing the angle of inclination which shows that by increasing the angle of inclination of inclined plane, the velocity increases. Figure 7 shows increase in temperature profile T as a result of increase in the value of gravitational parameter K for all values of β .

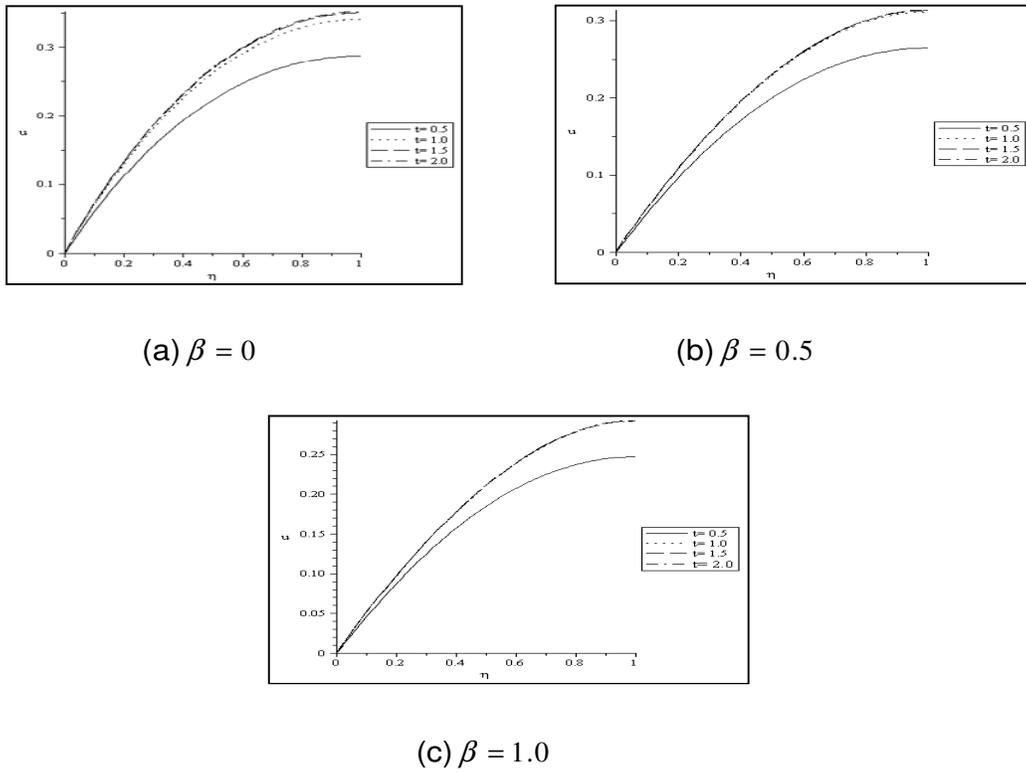


Figure 1. Time development of velocity profile u for various values of β and time t when $K = 1, M = 1$ and $\lambda = 0.1$.

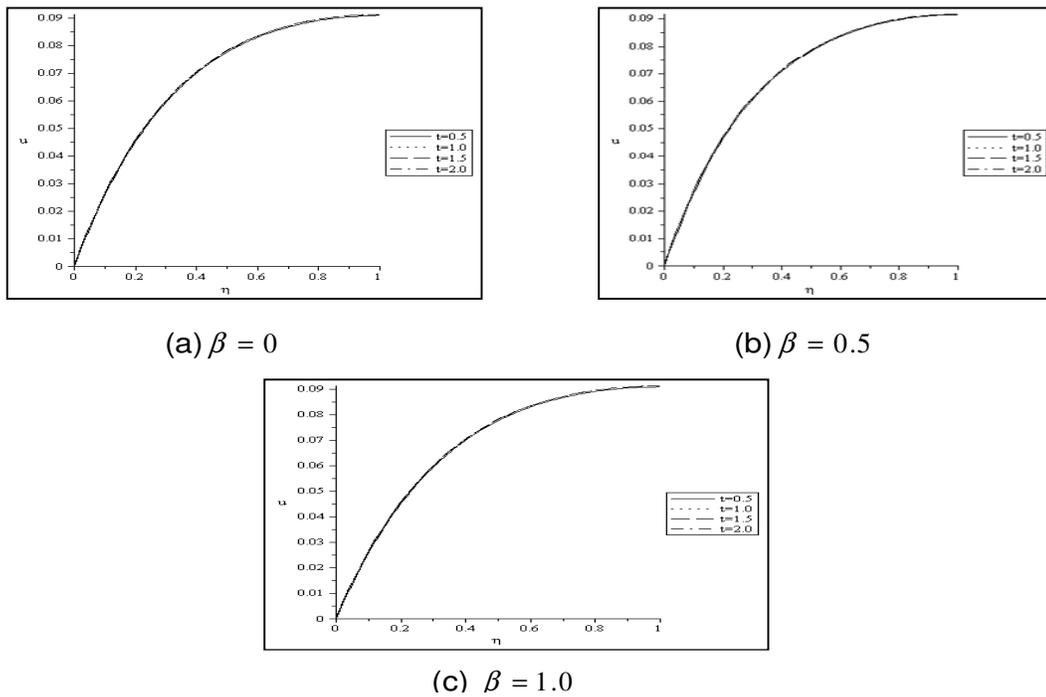


Figure 2. Time development of velocity profile u for various values of β and time t when $K = 1, M = 10$ and $\lambda = 0.1$.

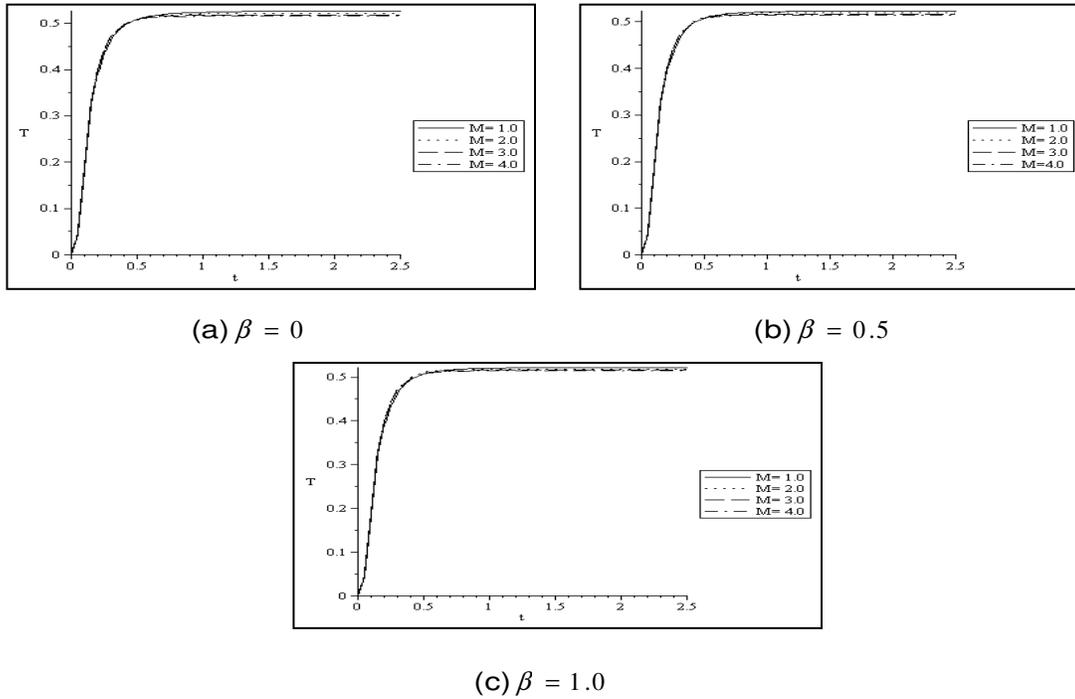


Figure 5. Time development of Temperature profile T for various values of β and M when $P_r = 1$, $E_c = 1, K = 1, M = 1$ and $\lambda = 0.1$.

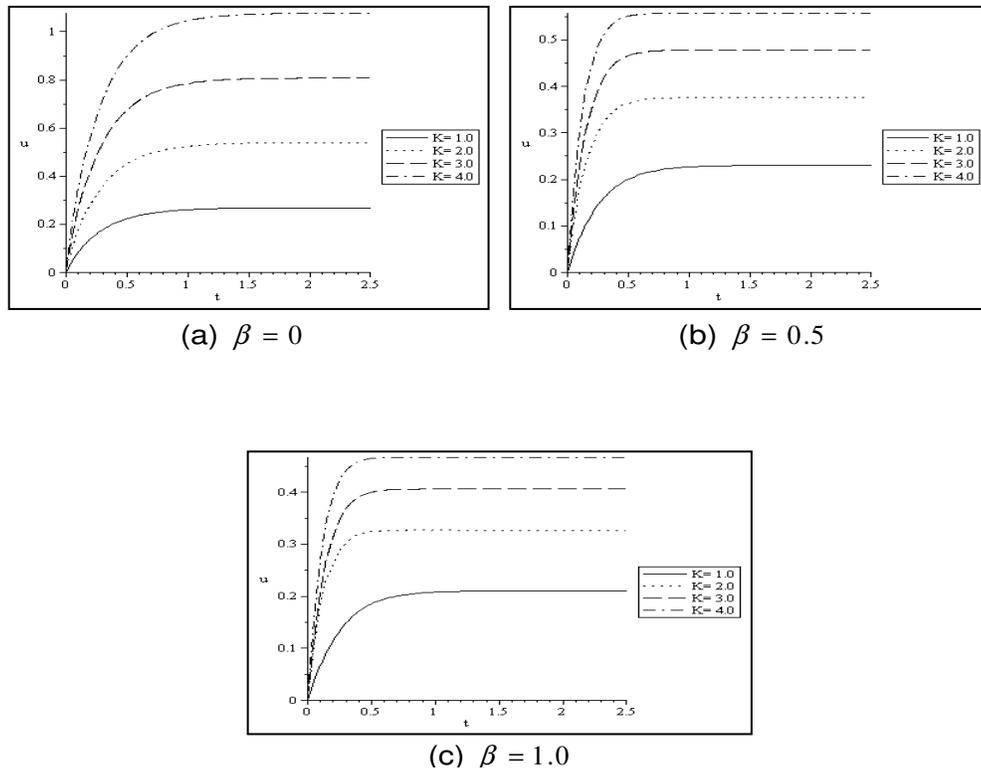


Figure 6. Time development of velocity profile u at $\eta = 0.5$ for various values of β and K when $M = 1, \lambda = 0.1$.

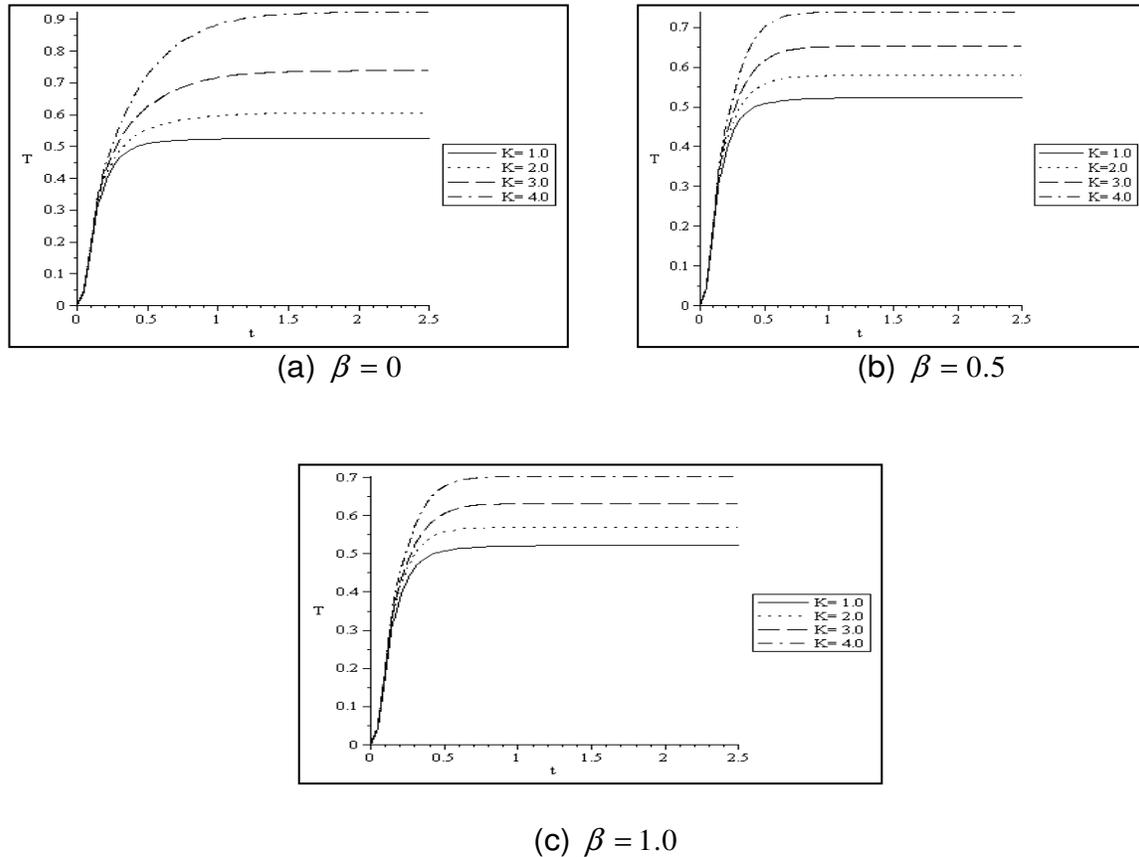


Figure 7. Time development of Temperature profile T at $\eta = 0.5$ for various values of β and K when $P_r = 1$, $E_c = 1$, $M = 1$ and $\lambda = 0.1$.

This can be attributed to the fact that an increase in K increases the angle of inclination which in turn increases u . And as u increases the Joule and viscous dissipation increases.

Figures 8 and 9 present the time evolution of u and T at $\eta = 0.5$ respectively for various values of Prandtl number and Eckert number for $\beta = 0, 0.5$ and 1 . In the figure $M = 1$ and $\lambda = 0.1$. In Figure 8, increase in P_r decrease T which in turn decreases the Joule and viscous dissipations. This accounts for crossing the curves of T with time for all values of β which become more apparent for higher values. The parameter β has no or negligible effect on the steady state time of T for all values of P_r . Figure 9 indicates the effect of Eckert Increasing in E_c cause the T to be steady at small, time number on temperature profile T depends on time and the increases at large time for all values of β .

The effect of increase in the values of magnetic parameter is to reduce the flow rate and thereby reducing the boundary layer thickness. This is exactly what happens in Figure 10, meaning that increase in magnetic parameter M induces an increase in the absolute value of the velocity gradient at the surface. The effects of Prandtl number and Eckert number on temperature profile are shown in Figure 11 and 12 respectively. Figure 11 reveal that the increase of Prandtl number P_r results in the decreasing of temperature profile at a certain time ($t = 0.5$) as the fluid is flowing down in an incline plane. The reason is that the thermal boundary layer thickness will decrease with increase in the values of Prandtl number P_r which also results to slow rate of thermal diffusion. The effect of increasing the values of Eckert number E_c is to increase the temperature profile T in an incline plane as shown in Figure 12. This behaviour of temperature enhancement can be seen as heat energy is stored in the fluid due to frictional heating.

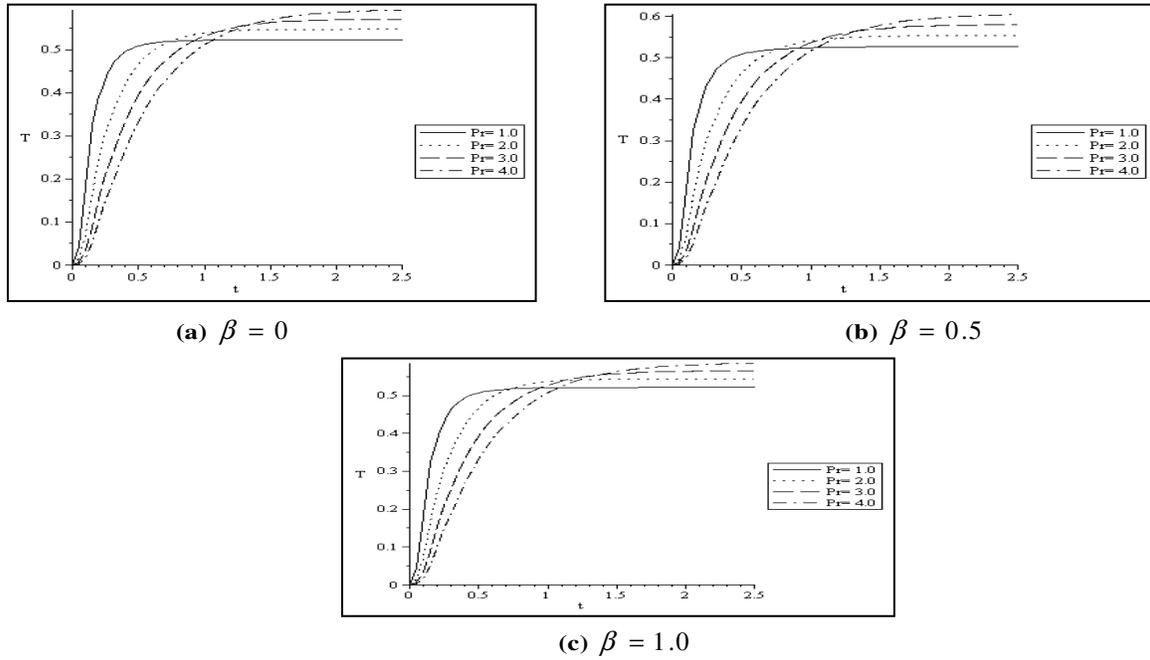


Figure 8. Time development of Temperature profile T at $\eta = 0.5$ for various values of β and P_r , when $K = 1$, $E_c = 1$, $M = 1$ and $\lambda = 0.1$.

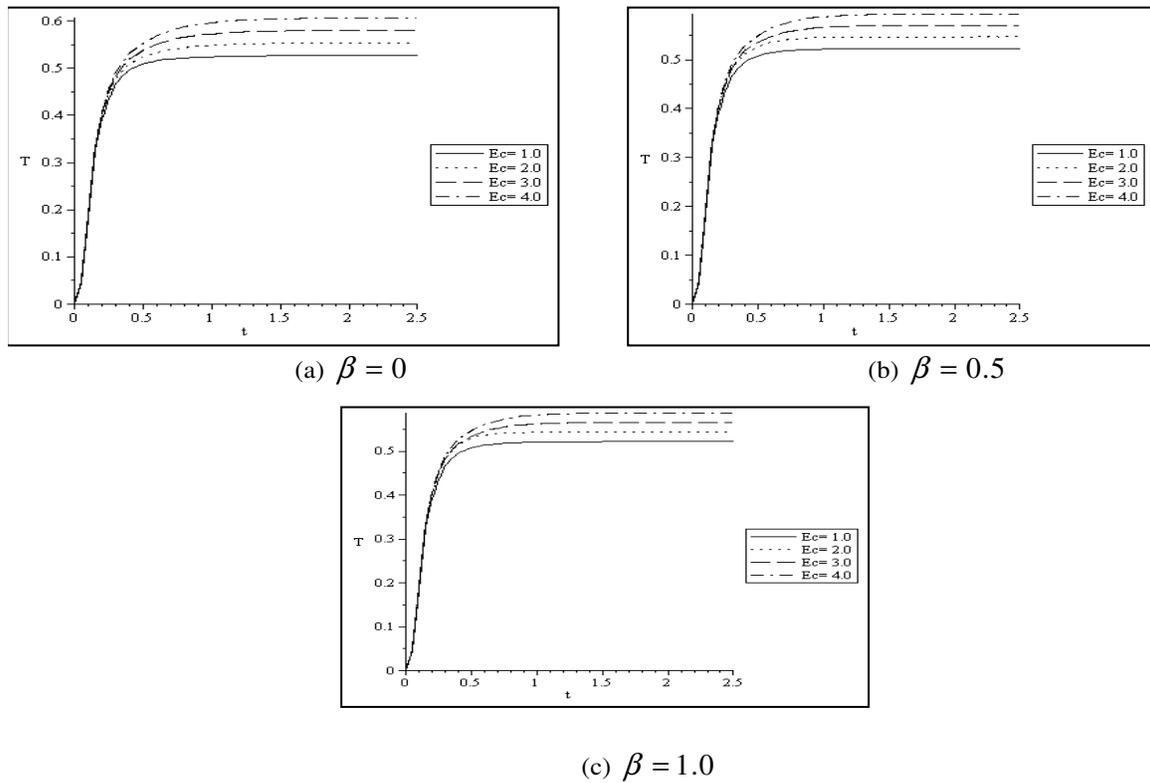


Figure 9. Time development of Temperature profile T at $\eta = 0.5$ for various values of β and E_c when $K = 1$, $P_r = 1$, $M = 1$ and $\lambda = 0.1$.

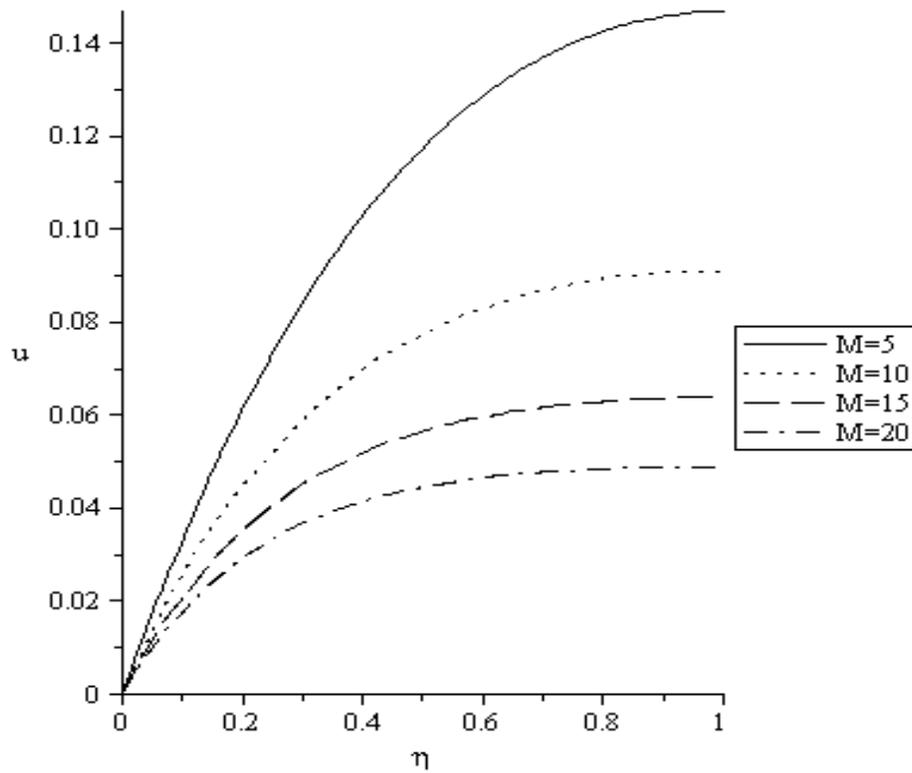


Figure 10. Effect of magnetic parameter (M) on velocity profile u at $t = 0.5$ when $K = 1$, $\beta = 0.5$, and $\lambda = 0.1$.

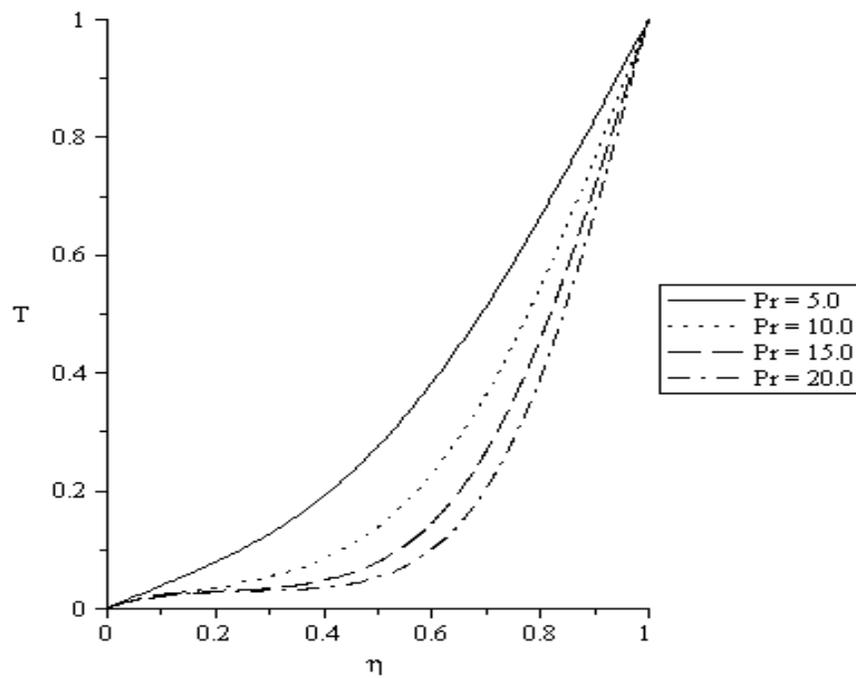


Figure 11. Effect of Prandtl number (Pr) on Temperature profile T at $t = 0.5$ when $E_c = 1$, $K = 1$, $M = 1$, $\beta = 0.5$, and $\lambda = 0.1$.

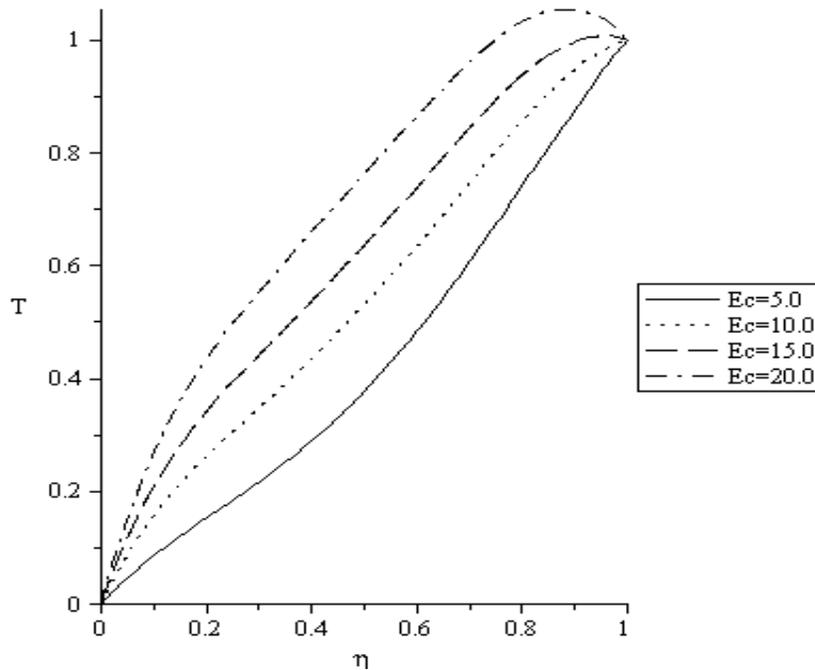


Figure 12. Effect of Eckert number (E_c) on Temperature profile T at $t = 0.5$ when $P_r = 5$, $K = 1$, $M = 1$, $\beta = 0.5$, and $\lambda = 0.1$.

Conclusion

The unsteady MHD thin film flow of a third grade fluid down an inclined plane with heat transfer is studied. The governing equations were developed and solved numerically using finite difference method. A systematic study on the effects of third grade parameter and other physical parameters controlling the flow and heat transfer characteristics is examined. The important findings of the graphical analysis of the results of the problem are as follows:

- (i) The effect of third grade parameter M and magnetic parameter is seen to decrease the velocity and temperature of the fluid at a small time but make them reach the steady state at large time;
- (ii) The effect of gravitational parameter is also seen to increase the velocity of fluid thereby increasing the Joule and viscous dissipation which in turn increases the temperature of the fluid;
- (iii) The Prandtl number and Eckert number decreases and increases respectively the temperature of the fluid thereby decrease its thickness.

REFERENCES

Hayat T, Mamboundou HM, Momuniat E, Mohamed FM (2008). "The Rayleigh problem for a third grade electrically conducting fluid in a magnetic field". *J. Nonlinear Math. Phys.* 15:77-90.

Chakraborty S (2001). "MHD flow and heat transfer of a dusty viscoelastic stratified fluid down an inclined channel in porous medium under variable viscosity." *Theor. Appl. Mech.* 26:1-14.
 Mohyuddin MR (2007). "Unsteady MHD flow due to eccentric rotating disks for suction and blowing". *Tur. J. Phys.* 31:123-135.
 Nayak I, Nayak AK, Padhy S (2012). "Numerical Solution for the flow and heat transfer of a third grade fluid past a porous vertical plate". *Adv. Stud. Theor. Phys.* 6(13):615-625.
 Gamal MA (2011). "Effect of Magnetohydrodynamics on thin films of unsteady micropolar fluid through a porous medium". *J. Modern Phys.* 2:1290-1304.
 Aiyesimi YM, Okedayo GT, Lawal OW (2012). "Viscous Dissipation effect on the MHD flow of a third grade fluid down an inclined plane with ohmic heating". *Math. Theory Model.* 2(9):133-161.