

Full Length Research Paper

Distribution of wind direction recorded at maximum wind speed: A case study of Malaysian wind data for 2005

N. A. B. Kamisan, A. G. Hussin*, Y. Z. Zubairi and S. F. Hassan

Centre for Foundation Studies in Science, University of Malaya, 50603 Kuala Lumpur, Malaysia.

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In this study, four types of circular probability distribution, namely circular uniform distribution, von Mises distribution, wrapped-normal distribution and wrapped-Cauchy distribution are considered in search of a circular probability distribution provides the best fit for Malaysian wind direction data recorded at maximum speed. The data collected were classified as annual, northeast monsoon and southwest monsoon. Seven sites were identified for this study based on their geographic location, that is, whether they are situated on the east coast or west coast of Malaysia. The objective of the study was to see whether there are any differences in the circular probability distribution of the wind direction based on these two coasts. Graphical measures were used to gauge which distribution gives the best fitting. To further support the findings, two performance indicators or goodness of fit tests are used in the analysis, namely the mean circular distance and the chord length. Generally, it was found that the monsoon-based data fits well with the von Mises distribution as compared to the annual dataset.

Key words: Wind data, circular distance, chord length, circular probability distribution, performance indicator.

INTRODUCTION

Malaysia has been fortunate in that extreme natural catastrophes rarely occur in the country, with the exception of the typhoon Utor that caused severe flooding in Johor in 2006 and recently in 2010, recurrent event of heavy rain downpour caused massive flooding in the northern, southern and eastern part of the peninsular. Such incidences have made us realize that something should be done to understand meteorological disasters like typhoons, for example. To understand what constitutes typhoons, we need to start with wind. Wind is when air flows, been caused by uneven heating on a surface. Different types of lands and water absorb heat at different rates. Thus wind varies in strength according to the different parts of the earth and different surfaces. A typhoon can be classified as an example of an extremely strong wind that can be disastrous. The objective of this paper is to model parametrically wind directions in

Malaysia recorded at maximum speed using circular probability distributions. Some of the benefits of this research are in terms of statistical contributions in predicting and modelling wind patterns which can be used for many purposes and for further research endeavour.

Wind direction is one of the features that should be considered in building wind turbines as well as structural and environmental design analysis (Wizelius, 2007). In many countries, project managers and architects have started to acknowledge the importance of ensuring a safe and comfortable wind environment in the vicinity of new buildings. The development of tall buildings in windy environments may lead to problems like "funneling" and "downwash". Other than that, wind direction is important in predicting the weather. This is possible because by predicting wind direction, one can have an idea of how the weather will change and forecast on the weather. Predictions of wind direction are also important for sports like golf and skydiving. Data on wind direction are comprised of angles measured in degrees or radian.

*Corresponding author. E-mail: ghapor@um.edu.my.

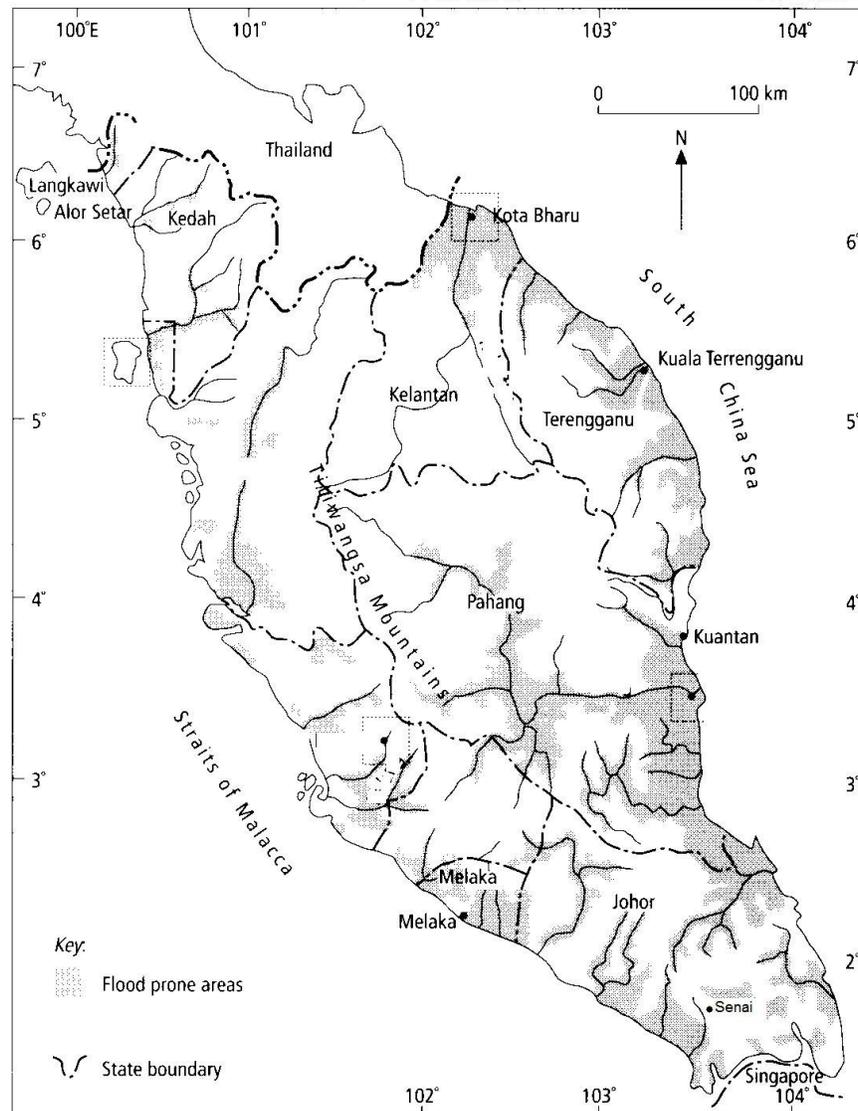


Figure 1. Locations of the maximum wind data that has been recorded daily in 2005. Source: www.fao.org/DOCREP/ARTICLE/WFC/XII/0452-B4.htm.

These types of data are known as circular data and thus wind direction can be modelled by using circular distributions; for example, Razali et al. (2008) in their study on surface wind direction for Bangi, a location in Malaysia, found that the von Mises seems to be the best distribution to describe the pattern of surface wind direction.

In this study, the wind directions at maximum wind speed have been recorded daily for seven locations and fitted with four circular distributions. The seven locations selected are Alor Setar, Langkawi, Melaka, Senai, Kota Bharu, Kuala Terengganu and Kuantan as shown in Figure 1. These places are selected based on their locations, that is, whether they are situated on the east coast or west coast of Malaysia. This is because the two

coasts are affected differently by the two monsoons that are prevalent in Malaysia, the northeast monsoon and the southwest monsoon. In the first stage of the study, the annual data on the wind directions were analysed as a whole. In the second stage, data were analysed based on the monsoons which relate to the locations. Only four circular distributions were selected as the software Axis, 2003 could only support these four distributions.

CIRCULAR PROBABILITY DISTRIBUTION

A circular probability distribution is a probability distribution whose total probability is concentrated on the circumference of a unit circle (Jamalamadaka and

SenGupta, 2001). The range of circular random variables can be measured in radian which can be defined as $[0, 2\pi)$ or $[-\pi, \pi)$. In this article, four circular probability distributions are discussed, which are the circular uniform distribution, von Mises distribution, wrapped-normal distribution and wrapped-Cauchy distribution.

Circular uniform distribution

Circular uniform distribution is the basic distribution on the circle which is invariant under rotation and reflection (Bogdan et al., 2002). If the total probability is spread out uniformly on the circumference, then we have circular uniformity (CU) distribution with the constant density function of

$$g(\theta) = \frac{1}{2\pi}, 0 \leq \theta \leq 2\pi$$

The von Mises distribution

The most useful distribution on the circle and was introduced in 1918 by von Mises in order to study the deviations of measured atomic weights from integral values (Mardia, 1972). This distribution is also known as circular normal distribution. A circular random variable θ is said to have a von Mises distribution if it has density function of

$$g(\theta; \mu, \kappa) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\theta - \mu)}, 0 \leq \theta < 2\pi$$

where $0 \leq \mu < 2\pi$ and $\kappa \geq 0$ are mean direction and concentration parameters, respectively. The function I_0 denotes the modified Bessel function of the first kind and order zero and is defined as:

$$I_0 = \sum_{r=0}^{\infty} \frac{1}{r!^2} \left(\frac{1}{2} \kappa\right)^{2r}$$

Wrapped-Cauchy distribution

The wrapped-Cauchy distribution is obtained by wrapping the Cauchy distribution on the real line with density of

$$f(x) = \left(\frac{1}{\pi}\right) \frac{\sigma}{\sigma^2 + (x - \mu)^2}, -\infty < x < \infty$$

around the circle. It has the probability density function

$$g(\theta; \mu, \rho) = \frac{1}{2\pi} \frac{1 - \rho^2}{1 + \rho^2 - 2\rho \cos(\theta - \mu)}, 0 \leq \theta < 2\pi$$

where $\rho = e^{-\sigma}$. The equality of the two aforementioned expressions is verified by equating the real parts of the geometric series identity $\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}$ with $a = \rho e^{-i(\theta - \mu)}$. The distribution is unimodal and symmetric.

Wrapped-normal distribution

A wrapped-normal distribution is obtained by wrapping a $N(\mu, \sigma^2)$ distribution around the circle where $\sigma^2 = -2 \log \rho$, i.e $\rho = e^{-\sigma^2/2}$. Its probability density function is given by

$$g(\theta; \mu, \rho) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{m=-\infty}^{\infty} \exp\left[\frac{-(\theta - \mu - 2\pi m)^2}{2\sigma^2}\right]$$

In particular, the mean direction is $\mu \pmod{2\pi}$ and the mean resultant length is ρ . From the theory of theta function, an alternate and more useful representation of this density can be shown to be

$$g(\theta; \mu, \rho) = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{p=1}^{\infty} \rho^{p^2} \cos p(\theta - \mu) \right\}, 0 \leq \rho \leq 1.$$

It is clear that the density can be adequately approximated by just the first few terms of the infinite series, depending on the value of σ^2 . It is unimodal and symmetric about the value of $\theta = \mu$. The wrapped-normal distribution possesses the additive property which is the convolution of two wrapped-normal variables is also wrapped-normal unlike the von Mises distribution.

PERFORMANCE INDICATOR

In measuring the suitability of a model, one could use performance indicators to check whether the data follows the specified model. Performance indicator is a basic measurement of how close a set of data follows a distribution. However, in the case of directional or circular data, one must not treat the data like linear data since directional data has distinctive features (Jammalamadaka and SenGupta, 2001). Two performance indicators are

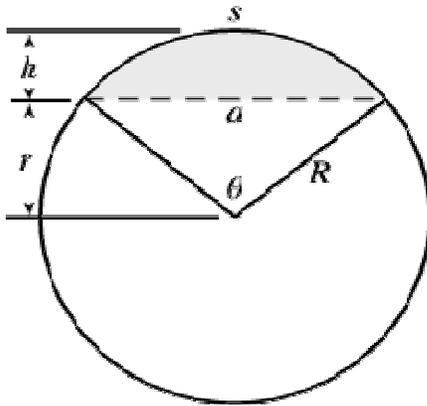


Figure 2. Chord length.

proposed for this study, namely the mean chord length and mean circular distance.

Mean chord length

Chord length is the length of the curve segment between two adjacent data points. As can be seen from Figure 2, 'a' (broken line) is the chord length.

Jammalamadaka and SenGupta (2001) proposed θ_{ij} as the smallest angle between θ_i and θ_j which is given by

$$\theta_{ij} = d_i(\theta_i, \theta_j) = \pi - |\pi - |\theta_i - \theta_j||$$

where $\theta_{ij} \in [0, \pi]$.

One can use this concept θ_{ij} to obtain the chord length between θ_i and θ_j . The length of a chord between two points θ_i and θ_j is calculated according to the formula

$$crd(\theta_{ij}) = 2r \sin\left(\frac{\theta_{ij}}{2}\right)$$

where $crd(\theta_{ij}) \in [0, 2]$ and r is the radius length. In this study.

The mean chord length is proposed and given by

$$A = \frac{1}{n} \sum \sin\left(\frac{\pi - |\pi - |\theta_i - \hat{\theta}_i||}{2}\right)$$

where θ_i is the observed, $\hat{\theta}_i$ is the predicted value and $A \in [0, 1]$.

If the chord length shows a small value, it means that the data follows the specified distribution. It can be seen as the weighted average of the length of the chord.

Mean circular distance

Circular distance calculates the length between any two points along the circumference and it takes the smaller of the two arc lengths between the points. Mean circular distance calculates the mean for all of the circular distances for each point in the data.

Let us assume that there are n points $\theta_1, \theta_2, \dots, \theta_n$, located on the circumference of a unit circle. Let d_{ij} be the circular distance between θ_i and θ_j for $i = 1, 2, \dots, n$, where d_{ij} is given by

$$d_{ij} = 1 - \cos(\theta_i - \theta_j)$$

and where $d_{ij} \in [0, 2]$.

In this study, a new measure of mean circular distance is proposed and given by

$$D = \frac{1}{2n} \sum (1 - \cos(\theta_i - \hat{\theta}_i))$$

where θ_i is the observed data and $\hat{\theta}_i$ is the predicted data and $D \in [0, 1]$.

Values that are approximately close to zero would indicate that the data follows the specified distribution.

Watson-Williams test

The Watson-Williams test is a circular analogue of two sample t -test. It assesses whether the mean directions of two or more groups are identical or not (Berens, 2009). The hypotheses are given below:

H_0 = All of s groups share a common mean direction, that is, $\bar{\alpha}_1 = \dots = \bar{\alpha}_s$

H_A = Not all s groups have a common mean direction

The test statistic (Mardia and Jupp, 2000) is given as

$$F = K \frac{(N-s)(\sum_{j=1}^s R_j - R)}{(s-1)(N - \sum_{j=1}^s R_j)}$$

where R is the mean resultant length when all samples are pooled and R_j the mean resultant vector length computed on the j th group alone (similar to total variance and within group variance in the ANOVA setting).

The correction factor K is computed from $K = 1 + \frac{3}{8\kappa}$, where κ is the maximum likelihood estimate of the concentration parameter of a von Mises distribution with resultant vector length r_w . We compute κ via the approximation given by Fisher (1993). Here r_w is the mean resultant length of the s resultant vector r_j computed for each group individually. The obtained value is then compared to a critical value at the δ level obtained from the F -table and a small p value means rejection of the null hypothesis, H_0 .

FITTING THE WIND DIRECTION WITH CIRCULAR PROBABILITY DISTRIBUTION

Parameter estimates

For all circular distributions, the three important parameters are mean angle, concentration parameter and mean resultant length respectively. The mean angle, μ is given by

$$\mu = \begin{cases} \tan^{-1}\left(\frac{S}{C}\right) & S > 0, C > 0 \\ \tan^{-1}\left(\frac{S}{C}\right) + \pi & C < 0 \\ \tan^{-1}\left(\frac{S}{C}\right) + 2\pi & S < 0, C > 0 \end{cases}$$

where $C = \sum_{i=1}^n \cos \theta_i$ and $S = \sum_{i=1}^n \sin \theta_i$.

The mean resultant length of circular distribution, ρ is given by

$$\rho = (\bar{C}^2 + \bar{S}^2)^{\frac{1}{2}},$$

where $\bar{C} = \frac{1}{n} \sum_{i=1}^n \cos \theta_i$ and $\bar{S} = \frac{1}{n} \sum_{i=1}^n \sin \theta_i$.

The concentration parameter of circular distribution κ (Fisher, 1993) is given by:

$$\kappa = \begin{cases} 2\rho + \rho^3 + \frac{5}{3}\rho^5 & \rho < 0.53 \\ -0.4 + 1.39\rho + \frac{0.43}{1-\rho} & 0.53 \leq \rho < 0.85 \\ (\rho^3 - 4\rho^2 + 3\rho)^{-1} & \rho \geq 0.85 \end{cases}$$

The parameters estimates for all seven locations in this study are given subsequently.

Graphical and numerical measures for annual wind direction

Table 1 shows the parameter estimated for each data set. After the parameters have been obtained from the observed data, a new set of data is generated from these parameters. Then the generated data is compared with the observed data graphically by using cumulative distribution function (cdf) plot and numerically by using three goodness-of-fit tests as previously discussed. One example of a cdf plot is shown subsequently for the location of Alor Setar in which the observed values and fitted values for all the distribution considered are plotted on the same graph. It can be seen that none of the cdf plots fits well with the observed cdf plot. In fact, similar plots done for the remaining six locations also show a similar result to the one in Figure 3. This indicates that none of these four circular distributions is a good fit for all of the specified locations for the annual dataset. To verify this outcome, three goodness-of-fit tests were used to calculate the fitness. The results are shown in Tables 2 to 4.

As can be seen from the Tables 2 to 4, the smaller the value of the mean chord length and mean circular distance, the better the specified distribution fits in with the wind direction. The same applies for the Watson-Williams test, the smaller the F value compared to the p -value indicates the better the fit with the wind direction. But as can be observed, both cdf plot and the three goodness-of-fit tests do not show a good fit. This could very well be due to the fact that the annual wind direction data comprise the wind direction for both monsoons, namely the southwest and northeast monsoons. As a result, the annual wind direction data have a mixture of both entirely two different two monsoons, thus hindering the search for the best circular distribution to describe the annual wind direction. The next step of the analysis to examine the data with respect to the two monsoons.

Table 1. Parameter estimates for all seven locations.

Location	Parameter			
	<i>n</i>	Mean angle (in radian)	Concentration parameter	Mean resultant length
Alor Setar	357	4.7922	0.782	0.364
Langkawi	363	4.7817	0.574	0.276
Melaka	365	3.9294	0.602	0.288
Senai	363	0.4007	0.052	0.026
Kota Bharu	365	1.5290	0.565	0.272
Kuala Terengganu	310	0.9730	1.217	0.522
Kuantan	365	1.8030	0.345	0.170

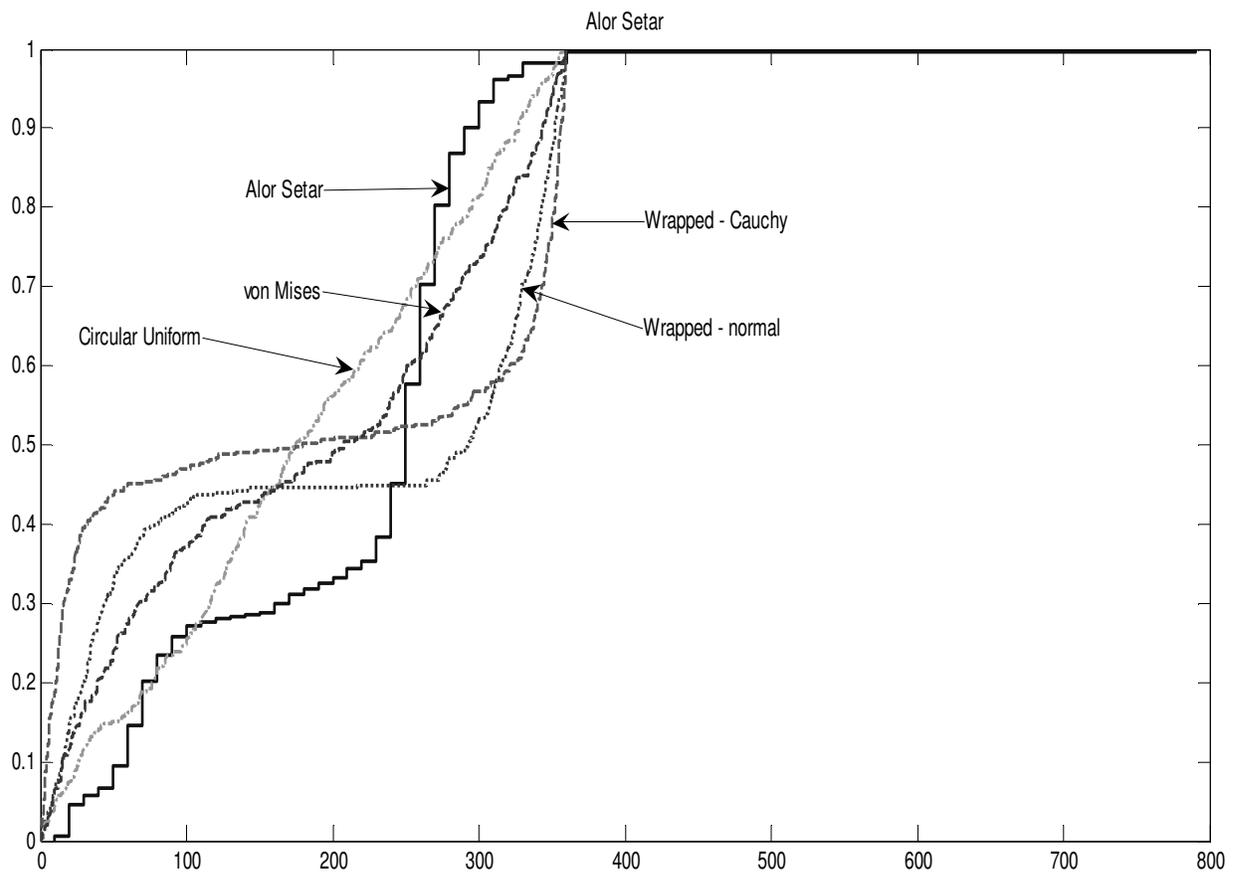


Figure 3. The cdf plot of Alor Setar.

Table 2. The mean chord length for annual wind direction.

Location	Circular distribution			
	Circular uniform	Wrapped Cauchy	Wrapped normal	von Mises
Alor Setar	0.6412	0.6565	0.6196	0.6366
Langkawi	0.6243	0.6304	0.6172	0.6586
Melaka	0.6238	0.6922	0.6833	0.6546
Senai	0.6249	0.6541	0.6266	0.6474
Kota Bharu	0.6627	0.6422	0.6418	0.6487
Kuala Terengganu	0.6459	0.5551	0.5792	0.5155
Kuantan	0.6157	0.6409	0.6382	0.6563

Table 3. The mean circular distance for annual wind direction.

Location	Circular distribution			
	Circular uniform	Wrapped Cauchy	Wrapped normal	von Mises
Alor Setar	0.5018	0.4888	0.4648	0.5082
Langkawi	0.4824	0.4797	0.4761	0.5278
Melaka	0.4793	0.5623	0.5534	0.5253
Senai	0.4924	0.5223	0.4902	0.5108
Kota Bharu	0.5276	0.5018	0.5005	0.5184
Kuala Terengganu	0.5076	0.4014	0.4338	0.3710
Kuantan	0.4724	0.5043	0.5039	0.5250

Table 4. The Watson-Williams test for annual wind direction.

Location	Circular distribution							
	Circular uniform		Wrapped normal		Wrapped Cauchy		von Mises	
	Statistical test							
	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>
Alor Setar	0.000	24.09	0.00	164.64	0.00	163.86	0.00	107.60
Langkawi	0.000	81.12	0.00	162.60	0.00	129.09	0.00	122.14
Melaka	0.721	0.13	0.00	276.24	0.00	299.70	0.00	291.86
Senai	0.807	0.06	0.00	62.23	0.00	86.56	0.00	29.85
Kota Bharu	0.000	41.41	0.00	133.49	0.00	158.24	0.00	129.31
Kuala Terengganu	0.012	6.40	0.00	95.89	0.00	100.50	0.00	46.25
Kuantan	0.054	3.17	0.00	91.31	0.00	97.76	0.00	79.31

Table 5. Parameter estimates for northeast (*N.e*) and southwest (*S.w*) monsoons.

Location	Parameter							
	<i>n</i>		Mean angle (radian)		Concentration parameter		Mean resultant length	
	Monsoon							
	<i>N.e</i>	<i>S.w</i>	<i>N.e</i>	<i>S.w</i>	<i>N.e</i>	<i>S.w</i>	<i>N.e</i>	<i>S.w</i>
Alor Setar	143	152	0.2968	4.5406	0.434	2.853	0.212	0.799
Langkawi	150	152	0.4305	4.4395	0.741	2.001	0.348	0.699
Melaka	151	153	0.2699	3.7691	0.598	2.225	0.287	0.732
Senai	151	151	0.1831	3.3210	1.233	1.184	0.527	0.511
Kota Bharu	151	153	1.4610	4.0290	1.549	0.232	0.610	0.115
Kuala Terengganu	151	98	0.8270	1.5160	2.172	0.700	0.725	0.331
Kuantan	151	153	0.9010	3.4370	1.413	0.903	0.576	0.412

FITTING MONSOON WIND DIRECTION WITH CIRCULAR PROBABILITY DISTRIBUTION

Parameter estimates

As mentioned earlier, since the results obtained from the cdf plot and the goodness of fit do not provide consistent results, the annual data were then categorised according

to the two major monsoons in Malaysia namely, the southwest monsoon and the northeast monsoon (the southwest monsoon hits the west coast of Malaysia from November till March and the northeast monsoon the east coast of Malaysia from May till September every year). The same procedure applied for studying the annual data was repeated for the monsoon-based data. Table 5 shows the parameters estimate for both monsoons

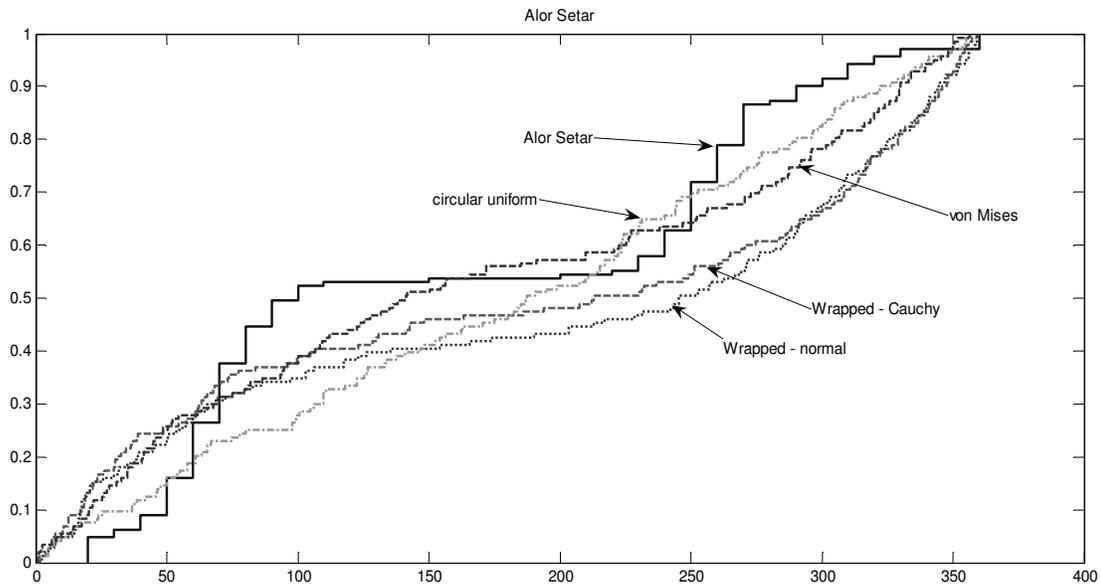


Figure 4. The cdf plot of northeast monsoon for Alor Setar.

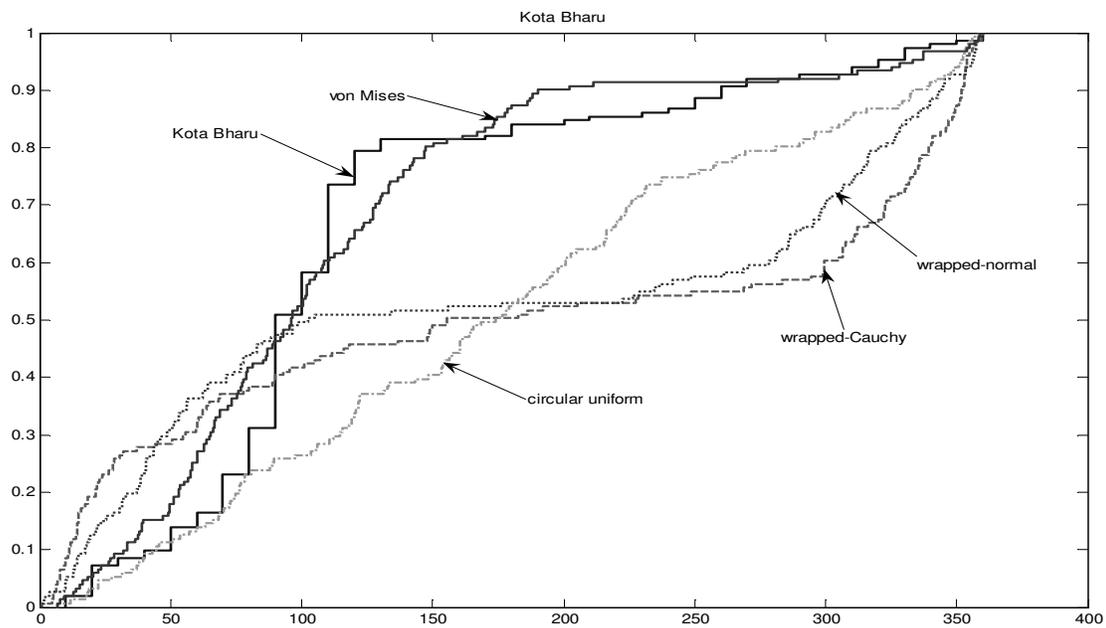


Figure 5. The cdf plot of northeast monsoon for Kota Bharu.

according to the locations.

As can be seen from Table 5, locations that faced a particular monsoon have larger concentration parameters compared to the ones that did not face the particular monsoon. For instance Alor Setar, Langkawi and Melaka have a larger concentration parameter during the southwest monsoon compared to Kota Bharu, Kuala Terengganu and Kuantan which have larger concentration during the northeast monsoon. Larger concentration parameter suggests that the data is closer to each other.

Graphical and numerical measures for monsoon wind direction

After the parameters had been obtained, new data were generated and compared with the previous data. Following are the results for both the cdf plot (for two locations) and the goodness-of-fit tests (for all the locations). As an illustration, the cdf plots for two locations namely Alor Setar and Kota Bharu is as given on Figures 4 to 7. We noted that a number of findings can be drawn. Fitted with probability circular distributions by

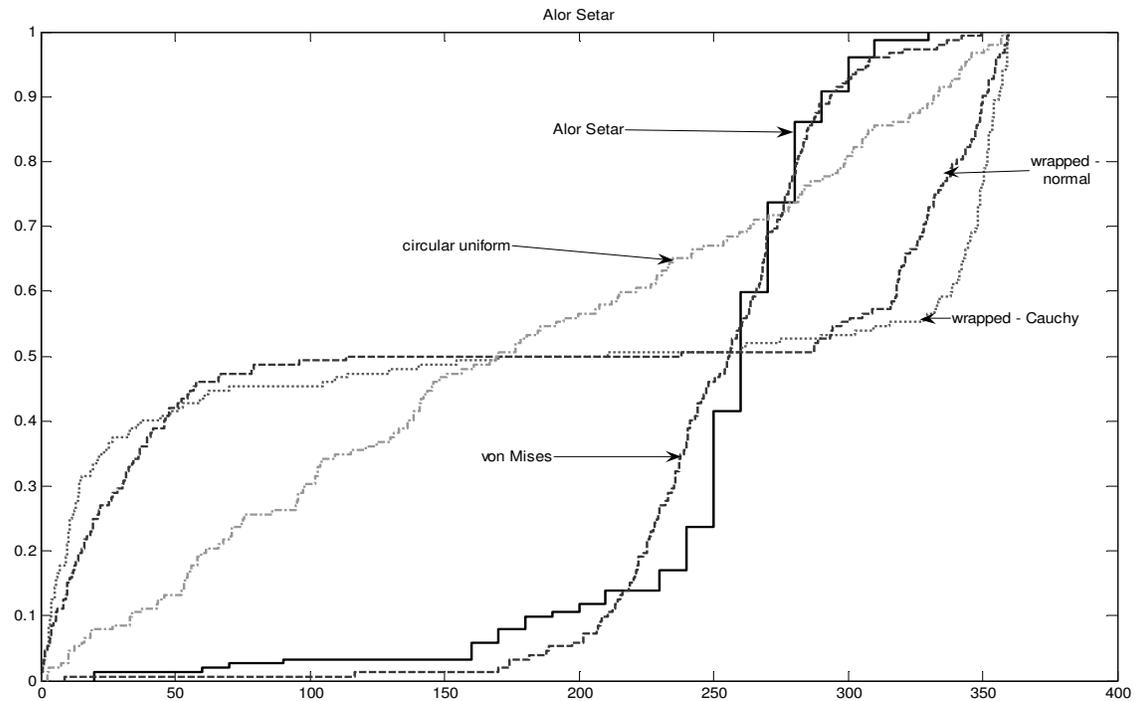


Figure 6. The cdf plot of southwest monsoon for Alor Setar.

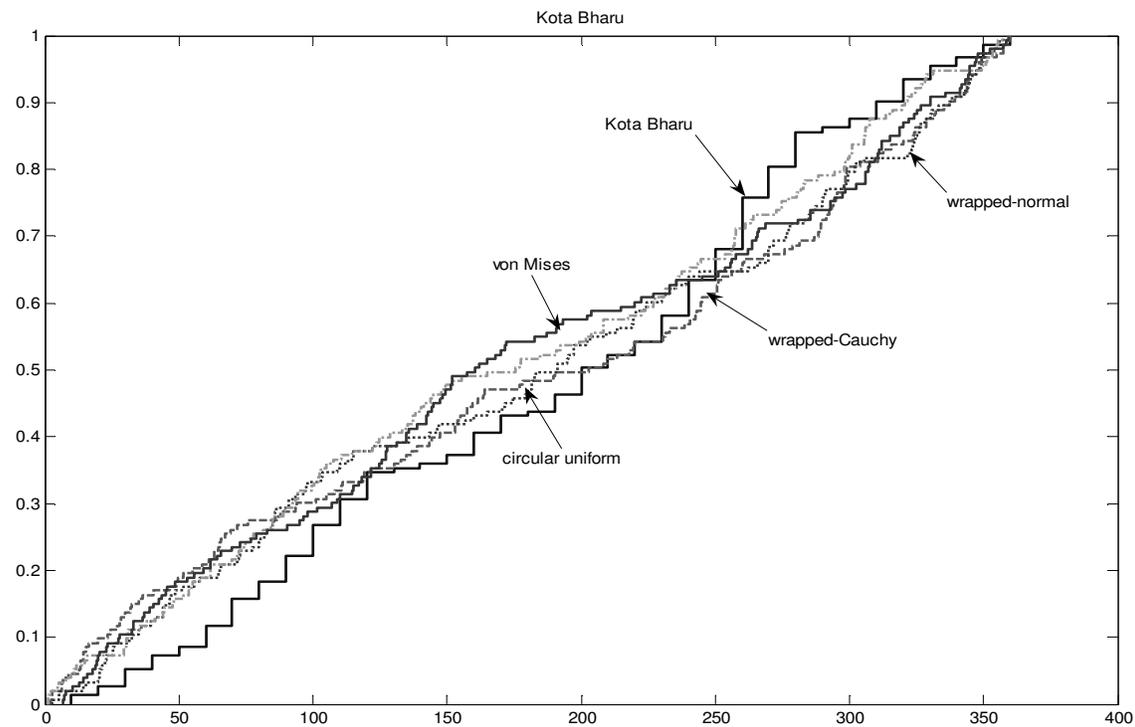


Figure 7. The cdf plot of southwest monsoon for Kota Bharu.

monsoons; locations on the east coast like Kota Bharu seems to fit in with von Mises during the northeast monsoon but it is not the case during the southwest monsoon (Figures 5 and 7). A similar pattern can be

detected during the southwest monsoon where von Mises seems to fit in with Alor Setar (Figure 6) but not for Kota Bharu. To support these findings, the three goodness-of-fit tests results are shown in Tables 6 to 9.

Table 6. Mean chord length and Mean circular distance of northeast (*N.e*) monsoon.

Location	Circular distribution							
	Circular uniform		Wrapped Cauchy		Wrapped Normal		von Mises	
	Performance indicator							
	Mean chord length	Mean circular distance	Mean chord length	Mean circular distance	Mean chord length	Mean circular distance	Mean chord length	Mean circular distance
Alor Setar	0.6241	0.4797	0.6011	0.4488	0.5870	0.4437	0.6346	0.4937
Langkawi	0.6275	0.4823	0.5514	0.3857	0.5434	0.3839	0.5841	0.4353
Melaka	0.6379	0.4950	0.5531	0.4062	0.5870	0.4344	0.6049	0.4610
Senai	0.6665	0.5265	0.5425	0.4011	0.5293	0.3701	0.5007	0.3454
Kota Bharu	0.6370	0.4960	0.6448	0.4993	0.5919	0.4510	0.4567	0.2944
Kuala Terengganu	0.6800	0.5510	0.4953	0.3208	0.4663	0.2952	0.3861	0.2203
Kuantan	0.6305	0.4926	0.5674	0.4081	0.5991	0.4482	0.4898	0.3216

Table 7. Watson-Williams test for northeast (*N.e*) monsoon.

Location	Circular distribution							
	Circular uniform	Wrapped-normal	Wrapped-Cauchy	Von Mises	Circular distribution	Circular uniform	Wrapped-normal	Wrapped-Cauchy
	Statistical test							
	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>
Alor Setar	0.000	55.18	0.000	509.50	0.000	550.59	0.172	1.87
Langkawi	0.613	0.26	0.005	8.14	0.051	3.84	0.414	0.67
Melaka	0.098	2.75	0.005	8.14	0.009	6.87	0.972	0.00
Senai	0.002	9.88	0.418	0.66	0.393	0.73	0.711	0.14
Kota Bharu	0.001	10.45	0.000	124.63	0.000	130.66	0.976	0.00
Kuala Terengganu	0.000	103.24	0.000	51.14	0.000	85.10	0.426	0.63
Kuantan	0.528	0.40	0.000	57.50	0.000	44.05	0.971	0.00

Table 8. Mean chord length and mean circular distance for southwest (*S.w*) monsoon

Location	Circular distribution							
	Circular uniform		Wrapped Cauchy		Wrapped Normal		von Mises	
	Performance indicator							
	Mean chord length	Mean circular distance	Mean chord length	Mean circular distance	Mean chord length	Mean circular distance	Mean chord length	Mean circular distance
Alor Setar	0.6465	0.5095	0.7260	0.5697	0.7088	0.5557	0.3237	0.1726
Langkawi	0.5843	0.4446	0.6894	0.5434	0.6935	0.5589	0.4334	0.2647
Melaka	0.6400	0.5176	0.8150	0.7185	0.8268	0.7326	0.4108	0.2550
Senai	0.6427	0.5160	0.7505	0.6425	0.7353	0.6289	0.5439	0.3959
Kota Bharu	0.6264	0.4884	0.6062	0.4705	0.6448	0.5066	0.6328	0.4830
Kuala Terengganu	0.6329	0.4860	0.6267	0.4757	0.6073	0.4615	0.6366	0.4948
Kuantan	0.6286	0.4930	0.7078	0.5781	0.7174	0.5959	0.6946	0.5722

The goodness-of-fit tests results are consistent with the cdf plot. Von Mises fits well with the locations along the

east coast of Malaysia during the northeast monsoon and locations along the west coast during the southwest

Table 9. Watson-Williams test for southwest (*S.w*) monsoon

Location	Circular distribution							
	Circular uniform		Wrapped-normal		Wrapped-Cauchy		Von Mises	
	Statistical test							
	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>	<i>p</i>	<i>F</i>
Alor Setar	0.000	12.68	0.433	0.62	0.003	9.31	0.650	0.21
Langkawi	0.000	27.96	0.000	764.26	0.000	673.42	0.867	0.03
Melaka	0.338	0.92	0.000	286.40	0.000	266.00	0.978	0.00
Senai	0.138	2.21	0.000	418.32	0.000	387.48	0.465	0.54
Kota Bharu	0.000	62.34	0.000	48.53	0.000	61.70	0.000	79.68
Kuala Terengganu	0.000	20.77	0.000	42.04	0.000	79.68	0.007	7.53
Kuantan	0.000	22.78	0.000	300.49	0.000	204.62	0.000	235.02

monsoon. Although for Kuala Terengganu, the Watson-Williams test does not show an acceptable *p*-value, this is understandable since the data for Kuala Terengganu contains a large amount of missing values during the northeast monsoon compared to the other locations. Where Senai is concerned, when it was tested using mean chord length and mean circular distance, for both monsoons, it shows a good fit though the values for both performance indicators are larger than for other locations when fitted with von Mises. However, when the Watson-Williams test was applied, Senai do not fit well with any of the distribution considered for both monsoons. This is because of Senai's particular location. It is situated at the southern part of Malaysia where it is exposed to both monsoons. Two approaches of performance indicators, the mean chord length and the mean circular distance are consistent with the graphical and Watson-Williams test which mean that the two performance indicators can be used to test the goodness-of-fit for a comparison between two sets of circular data.

Conclusion

The annual wind direction recorded at maximum wind speed in 2005 did not fit in with any of the four circular distributions considered for all the seven locations. This is because the data gathered were spread between the two major monsoons in Malaysia. When the data were divided according to the two monsoons which are the northeast and the southwest monsoons, von Mises distribution seems to be the best circular distribution that fits well with the data and depending on the location and the monsoon. In conclusion, it can be said that von Mises distribution is the best circular distribution to describe the wind direction recorded at maximum wind speed and it depends on the location and the monsoon.

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