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Analytical and numerical study of a pulsatile flow in presence of a magnetic field

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This paper deals with the effects of magnetic field on heat transfer in a pulsatile flow. A mathematical model is developed to investigate the impact of magnetic field on the velocity and the temperature distributions between two concentric ducts. Finite differences method is used in order to solve the dimensionless governing equations, and implicit schemes for velocity and temperature are obtained. The effects of magnetic field on the velocity are represented by the Hartmann number. It is found that the increase of magnetic field leads to eliminate the annular effect of the pulsatile flow. It is also found that the velocity can be controlled by the external magnetic field which leads to affect the temperature profiles and so the heat transfer that could be improved or reduced by mastering the magnetic field.

Key words: Pulsatile flow, magnetohydrodynamics (MHD), concentric ducts, finite differences, blood flow.

INTRODUCTION

The study of pulsatile flow has been the subject of numerous investigations. The first work dates back to 1929 when Richardson and Tyler revealed by experimental measurements the existence of one of the main features of the oscillating flow which is called the annular effect. This effect is characterised by the presence of velocity maximums near the wall of the pipe. Later analyses of Womorsley (1955) and Uchida (1956) confirmed this result by analysing the sinusoidal motion of an incompressible fluid oscillating in a horizontal pipe. Atabek and Chang (1961) studied the unsteady flow in cylindrical pipe; they have developed an analytical solution for the velocity profile by assuming that the flow is established with far inputs.

Yakhot and Grinberg (2003) investigated the influence of the pressure gradient frequency on the velocity amplitude and the phase difference between the pressure gradient and the axial velocity. This phase difference varies from 0° for the slow frequencies to 90° for the high frequencies. Kakac and Yenner (1973) obtained an exact solution in the case of a forced flow between two parallel plates. Suces (1981) numerically investigated the response functions of the wall temperature and the average temperature between a laminar fluid flow and a flat plate by using a finite difference method. Zhao (1995) performed numerical and experimental studies on a

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laminar air flow oscillating in a cylindrical pipe, heated by a uniform heat flux. From temperatures measured on several positions and at the inner wall of the heater, they obtained a correlation of average Nusselt number. Majdalani (2002) determined the exact solution of the Navier-Stokes equations governing the pulsatile flow in a cylindrical pipe where the pressure gradient was replaced by a sum of pulses expressed in terms of Fourier coefficients.

The application of magnetic field to a moving and electrically conducting liquid induces both electric and magnetic fields. A body force known as the Lorentz force is produced as a result of the interaction between the induced magnetic and electric fields. This force tends to oppose the movement of the liquid which leads to decrease the flow rate. Agrawal and Anwaruddin (1984) proposed a mathematical model for the effect of magnetic field on blood flow through an equally branched channel with flexible walls. They found that the magnetic field can be used as a blood pump in carrying out cardiac operations to cure some arterial diseases such as arteriosclerosis and arterial stenosis. Stud et al. (1977) examined the effect of a moving magnetic field on blood flow, and found that the application of a suitable magnetic field increases the blood flow rate.

In current study, we investigate analytically and numerically the effect of magnetic field on velocity and temperature distributions in case of pulsatile flow across a cylindrical duct. The importance of this study could be so sensible in the knowledge of blood behavior when subjected to a magnetic field and therefore offering best platform to reduce some arterial diseases. Finite differences method with an implicit scheme is used in order to solve the dimensionless governing equations. Velocity and temperature profiles are presented for different Womersley and Hartmann numbers.

MATHEMATICAL FORMULATIONS

Physical problem

The dynamic and thermal behaviors of a viscous and electrically conducting fluid flow between two cylindrical ducts, is presented in Figure 1. The fluid flow is subjected to a constant magnetic field and a pulsatile pressure gradient parallel to the axis.

$$\frac{\partial P}{\partial z} = -A\,cos(\omega\, t)$$

Initially, the internal duct is at a temperature of Tint=400 K, the external duct is supposed adiabatic and the fluid is at a temperature of 300 K and atmospheric pressure.

Governing equations

Simplifying assumptions

1. The fluid is incompressible, viscous and electric conductor,
2. The flow is laminar and axisymmetric,
3. The energy losses due to viscosity are negligible,
4. The magnetic field is constant and radial.

Under the mentioned assumptions, the governing equations are:

Continuity equation:

$$\nabla\vec{U} = 0 \tag{1}$$

Momentum equation:

$$\rho\left(\frac{\partial\vec{U}}{\partial t} + \vec{U}.\nabla\vec{U}\right) = -\nabla P + \mu\nabla^2\vec{U} + \vec{J}\wedge\vec{B} \tag{2}$$

Energy equation:

$$\rho\,C_p\left(\frac{\partial T_f}{\partial t} + \left(\vec{U}.\nabla T_f\right)\right) = \lambda\nabla^2 T_f \tag{3}$$

By introducing the following dimensionless variables:

$$r = \frac{r'}{R_e},\quad z = \frac{z'}{R_e},\quad t = \omega\,t',\quad w = \frac{w'}{\omega\,R_e},\quad T = \frac{T'-T_f}{T_i-T_f},$$
$$P = \frac{p'}{\rho\,R_e^2\,\omega^2},\qquad Re_\omega = \alpha^2 = \frac{R_e^2.\omega}{\nu},\qquad Pr = \frac{\mu.C_f}{k_f},$$
$$Ha = R_e\frac{H}{C}\sqrt{\frac{\sigma}{\nu.\rho}}.$$

The explicit form of the governing equations can be written as follows:

$$\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = \frac{\partial P}{\partial r} + \frac{1}{Re_\omega}\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right] \tag{5}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} + \frac{1}{Re_\omega}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] - \frac{Ha}{Re_\omega}w \tag{6}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{1}{Re_\omega\,Pr}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] \tag{7}$$

Initial conditions:

t=0

$$u(r,z,0) = w(r,z,0) = 0$$
$$p(r,z,0) = T_f(r,z,0) = 0$$

Boundary conditions:

At the external duct:

$$u(1,z,t) = w(1,z,t) = 0$$
$$\frac{\partial T_f}{\partial r}(1,z,t) = 0$$

At the internal duct:

$$u\left(\frac{R_i}{R_e}, z, t\right) = w\left(\frac{R_i}{R_e}, z, t\right) = 0$$
$$T_f\left(\frac{R_i}{R_e}, z, t\right) = 1$$

ANALYTICAL SOLUTION

In order to solve the problem analytically, we assume that the flow is fully developed:

The governing equations of the fluid flow become:

Momentum equation:

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \vartheta\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)\right] - \frac{\sigma H^2}{\rho C^2} w \tag{8}$$

Energy equation:

$$\frac{\partial T(r,z,t)}{\partial t} + w\frac{\partial T}{\partial z} = \frac{k}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] \tag{9}$$

Due to the axis-symmetry of the problem, the study can be reduced to the annular space between the two ducts.
The dimensionless equations (8) and (9) become:

$$\frac{\partial w}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{\alpha^2}\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right] - \frac{Ha^2}{\alpha^2} w \tag{10}$$

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{1}{\alpha^2 . Pr}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}\right] \tag{11}$$

The pressure gradient can be written as:

$$\frac{\partial P}{\partial z} = -\tilde{A}.\mathrm{Cos}(\omega.t) = \mathrm{Re}\left(-\tilde{A}.e^{it}\right) \tag{12}$$

The velocity solution is sought in the form:

$$w(r,t) = Real\left(f(r).e^{i.t}\right) \tag{13}$$

By introducing this solution in equation (10), we obtain the following modified Bessel equation:

$$\frac{\partial^2}{\partial r^2}\bar{f} + \frac{1}{r}\frac{\partial}{\partial r}\bar{f} - [Ha^2 + i.\alpha^2]\bar{f} = -\tilde{A}\,\alpha^2 \tag{14}$$

Where the solution is a combination of Bessel functions I_0 and K_0 of first and second kind respectively:

$$f(r) = C_1 I_0(\eta r) + C_2 K_0(\eta r)$$

Where: $\eta = [Ha^2 + i\,\alpha^2]^{1/2}$

The constants C_1 and C_2 that are determined from the no slip boundary conditions:

$$r = \frac{R_i}{R_e} \quad w = 0 \tag{15}$$

$$r = 1 \quad w = 0 \tag{16}$$

In order to ease the form of equations, we pose:

$$R_i^* = \frac{R_i}{R_e}$$

Therefore:

$$C_1 = \left[\frac{\frac{\tilde{A}^*.\alpha^2}{\eta^2}\left(K_0(\eta R_i^*) - K_0(\eta)\right)}{I_0(\eta R_i^*).K_0(\eta) - K_0(\eta R_i^*).I_0(\eta)}\right] \tag{17}$$

and

$$C_2 = \left[\frac{\frac{\tilde{A}^*.\alpha^2}{\eta^2}\left(I_0(\eta) - I_0(\eta.R_i^*)\right)}{I_0(\eta R_i^*).K_0(\eta) - K_0(\eta R_i^*).I_0(\eta)}\right] \tag{18}$$

Thus, the evolution of the velocity profile can be written as follows:

$$w(r,t) = Real\left\{C_1 I_0(\eta r) + C_2 K_0(\eta r) + \frac{\tilde{A}^*\alpha^2}{\eta^2}\right\} e^{it} \tag{19}$$

In order to solve analytically the equation (11), we assume that the temperature solution profile can be written as:

$$T_f^*(r,z,t) = Real\left(-\gamma^*.z + \gamma^* g(r) e^{it} + 1\right) \tag{20}$$

With: $\gamma^* = \frac{R_e}{L}$

Therefore, we obtain the following differential equation:

$$\frac{\partial^2}{\partial r^2} g + \frac{1}{r}\frac{\partial}{\partial r} g - i.\alpha^2 Pr.g = -\alpha^2.Pr.f \tag{21}$$

By using the following boundary conditions:

$$r = R_i^* \Rightarrow T = 1 \tag{22}$$

$$r = 1 \Rightarrow \frac{\partial T}{\partial r} = 0 \tag{23}$$

The temperature solution profile can be written as:

$$T(r,z,t) = Real\left\{-\gamma^* z + \gamma^*\left[A_1 I_0(\delta r) + A_2 K_0(\delta r) - iC_1 I_0(\eta r) - iC_2 K_0(\eta r) - \frac{i\tilde{A}\alpha^2}{\eta^2}\right] e^{it} + 1\right\} \tag{24}$$

Where: $\delta = \alpha\sqrt{i.Pr}$

$$A_1 = \left[\frac{\left(\psi_1.\delta.K_1(\delta) + \psi_2 K_0(\delta R_i^*)\right)}{I_0(\delta R_i^*).\delta.K_1(\delta) + K_0(\delta.R_i^*).\delta.I_1(\delta)}\right]$$

$$A_2 = \left[\frac{\left(\psi_1.\delta.I_1(\delta) - \psi_2 I_0(\delta R_i^*)\right)}{I_0(\delta R_i^*).\delta.K_1(\delta) + K_0(\delta.R_i^*).\delta.I_1(\delta)}\right]$$

$$\psi_1 = i.\left[C_1 I_0(\eta.R_i^*) + C_2 K_0(\eta.R_i^*) + \frac{\tilde{A}^*.\alpha^2}{\eta^2}\right] + \frac{x}{e^{it}}$$

$$\psi_2 = i.\left[C_1.\eta.I_1(\eta) - C_2.\eta.K_1(\eta)\right]$$

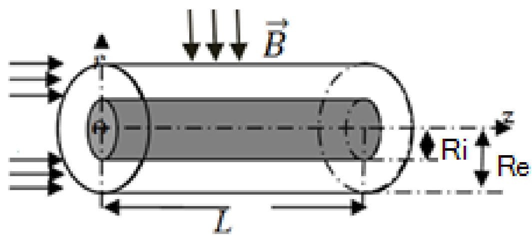

Figure 1. Flow field geometry.

NUMERICAL ANALYSIS

The system of Equation (4) to (7) with the corresponding initial and boundary conditions is solved numerically by finite differences method using implicit scheme. The obtained solution at the fully developed regime will be compared to the analytical solution (19) for the axial velocity and (24) for the temperature.

At each new time, the system of the algebraic equations resulting from the FDM discretization have tri-diagonal matrix form which is solved by TDMA Algorithm.

Because the problem of this study is axisymmetric, the computational domain is reduced to the mesh grid domain illustrated in Figure 2a.

In the vicinity of the ducts, the mesh is refined by replacing the mesh situated near the wall of the internal duct by sub decreasing mesh size following geometric sequences of G (Figure 2b). Where the sum of sub-mesh sizes is equal to the size of a mesh grid. Other meshes that are far from the ducts remain the same size.

RESULTS AND DISCUSSION

Figures 3 to 8 show the analytical and numerical solutions obtained for the velocity profiles for $\frac{R_i}{R_e} = 0.3$ and Ha=0.

It can be seen that there is a large similarity between the analytical and numerical results, which validate the numerical method used in this study. Some differences exist because the analytic solution takes into consideration one-directionality of the problem.

In order to show the effect of magnetic field on the velocity profiles, the following results are shown in Figures 9 to 11 without magnetic field (Ha=0) for $\frac{R_i}{R_e} = 0.3$ and different Womersley (Re_ω).

However, the following results that are shown in Figures 12 to 14 illustrate the effect of magnetic field on the velocity profiles for a wide range of Hartmann numbers (Ha=1, 15, 30).

This results show that the maximum of velocity in a pulsatile flow is situated near the walls of the ducts, which is called the annular effect, revealed by experimentally by Richardson and others and developed analytically by Atabek and Chang (1961). This annular effect increases by the increase of Womersley number Re_ω. The Figure 15 shows the influence of Womersley number on the situation of velocity maximums in the annular space between the two ducts.

In the other hand, it can be seen from Figures 12 to 14 that the magnetic field acts as a retardant against the flow which leads to decrease the flow rate. Furthermore, the magnetic field leads to eliminate the annular effect which is considered as a characteristic of the pulsatile flow.

The flow area also influences on the velocity profiles by eliminating the annular effect as it is shown in (Table 1) where the dimensionless radius of the internal duct varies from 0.3 to 0.8

The results show that the decrease in the flow area from 0.7 to 0.5 leads to decrease gradually the annular effect. Nevertheless, the annular effect is almost absent when the flow area is in the vicinity of 0.4 to 0.2.

Figure 16 illustrates the influence of flow area reduction on velocity profile and its impact on the annular effect. The reduction of flow area leads to increase the velocity as it is shown in Figure 16, where the velocity increases from about 0.4 to 1.2 for the same Re_ω, t and Ha. However, it can be seen that the velocity maximums are in the vicinity of the ducts for the flow areas that vary from 0.7 to 0.5 and the reduction in the flow area leads to reduce the annular effect until its disappearance for the flow areas that vary from 0.4 to 0.2 where the velocity maximum is situated in the middle of the annular space.

Figures 17 and 18 shows a comparison between results of the vortex profiles obtained by the present study and those obtained by Majdalani (2008).

It can be seen that the results of vortex profiles obtained by the present study resemble to the results obtained by Majdalani (2008). However, the existence of slight differences is due to the fact that Majdalani (2008) worked on a pulsatile flow in a rectangular duct.

The Figure 19 shows the temperature profiles for a moderate flow regime (Re_ω=10) and without the application of magnetic field (Ha=0). However, the Figure 20 shows the temperature profiles for the same regime but in presence of magnetic field (Ha=15).

At the light of the results presented in Figures 19 and 20, it appears that the application of an external magnetic field has improved the heat transfer between the two ducts. In addition, the velocity can be controlled by managing the magnetic field which means that the heat transfer can be reduced or enhanced depending on the application required.

Conclusion

The effect of magnetic field on heat transfer has been studied analytically and numerically. An exact solution for velocity and temperature distribution across the annular space between two cylinders in case of a pulsatile flow is developed, which is precious for the knowledge of blood behavior when subjected to a magnetic field which will lead to further improve the reduction of some arterial

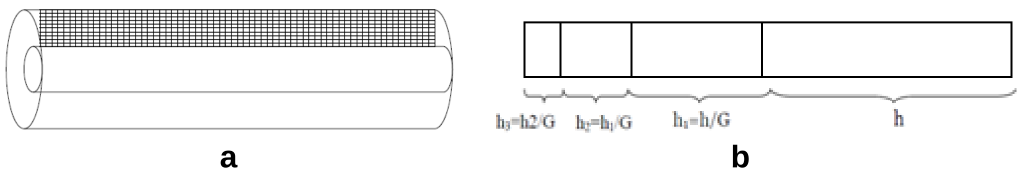

Figure 2. (a) Computational domain, (b) Scheme of the grid independency analysis.

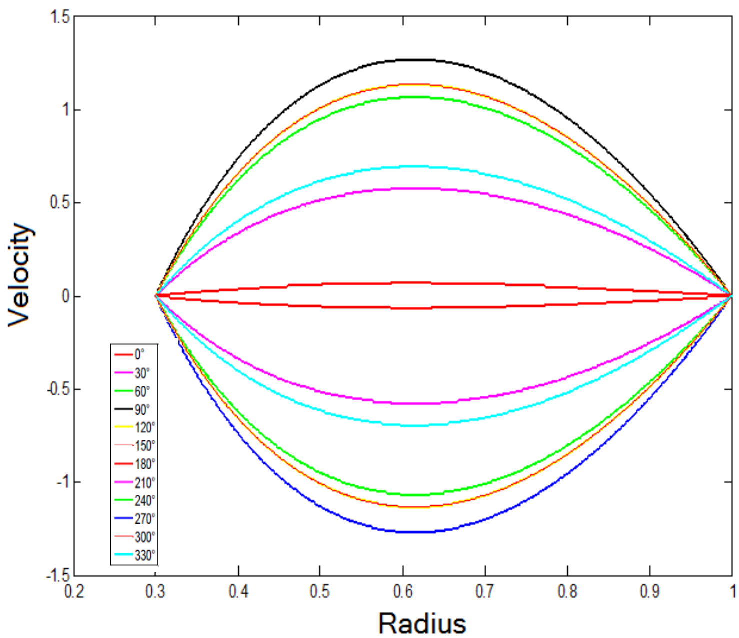

Figure 3. Analytical velocity profiles for Reω=1.

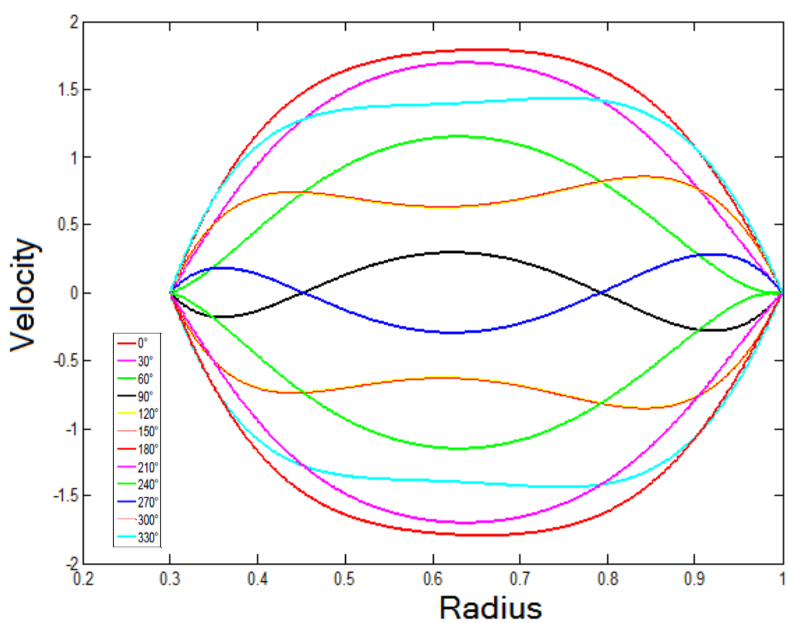

Figure 4. Analytical velocity profiles for Re_{ω}=10.

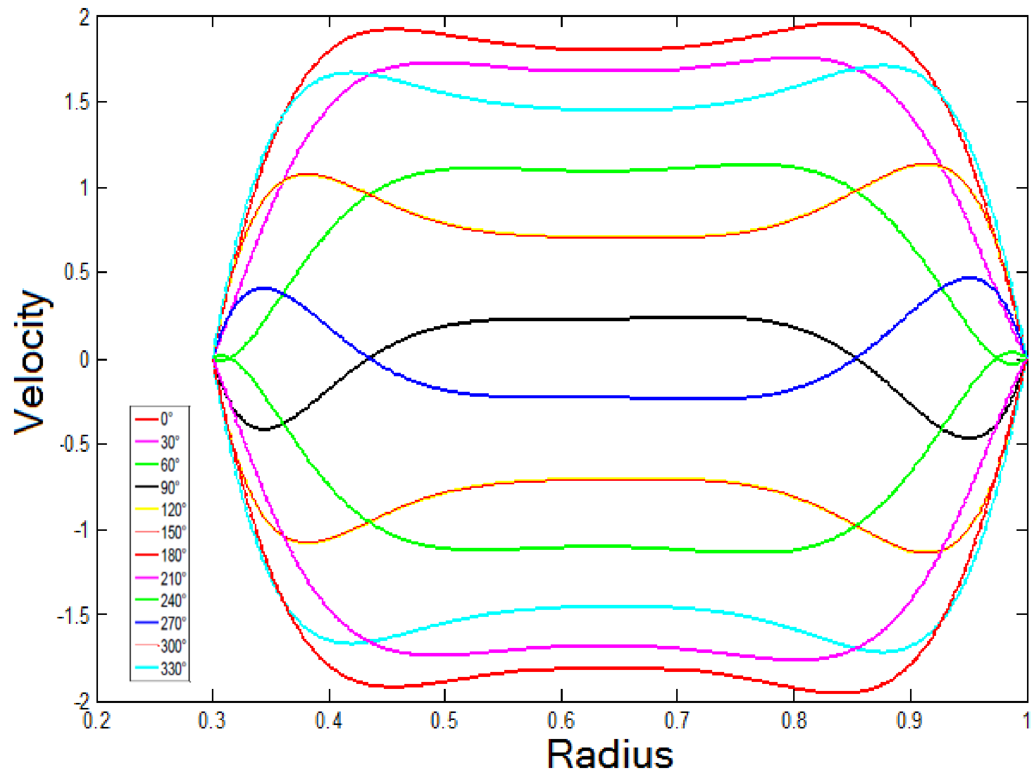

Figure 5. Analytical velocity profiles for Re_ω=30.

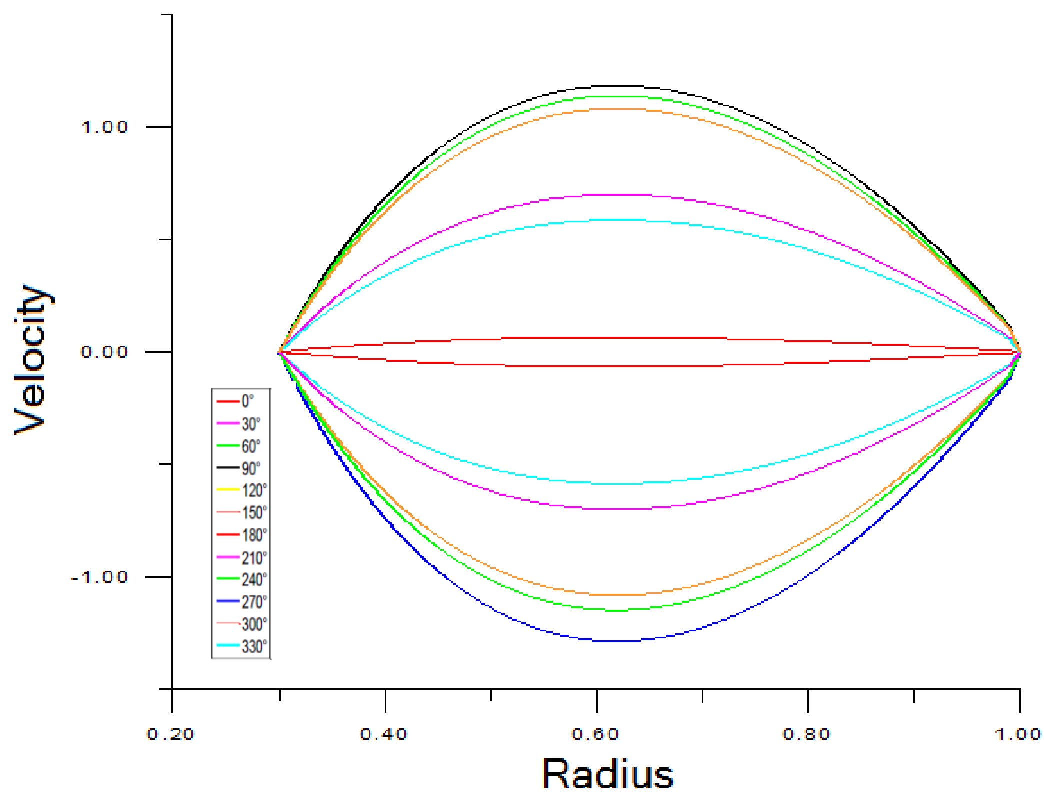

Figure 6. Numerical velocity profiles for Re_ω=1.

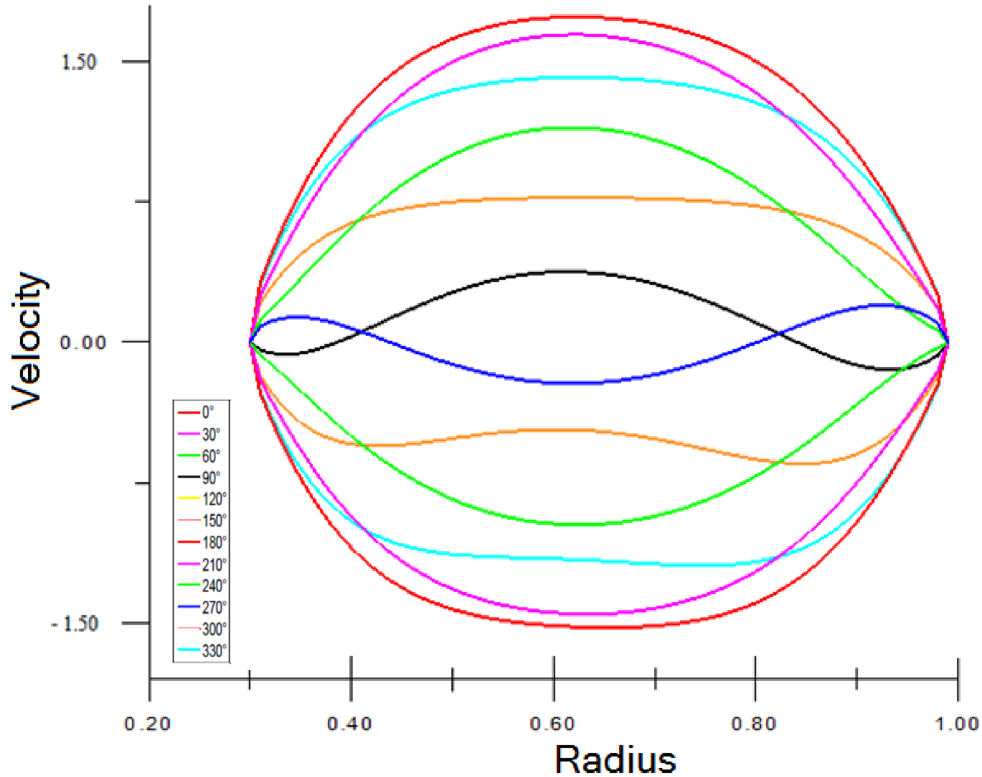

Figure 7. Numerical velocity profiles for Re_{ω}=10.

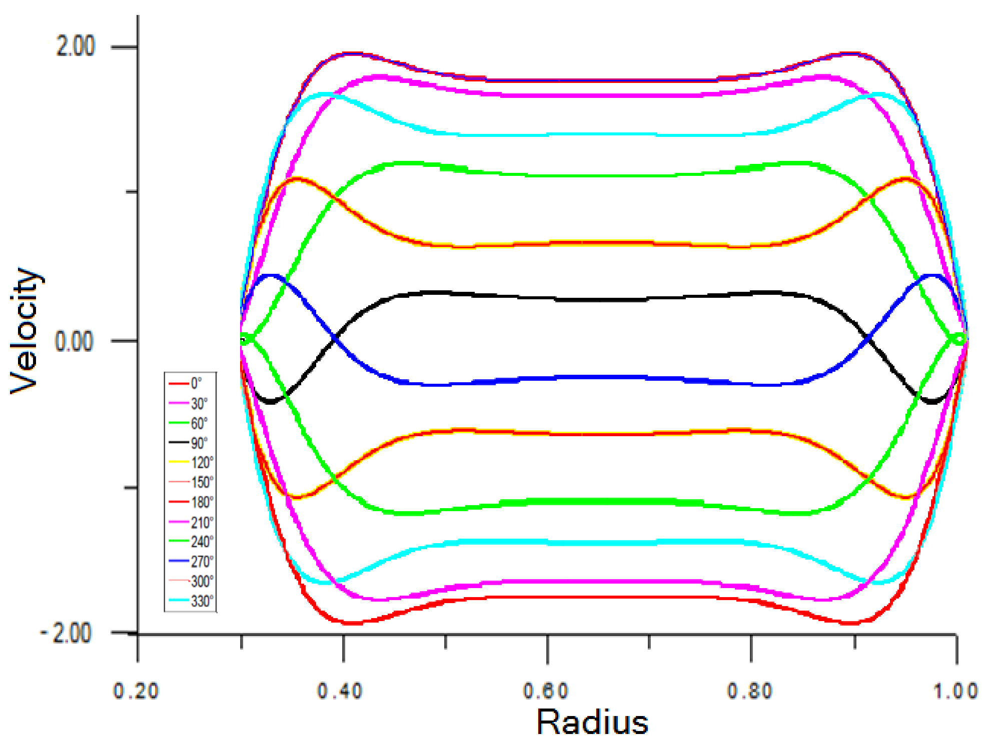

Figure 8. Numerical velocity profiles for Re_{ω}=30.

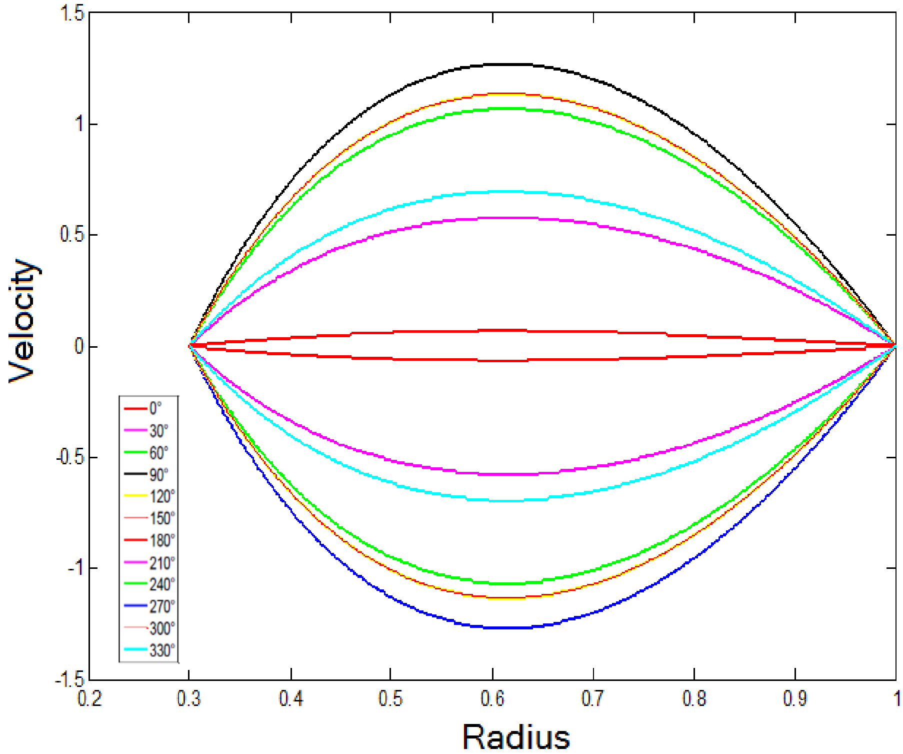

Figure 9. Velocity profiles for Re_ω=1, Ha=0.

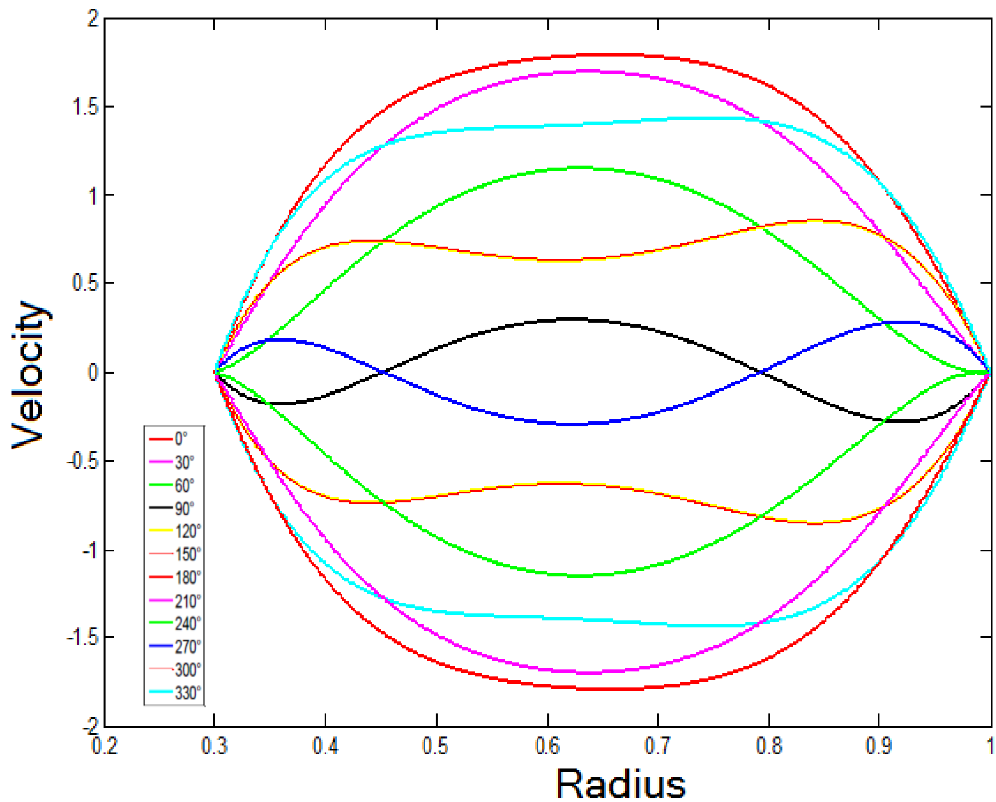

Figure 10. Velocity profiles for Re_ω=10, Ha=0.

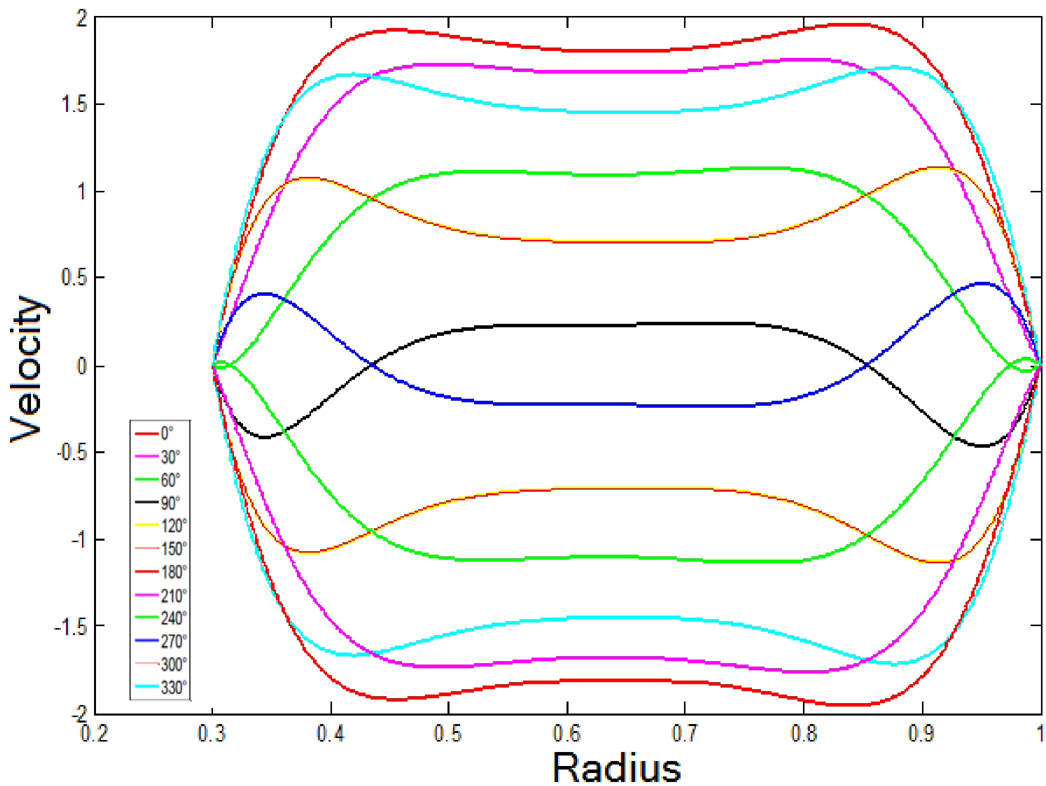

Figure 11. Velocity profiles for Re□=30, Ha=0.

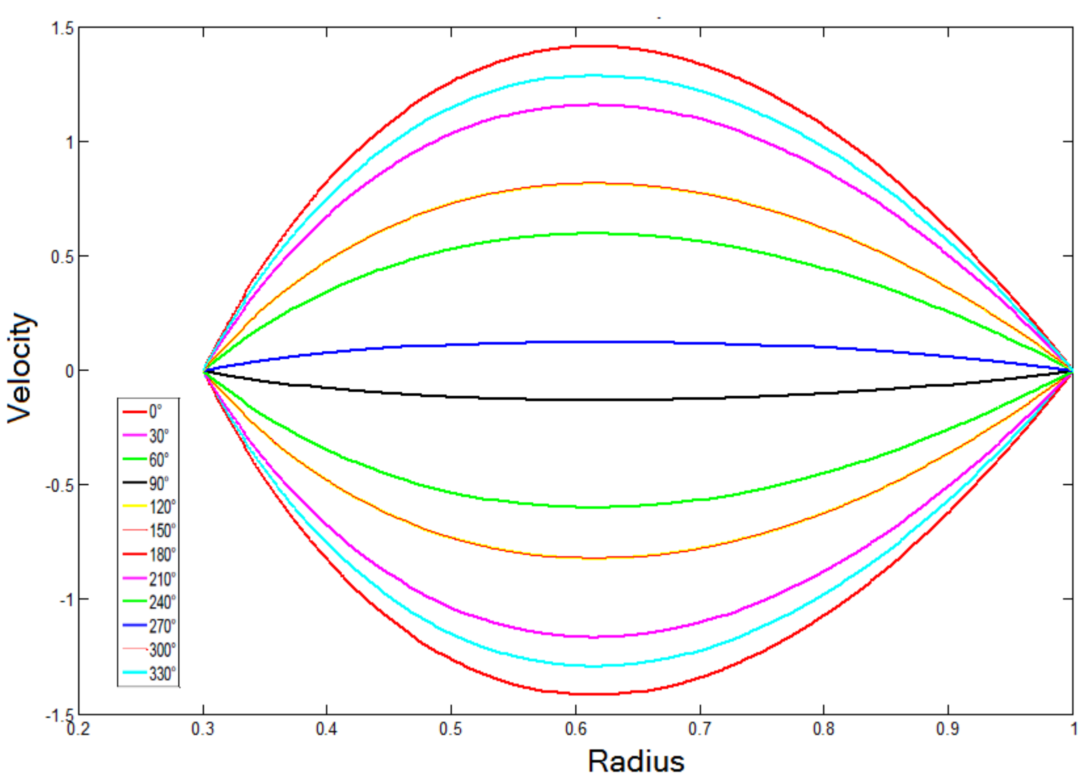

Figure 12. Velocity profiles for Re_{ω}=1, Ha=1.

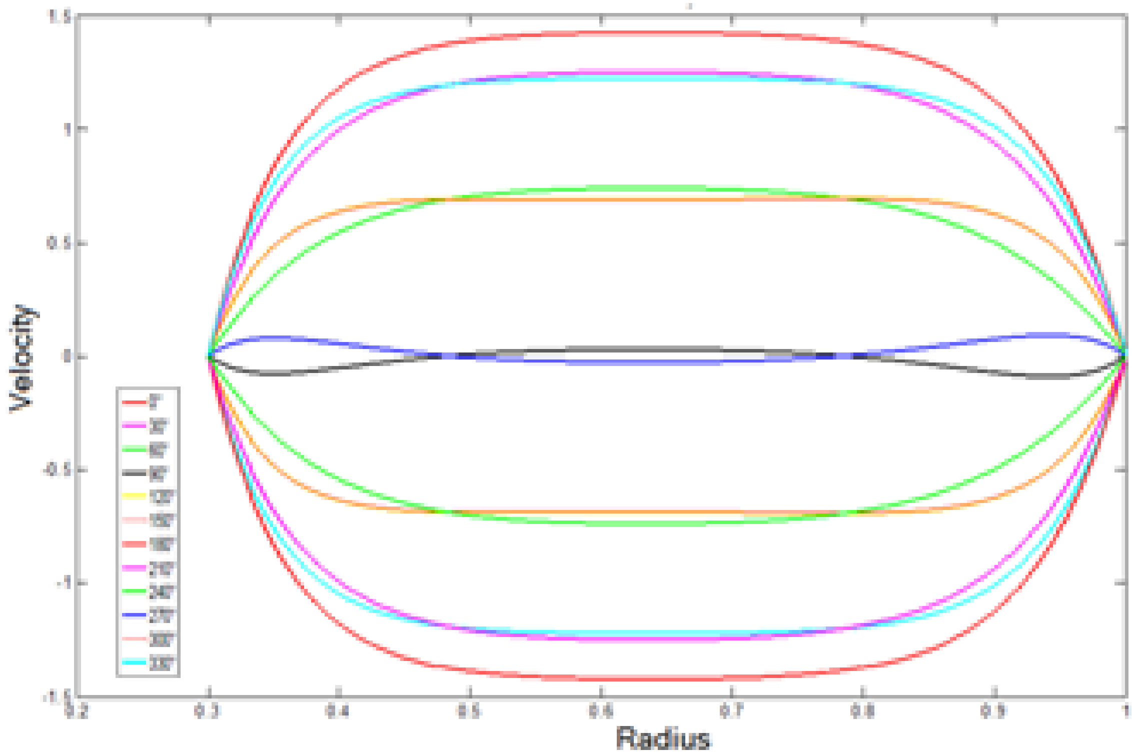

Figure 13. Velocity profiles for Re_{ω}=10, Ha=15.

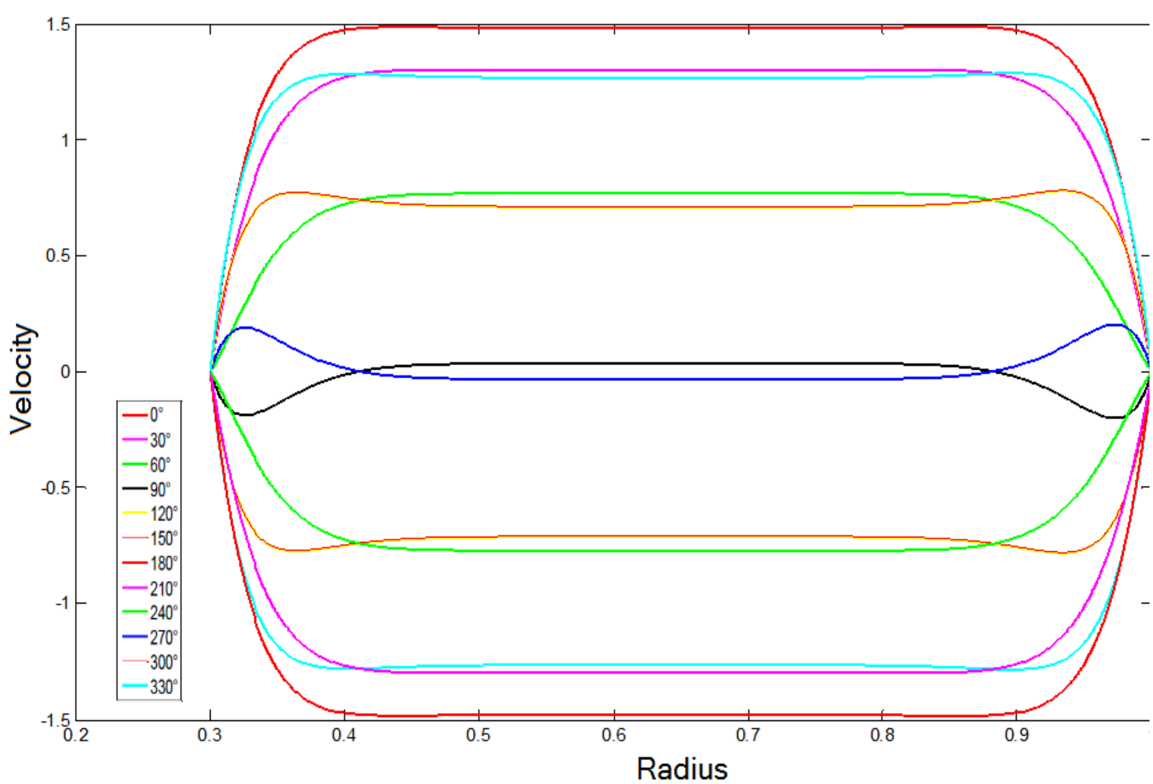

Figure 14. Velocity profiles for Re_{ω}=30, Ha=30.

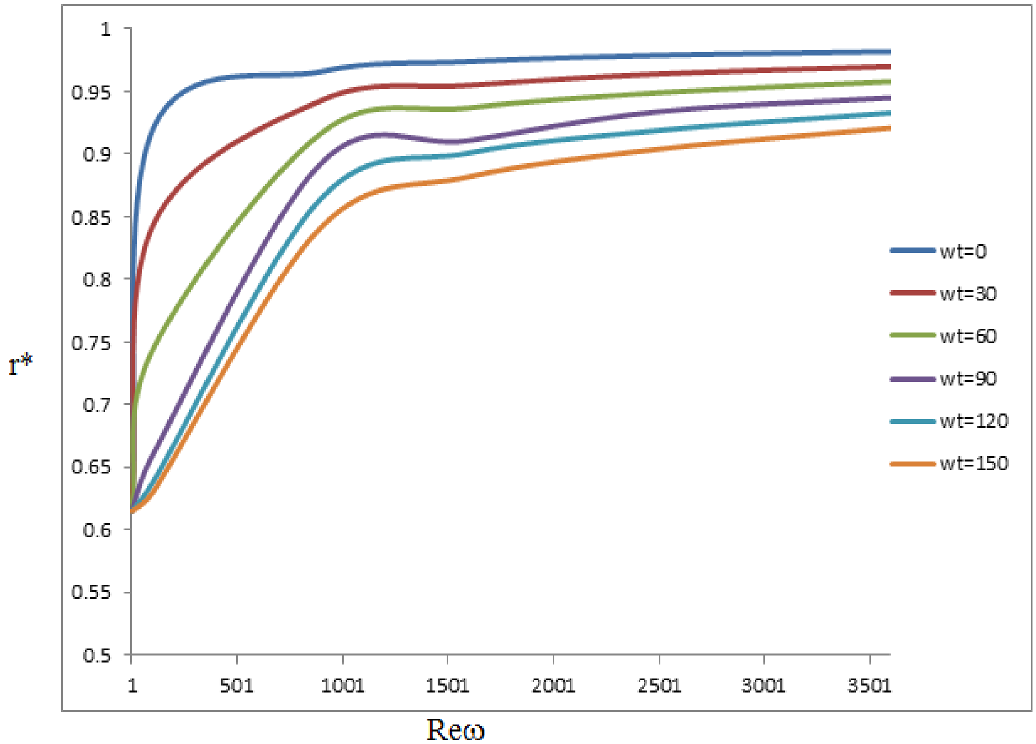

Figure 15. Position of velocity maximums for different Womersley numbers and Ha=0.

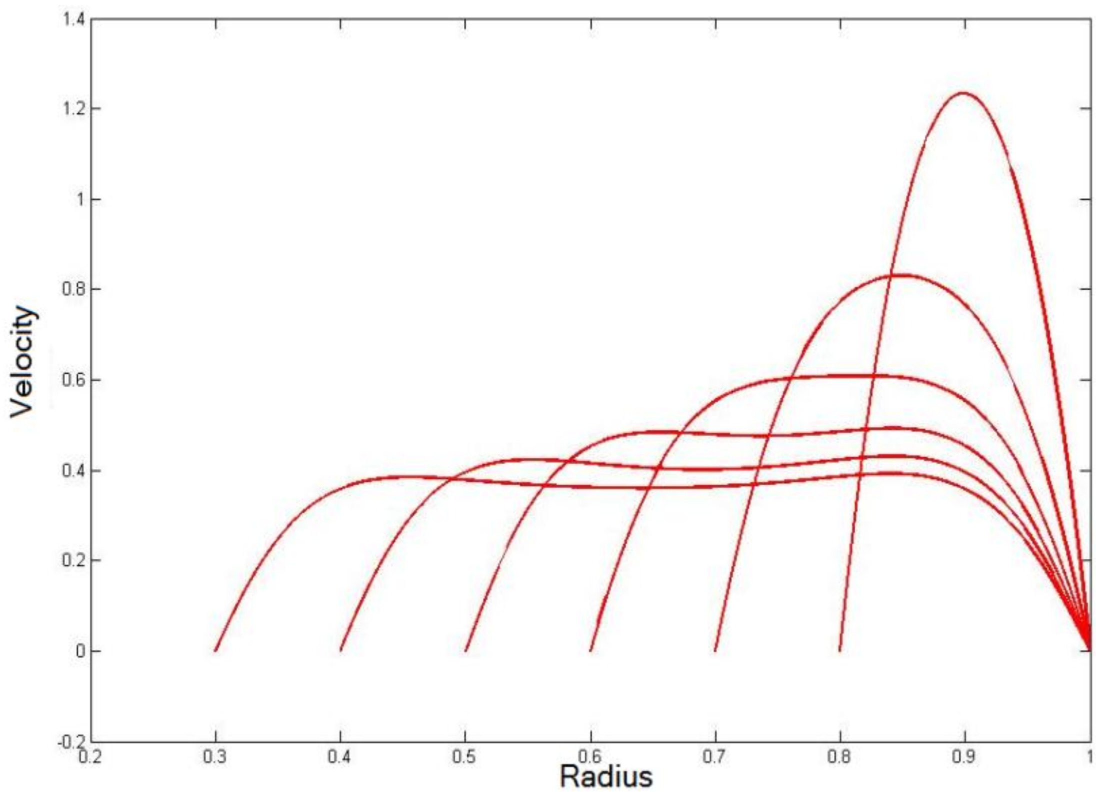

Figure 16. Influence of the flow area (from 0.3 to 0.8) on the velocity profiles for Re_{ω}=20, t=30^{o}, Ha=0.

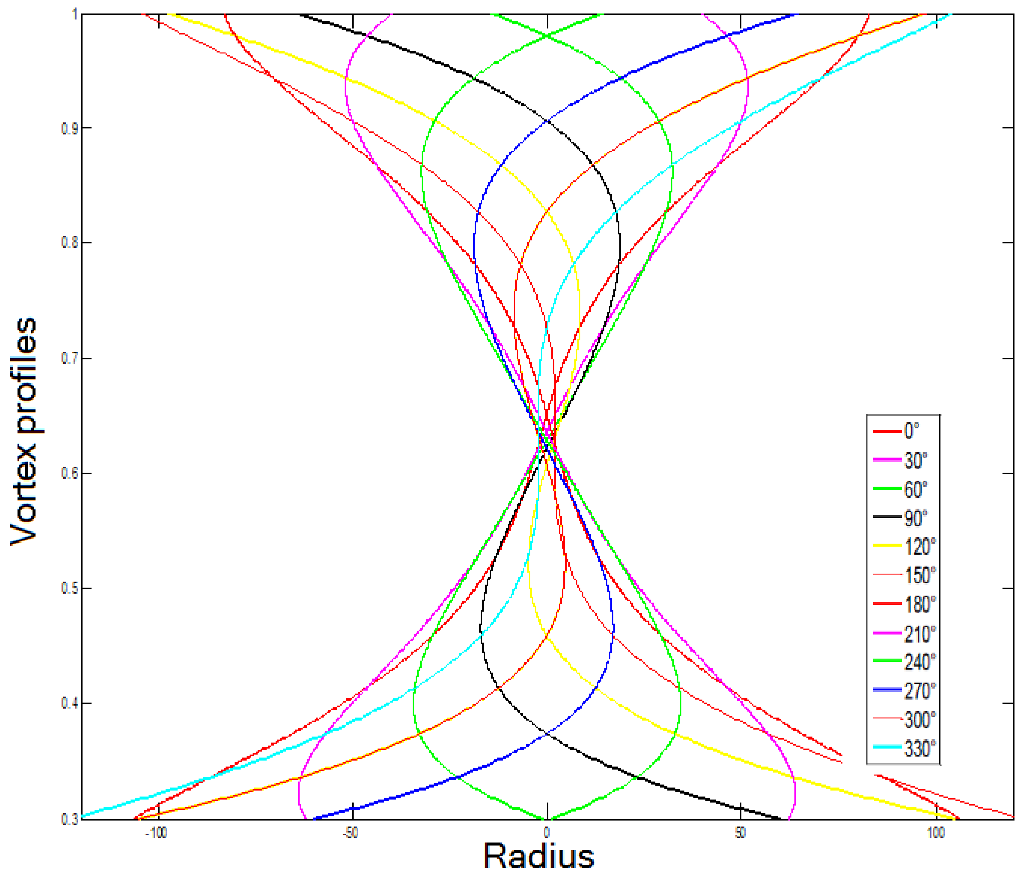

Figure 17. Vortex profiles for Re_ω=10 obtained by the present study.

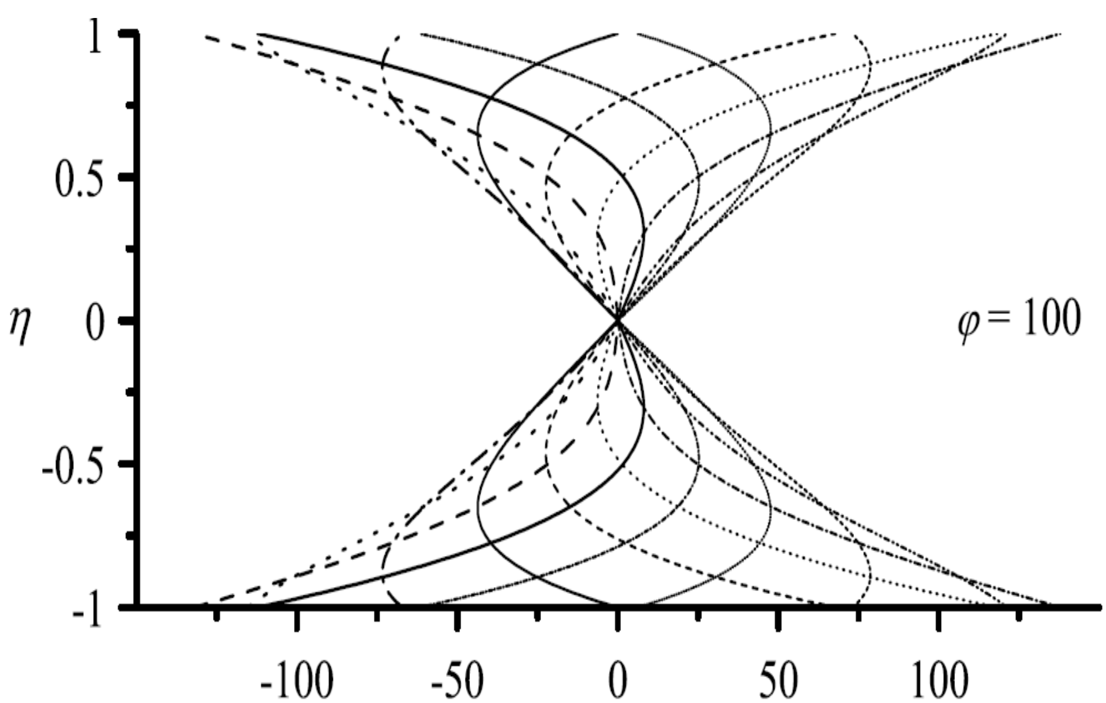

Figure 18. Vortex profiles for Re_ω=10 obtained by Majdalani (2008).

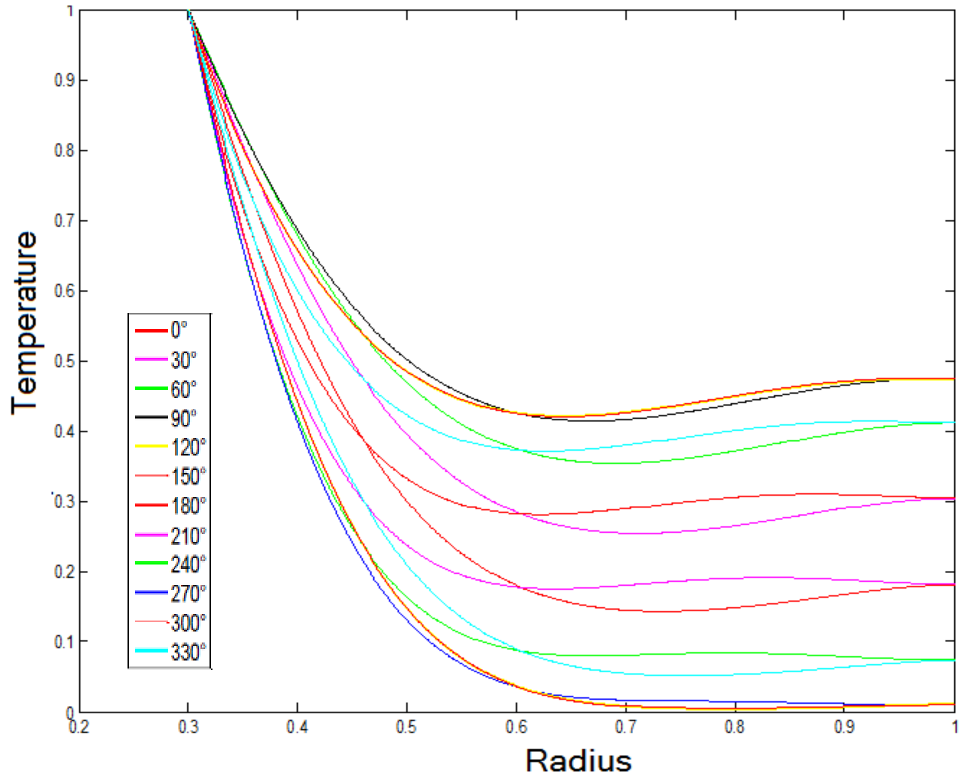

Figure 19. Temperature profiles for Re□=10 and Ha=0.

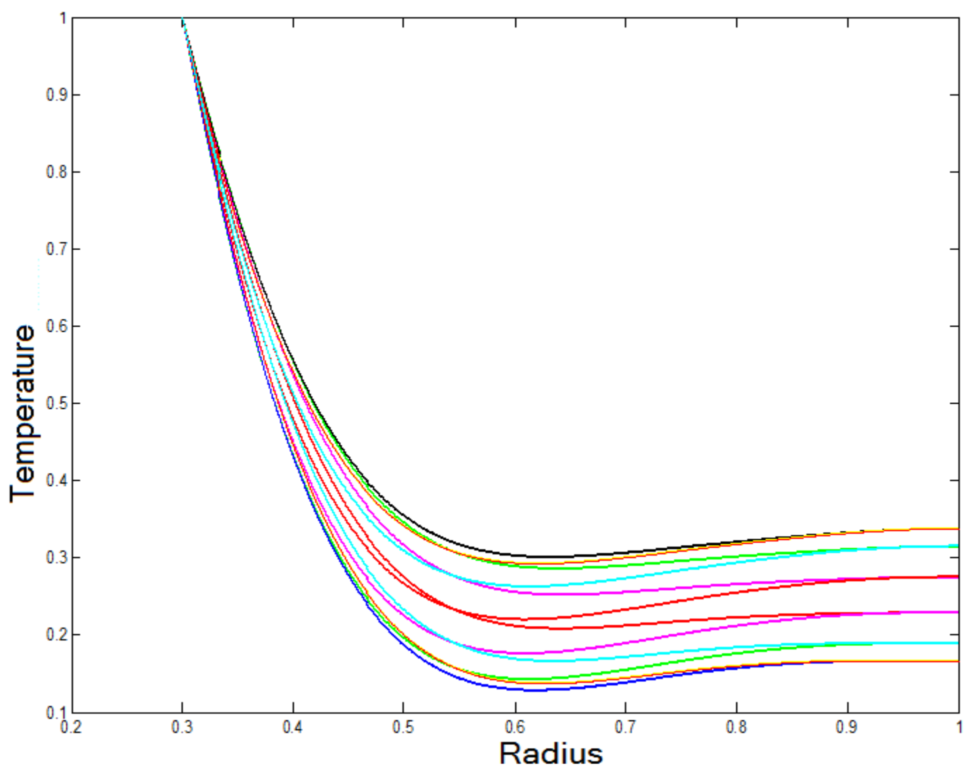

Figure 20. Temperature profiles for Re_{ω}=10 and Ha=15.

Table 1. Position of velocity maximums for Re□=20, t=30°, Ha=0.

Dimensionless radius	Position of velocity maximum	Percentage
0.3	0.455	22.14
0.4	0.551	25.17
0.5	0.663	32.6
0.6	0.791	47.75
0.7	0.849	49.66
0.8	0.900	50.00

diseases. The developed analytical solutions for the velocity and temperature are shown graphically for a wide range of Womersley and Hartmann numbers. The results showed that the constant magnetic field imposed to the pulsatile flow leads to eliminate the annular effect, which is a characteristic of this type of flow. Furthermore, the results showed that the velocity could be controlled by the external magnetic field and also the temperature and so the heat transfer could be reduced or improved by mastering the intensity of the magnetic field.

Conflict of Interest

The authors have not declared any conflict of interest.

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